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Stochastic Processes as Tools for Managing Uncertainty in Real-World Systems

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Abstract- — Systems in reality are characterized by uncertainties. There are uncertainties associated with nature, economics, communications, healthcare delivery, production systems, and social systems, among others, which cannot be modeled using deterministic equations. Stochastic processes facilitate the formulation of mathematical models of systems whose behavior is affected by some random elements. They help in assessing risks, optimizing resource allocations, forecasting future behaviors, and enhancing system robustness. The paper centers on stochastic processes as techniques for dealing with uncertainty in systems. First, the concept of stochastic processes will be defined. Major types of stochastic processes, including Markov chains, Poisson processes, Brownian motion, random walks, and queueing models, will be discussed. Then, applications of stochastic models in various areas, such as financial modeling, engineering, health care, climate studies, operations management, telecommunications, and machine learning, will be explored. Additionally, this paper will examine advantages and disadvantages associated with stochastic modeling. In particular, problems associated with assumptions used in building stochastic models and computational complexities of such models will be analyzed. It will be concluded that stochastic processes represent powerful tools for studying and managing uncertain systems because they enable turning randomness from an issue into a quantifiable phenomenon.

Keywords: stochastic processes, uncertainty, Markov chains, Poisson process, Brownian motion, queuing theory, risk analysis, probabilistic modelling.

I. INTRODUCTION

Modern societies thrive on fast-changing systems bound to other systems that are not only constantly evolving but also interrelated and inherently uncertain. Weather patterns ebb and flow unpredictably, stock markets respond to infinite factors, disease outbreaks occur unevenly and supply chains are thrown off-balance by random shocks. Most of the time, deterministic models, which assume that fixed inputs lead to fixed outputs, do not reflect the variety that we observe. Deterministic methods retain some of their usefulness in describing idealized systems but are usually inadequate to problems in which random events heavily affect the outcome. This challenge has driven the evolving and wide reliance on stochastic processes. A stochastic process is a way to understand the way a system behaves under uncertainty as a string of random variables indexed by a time or space. Instead of attempting to eliminate randomness, stochastic modelling uses it as part of the structure of analysis itself. In this way researchers and practitioners are able to estimate not only the expected outcomes but also variability, extremes, transition probabilities and long-term distributions. We consider the complexity of stochastic processes because of their flexibility. They can be applied to modelling customer arrivals at a service center, packet transmission in a communication network, variations in financial asset prices, machine failures in industrial systems, and epidemics in populations. In all these contexts, uncertainty is not an accessory but rather the heart of the construct. Stochastic tools thus represent a more realistic and often more applicable foundation for forecasting and decision-making. This paper investigates how stochastic processes offer a tool for dealing with uncertainty in real-world systems. It begins by introducing the concept of stochastic modeling, followed by analyzing the processes involved in stochastic modeling and their significance from a mathematical perspective. The next part discusses the areas in which the use of such models has shown to be most beneficial. Lastly, it looks into the restrictions and future prospects of stochastic modeling, especially in the world of data analysis.

II. CONCEPTUAL FOUNDATIONS OF STOCHASTIC PROCESSES

A stochastic process can be mathematically described as a set of random variables \{X_t:t∈T\} that describe a process, whereby the variable t represents time or an index and X_t is the state of the process at the particular index. States can be discrete or continuous, depending on what one desires as well as time can be discrete or continuous. This makes stochastic processes useful in numerous real-world applications. The most crucial thing about the state of the system is that it is always uncertain (though previous states may exist), and a stochastic process is one kind of process characterized by its unpredictability. This is because such processes are probabilistic and hence not deterministic. As such, stochastic processes bridge the two important viewpoints–the process as a function of time (the temporal view) and the probabilistic approach to modeling uncertainty. There is a great difference in terms of predictive approach between these two kinds of modeling. While deterministic models produce the same output from the same initial state, the latter does not. For example, when the factory machine undergoes random wear that leads to random machine breakdowns; then the factory machine cannot be predicted accurately because there is no deterministic way of maintaining it so breakdowns cannot be predicted accurately. On the other hand, the stochastic reliability model has the ability to predict failures and therefore help implement flexible preventive maintenance measures. Another important point here is the difference between stationary and non-stationary. Stationary processes are the statistical processes where the statistical properties mean and variance remain the same as a function of time. However, in the real world, most of the processes are non-stationary as they are influenced by seasonal variations, policy changes, or structural shifts. It is important to know about such difference to build models accordingly. The second type of difference arises in relation to dependence. There are some stochastic processes whose observations are independent while others depend on each other over time. Many times, we see systems having memory whereby the state is created from the past. We can incorporate these dependencies in our stochastic models by means of autocorrelation, transition probability, and diffusion function. This is the reason why stochastic processes as a whole possess greater practical utility in the depiction of uncertainty, temporal patterns, and dependencies into a unified framework. Not only can they be used to forecast theoretical probabilities, but also assist in decision-making in a temporally dependent setting.

2.1 Mathematical Foundations of Stochastic Processes

The usefulness of stochastic processes in managing uncertainty is rooted in their mathematical structure. A stochastic process is typically defined as a family of random variables

$$\{X(t): t \in T\},$$

where T is the index set for time and X(t) represents the state of the system at time t. In case the index set T is discrete, the process will proceed in a stepwise manner, but when T is continuous, then the process will proceed in a continuous manner.

One of the most important quantities in stochastic analysis is the probability distribution of the state of the system. When the state space of the system is discrete, we can express this as

$$P(X_t = j \mid X_0 = i),$$

This provides the probability that the process is in state j at time t, knowing that the process was initiated in state i. This approach helps us represent the uncertainty mathematically and precisely.

The simplest stochastic process to comprehend and analyse is the Markov chain. For a process to be defined as a Markov chain, it must have the Markov property:

$$P(X_{n+1} = j \mid X_n = i, X_{n-1}, \dots, X_0) \\ = P(X_{n+1} = j \mid X_n = i).$$

This implies that the further evolution of the process is dependent only on the present state and not the entire history of the states up until now. The probability of moving from one state to another i to j is referred to as p_ij. Thus, the set of all these probabilities constitute the transition matrix

$$P = \begin{pmatrix} P11 & p12 & p1m \\ p21 & p22 & p2m \\ \dots & \dots & \dots \\ pm1 & pm2 & pmm \end{pmatrix}$$

where each row sums to 1:

$$\sum_{j=1}^{m} p_{ij} = 1.$$

This matrix representation is extremely useful in practical systems. For example, in a reliability model, a machine may move between states such as operational, degraded, and failed. The transition matrix then gives the probability of moving from one condition to another in each time period. If the initial distribution of the system is represented by a row vector π^((0)), then after \(n\) steps the distribution becomes

$$\pi^{(n)} = \pi^{(0)} P^n.$$

The above equation can be employed by analysts for determining the long-run outcomes of uncertain systems and predicting the probability of certain conditions.

A Poisson process is one more stochastic process, often used to describe a situation when a particular event occurs randomly. Suppose that N(t) stands for the number of such events until the moment t. Then a Poisson process with parameter λ> 0 must satisfy

$$P(N(t)=k)=\frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k=0,1,2,$$

The parameter \(\lambda\) represents the average number of events per unit time. The expected value and variance of a Poisson process are both equal to

$$E[N(t)]=\lambda t, \qquad \mathrm{Var}\big(N(t)\big)=\lambda t.$$

This property makes the Poisson process especially useful in applications such as modeling customer arrivals at a service center, phone calls in a telecommunications network, or machine breakdowns in a factory.

In continuous-time modelling, Brownian motion plays a foundational role. A standard Brownian motion \{W(t),t≥0\} satisfies the properties:

1. $(W(0)=0)$,
2. increments are independent,
3. $(W(t)-W(s)\sim N(0,t-s)) for (0\le s<t)$,
4. sample paths are continuous.

Brownian motion is central to stochastic differential equations, which are used to describe systems influenced by both deterministic trends and random shocks. A common form of a stochastic differential equation is

$$dX_t=\mu X_t\,dt+\sigma X_t\,dW_t,$$

where μ is the drift term representing systematic change, σ is the volatility coefficient representing uncertainty, and W_t is Brownian motion. This equation is widely used in finance to model stock prices, but it can also describe population growth, physical diffusion, and noisy engineering systems.

Another mathematically important concept is the expected value of a stochastic process, which gives the average behavior of the system over time. For a process X(t), the mean function is

$$m(t)=E[X(t)],$$

while the covariance function is

$$C(s,t)=E\big[\big(X(s)-m(s)\big)\big(X(t)-m(t)\big)\big].$$

This provides an overview of the distribution of the central tendency and dependencies present in the process. In terms of managing uncertainty, the covariance function plays an important role, in that it gives an indication of how dependent the values are on one another.

Queuing models also depend on mathematical stochastic principles. In a basic M\/M\/1 queue, arrivals follow a Poisson process with rate \(\lambda\), service times are exponentially distributed with rate μ, and there is a single server. One key performance measure is the expected number of customers in the system:

$$L=\frac{\lambda}{\mu-\lambda}, \qquad \text{for } \lambda<\mu.$$

The expected waiting time in the system is

$$W=\frac{1}{\mu-\lambda}.$$

In these equations, one can observe the relationship between the arrival rate and the service rate and how their similarity leads to congestion. These findings can be very useful for hospitals, call centers, transportation terminals, and computer systems since uncertainty should be handled effectively within these areas.

In conclusion, the aforementioned mathematical models illustrate that stochastic processes are not only theoretical concepts but also effective analytical methods. Uncertainty can be described via probabilistic terms, transition rules, rates, expectations, and variations. With the help of transition law, Poisson probability, stochastic differential equation, and queueing models, it is possible to analyze unpredictable behavior.

III. MARKOV CHAINS

One of the most commonly used stochastic processes is the Markov chains. Markov chains are characterized by the fact that the next state depends only on the current one rather than the entire sequence of states up until that point. This particular feature called the Markov property allows analyzing the process without complicating computations too much.

Markov chains help effectively model problems where the state space is discrete. States may include weather, credit rating, machine operating condition, customer behavior type, etc.

Transition probabilities indicate the probability of changing from one state to another and are conveniently collected in a transition matrix. With such representation of the problem, analysts have the ability to study both short-term and equilibrium behaviors.

For instance, in a hospital, patients' conditions may be described as a sequence of stable, critical, recovered, or dead states. Thus, through finding out the transition probabilities for this process, one can make predictions about the demand for hospital beds.

3.1 Poisson Processes

The Poisson process describes the distribution of random events over time when events occur independently and at a relatively stable rate. The process is typically used to describe arrivals, failures, calls, accidents, and other occurrences.

A Poisson process is popular due to its mathematical simplicity. If customers arrive at the bank following a Poisson process, then the probability of the number of arrivals within a certain period of time can be calculated easily. Thus, the Poisson process finds applications in services, traffic analysis, telecommunication, and reliability studies.

Variations, including the non-homogeneous Poisson process, can incorporate changes in the arrival rate. For example, people visiting an emergency room might arrive at a higher rate during certain periods than others, and web traffic on a website could significantly increase during promotion days.

3.2 Brownian Motion

The Wiener process is also referred to as Brownian motion. It is a continuous stochastic process whose main aim is to model random fluctuations over continuous time periods. The stochastic process possesses independent increments, normal distribution, and continuous paths. The Brownian motion process is fundamental in stochastic calculus and is applied widely across the financial, physics, and engineering fields.

Financially, stock price movements are modeled using the Geometric Brownian Motion. The stock prices in the model are characterized by a stochastic differential equation. Brownian motion is applied in physics through modeling random movements of suspended particles within a fluid. In engineering, the stochastic process is used to model noise or uncertainties.

3.3 Random Walks

In a random walk, each step is produced by some form of randomness. This can happen in both discrete and continuous versions. Random walks are strongly connected to Markov processes and Brownian motion and find use in economics, biology, physics, and computer science.

The movement of stock prices is sometimes modeled using a random walk assumption in economic theories. The movement of animals, algorithms for searching, and diffusion across networks are all examples where random walk concepts can be applied. The beauty of random walks is that they can produce highly complicated collective behavior through simple randomness.

3.4 Queuing Processes

Queuing theory employs stochastic processes to consider systems in which entities arrive, wait, receive service, and depart. Examples are bank customers, data packets from routers, patients at hospitals, and jobs in computer systems. Key performance indicators include waiting times, queue lengths, utilization rates, and service capacity requirements. Stochastic queuing models are important as arrival and service times are not predictable consistently. A deterministic timetable may significantly underestimate congestion, while a stochastic queuing model can estimate delays and assist capacity planning. As an example, hospital administrators can use queuing theory to find out how much staff is required in order to keep patient wait times within acceptable limits.

IV. WHY STOCHASTIC PROCESSES MATTER IN MANAGING UNCERTAINTY

The greatest importance of stochastic processes stems from their capability to transform unstructured uncertainty into probabilistic knowledge. In practice, uncertainty emerges due to incomplete data, environmental factors, human activities, or inherent randomness. If the stochastic approach is not applied, the uncertainty can be disregarded, simplified, or seen as a source of error. Instead, stochastic processes consider uncertainty as a property to be quantified and studied.

First, stochastic processes enhance prediction. Instead of offering one forecasted result, they produce a range of potential results in the form of distributions. Such an approach proves especially useful for predicting risky scenarios when extremes, rather than averages, are important. An operator of a power system does not care only about the expected demand but also about the probability of occasional peaks that might disrupt the system's stability.

Secondly, stochastic models enable risk analysis. The use of stochastic models enables decision makers to compute the probabilities of failure, congestion, losses, or any other negative events. For example, financial risk is related to the distribution of returns, rather than expected returns. Meanwhile, for public health emergencies, the probability of outbreak spreading is important, rather than just the number of daily cases.

Thirdly, stochastic models allow optimal decision-making under uncertainty. In operational research and control theory, one can make an optimal decision that seeks to maximize or minimize the expected cost or reward respectively. This is useful when dealing with problems such as inventory management, maintenance, and routing problems.

Fourth, stochastic models aid in scenario analysis and resilience planning. Since stochastic models account for uncertainty explicitly, they can be used to generate shocks and examine their impacts on the systems and infrastructure.

In short, stochastic processes matter because they provide a disciplined method for thinking about uncertainty. They do not eliminate unpredictability, but they make it analyzable and actionable.

V. APPLICATIONS IN REAL-WORLD SYSTEMS

5.1 Finance and Economics

The finance sector is the most important area in the application of stochastic processes. The stock prices, interest rates, foreign exchange rates, and volatilities are all random variables, which need to be modeled to analyze them. Pricing options is achieved by modeling the stochastic property of the stock price. Risk evaluation in finance needs an estimate of the probabilities of losses, Value at Risk, and other extreme cases, which need stochastic processes modeling. Stochastic modeling of insurance entails stochastic arrivals of claims, claim sizes, and reserves adequacy.

Some macroeconomic problems like business cycles and productivity shocks have been analyzed using stochastic processes models. Economists have used stochastic processes to examine labor market dynamics, consumption behavior, and risky corporate activities.

5.2 Engineering and Reliability

Engineered systems are always exposed to disturbance from the outside world, system failures, and variable loading. Stochastic models help engineers design a reliable, safe, and efficient system despite uncertainties that arise from time to time.

The stochastic model is extensively utilized in reliability engineering. The Markov model can be used to represent the various states of the system such as operating mode, deterioration mode, and breakdown mode. A Poisson process is often used to model the failure process, whereas renewal processes model the maintenance and repair process. Control engineering involves stochastic differential equations representing stochastic processes. In addition, control engineering uses stochastic filters such as the Kalman filter to estimate the stochastic processes.

Stochastic models have broad applications in communication engineering because of the inherent randomness in communications. Examples include the random arrival of packets in networks, interference, and fading channels. Queueing theory can be used to estimate the necessary buffer size and routing strategy.

5.3 Healthcare and Epidemiology

There are numerous sources of randomness in the healthcare industry, from patient arrivals to disease progression, treatment outcomes, and resource availability. One can employ the stochastic process theory within mathematics to deal with these matters.

Queuing theory is applied within hospitals to forecast patient waiting time, bed occupancy, and staff needs. Markov models are applied to model disease development and treatment processes. Events related to survival analysis or counting processes include disease relapse and mortality. Stochastic compartmental models of disease dynamics are often used in epidemic models, especially when the number of cases is small or where random events could significantly affect epidemic dynamics.

Stochastic methodologies have been very effective in dealing with health emergencies, where epidemics must be predicted, incorporating uncertainty around infection rates, hospitalization, and interventions.

5.4 Environmental and Climate Systems

The behavior of environmental systems displays an element of randomness with respect to precipitation, temperature, river flows, dispersion of pollutants, and biological interactions. Using stochastic models allows scientists to understand this variability and manage the associated risks.

Environmental planners often rely on stochastic time series modeling and stochastic differential equations when studying floods, droughts, and reservoir behavior. Stochastic parameterization is used in atmospheric and oceanic science for representing sub-grid-scale processes in weather and climate models. Risk assessments within the environment often require assessing the likelihood of extreme events, such as hurricanes, wildfires, and heatwaves.

As environmental planning is inherently forward-looking and has severe consequences, stochastic models are especially useful.

5.5 Operations Research and Supply Chains

Supply chains face uncertain demands, variable lead times, disruptions, and transportation uncertainties. The application of stochastic models is fundamental in inventory management, production planning, and logistics management.

Inventory management models assume that there is randomness in demand and employ stochastic dynamic programming to come up with optimal reorder points. Queueing models represent warehousing and transport delays. Decision processes under uncertainty allow for sequential decisions when uncertainties are present and managerial decisions have to be made.

For example, a company that operates within a global supply chain will apply stochastic models to determine the safety stock, select the suppliers, and respond to any disruption.

5.6 Artificial Intelligence and Machine Learning

Although the concept of machine learning is often isolated from the field of probability theory, there are several AI techniques which require stochastic processes. These include hidden Markov models, Bayesian filtering, stochastic gradient, Monte Carlo simulation, and reinforcement learning. In the realm of natural language processing, probabilistic models of sequence address the issue of uncertainty in word sequences and speech signals. In robotics, stochastic localization and path planning incorporate noisy data and uncertain motions. In reinforcement learning, the Markov Decision Process forms the basis for AI agents in dealing with the transition uncertainties among states and rewards.

Real-life application of artificial intelligence requires stochastic techniques. Real-life data contains uncertainties and noisy observations, and therefore, it requires appropriate means to address such issues within the scope of artificial intelligence techniques.

VI. STRENGTHS OF STOCHASTIC MODELING

First, stochastic processes are realistic. There exist numerous examples where systems cannot be modeled deterministically due to random disturbances. Second, stochastic processes are adaptable. These processes can be used in modeling any system that can be described in terms of discrete or continuous, short or long period of time, strongly or weakly correlated events, and systems of small or big scale. Third, stochastic processes allow decision makers to evaluate risks. Stochastic processes produce probability distribution; therefore, their potential risk may be estimated.

Another benefit of using stochastic processes is that these processes may be used for constructing models in a manner similar to simulation methods. In some cases, an analytic approach fails to solve a specific problem, but a simulation technique can be helpful in finding an approximate solution for the model under consideration. Simulations become especially handy when working with complex processes that involve many states. Finally, stochastic approaches can assist researchers in learning from empirical data. Parameters for transition probabilities, arrivals, diffusions, etc., may be estimated from existing data sets.

VII. LIMITATIONS AND CHALLENGES

However, there are limitations to the use of stochastic processes in practical applications. First, model specification may pose problems. Using the wrong type of stochastic process will lead to inaccurate forecasting and poor decision-making. For example, assuming that events are independent in situations where strong dependency occurs will yield incorrect predictions and risks.

Second, collecting data for a stochastic model is a problem. It generally takes a lot of data to estimate the parameters of the model, but in many real-life scenarios, the data may be insufficient, noisy, biased, or non-stationary.

The third challenge associated with using stochastic processes is their computational complexity. Some types of stochastic models are difficult to solve because of their high dimensionality or continuous time frame.

Fourth, stochastic modeling requires certain assumptions to hold true. The assumptions made about the process may not hold true in practice due to external factors affecting the system or measurement errors. Ignoring this issue can make the results of the stochastic process overly precise and inaccurate.

Finally, presenting results based on stochastic processes is challenging. Often, decision makers prefer concrete numbers instead of probability distributions.

VIII. EMERGING DIRECTIONS

Future directions in stochastic processes will largely depend on trends in computation, data science, and interdisciplinary modeling. One of them might be the intersection of stochastic models and machine learning. The combination of hybrid techniques utilizes the advantages of probabilistic models, such as their transparency, while ensuring precision of computational models. Real-time uncertainty quantification will be yet another important trend. Using modern tools of sensors, streaming, and digital twins, it will become possible to continually update stochastic models based on the changes in the systems' states. Possible applications include smart cities, autonomous cars, manufacturing, and diagnostic medicine.

Furthermore, stochastic control and optimization is currently enjoying growing popularity. With increasing interconnectivity and adaptability, decision making in the presence of uncertainty becomes more and more important. This field covers problems like renewable energy grids, cyber-physical systems, and autonomous systems.

Lastly, research into rare events and resilience will also gain traction. Modern complex systems face not only fluctuations but potentially catastrophic scenarios rarely occurring. In order to investigate such features of modern systems and develop protection against them, stochastic approaches are necessary.

IX. CONCLUSION

One of the methods used to explore and control the uncertain aspects of any system is stochastic process theory. The uniqueness of stochastic process theory compared to other approaches is its consideration of the element of randomness and its intrinsic nature in real life. Modeling and prediction, optimization of system behavior, and management of the associated risks and vulnerabilities are possible with the help of probability distributions offered by stochastic processes. One of the advantages of stochastic processes is their ability to convert the uncertainty into knowledge that could be controlled. They enable organizations to forecast the behavior of the systems, plan against potential threats, optimize resource utilization, and make more informed decisions. However, at the same time, the usefulness of stochastic processes is related to their proper application and data availability. Increasingly complicated reality and access to a great amount of data increase the significance of analyzing systems by means of stochastic process analysis techniques. With the recent developments in computer algorithms and machine learning, it becomes possible to apply this method in relation to complicated data environments. Therefore, stochastic processes become a good method of reasoning under uncertainty conditions.

REFERENCES

1. Adamson, J., Evans, M. M., & Stein, R. (Eds.). (2002). The environmental justice reader: Politics, poetics, and pedagogy. University of Arizona Press.
2. Alaimo, S. (2010). Bodily natures: Science, environment, and the material self. Indiana University Press.
3. Asmussen, S. (2003). Applied probability and queues (2nd ed.). Springer.
4. Bachelier, L. (1900). Théorie de la spéculation. Gauthier-Villars. (Standard historical reprint/reference in stochastic finance).
5. Ban, G. Y., & Rudin, C. (2019). The big data newsvendor: Practical insights from machine learning. Operations Research, 67(1), 90–108. https://doi.org/10.1287/opre.2018.1757
6. Beichelt, F. (2006). Stochastic processes in science, engineering and finance. CRC Press.
7. Berlant, L. (2011). Cruel optimism. Duke University Press.
8. Bertsimas, D., & Kallus, N. (2020). From predictive to prescriptive analytics. Management Science, 66(3), 1025–1044. https://doi.org/10.1287/mnsc.2018.3253
9. Braidotti, R. (2013). The posthuman. Polity Press.
10. Buell, L. (2005). The future of environmental criticism: Environmental crisis and literary imagination. Blackwell.
11. Buizza, R., Miller, M., & Palmer, T. N. (1999). Stochastic representation of model uncertainties in the ECMWF ensemble prediction system. Quarterly Journal of the Royal Meteorological Society, 125(560), 2887–2908. https://doi.org/10.1002/qj.49712556006
12. Chakrabarty, D. (2009). The climate of history: Four theses. Critical Inquiry, 35(2), 197–222. https://doi.org/10.1086/596640
13. Cinlar, E. (2011). Probability and stochastic processes. Springer.
14. Collins, P. H. (2000). Black feminist thought: Knowledge, consciousness, and the politics of empowerment (2nd ed.). Routledge.
15. Durrett, R. (2010). Probability: Theory and examples (4th ed.). Cambridge University Press.

16. Ethier, S. N., & Kurtz, T. G. (2005). Markov processes: Characterization and convergence. Wiley.
17. Gardiner, C. W. (2009). Stochastic methods: A handbook for the natural and social sciences (4th ed.). Springer.
18. Garrard, G. (2011). Ecocriticism (2nd ed.). Routledge.
19. Grimmett, G., & Stirzaker, D. (2001). Probability and random processes (3rd ed.). Oxford University Press.
20. Gupta, A., & Maranas, C. D. (2003). Managing demand uncertainty in supply chain planning. Computers & Chemical Engineering, 27(8-9), 1219–1227. https://doi.org/10.1016/S0098-1354(03)00048-6
21. Heise, U. K. (2008). Sense of place and sense of planet: The environmental imagination of the global. Oxford University Press.
22. Kolbert, E. (2014). The sixth extinction: An unnatural history. Henry Holt and Company.
23. Øksendal, B. (2003). Stochastic differential equations: An introduction with applications (6th ed.). Springer.
24. Plumwood, V. (2002). Environmental culture: The ecological crisis of reason. Routledge.
25. Sahinidis, N. V. (2004). Optimization under uncertainty: State-of-the-art and opportunities. Computers & Chemical Engineering, 28(6-7), 971–983. https://doi.org/10.1016/j.compchemeng.2003.09.017