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MHD Flow Through Long Elastic Porous Tube with Heat Transfer

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Abstract- In this work, the three-dimensional MHD flow and the heat transfer of an incompressible viscous fluid through a porous medium with two parallel porous plates is studied. The top plate is kept at the same suction velocity and moves in the same direction along the longitudinal direction, and the lower stationary plate is kept at a transverse sinusoidal injection velocity. The plates are normally magnetically applied and the two plates are kept at different constant temperatures. The continuity, momentum, and energy equations for the medium are given by the magnetic field, permeability of a porous medium, suction/injection, and thermal buoyancy. This dimensional equation can be written in terms of the Hartmann number, suction/injection parameter, permeability parameter, Prandtl number, and Grashof number. The resulting nonlinear dimensionless equations are solved numerically using a finite difference scheme based on central and forward difference approximations. We use the numerical results to investigate the influence of the governing parameters on the velocity components and temperature distribution. The field effect of the magnetic field significantly influences the flow through the electromagnetic damping effect, and suction/injection of energy changes the velocity distribution in the porous region. The permeability of the porous medium generally enhances the fluid motion by decreasing the resistance of the porous matrix. The thermal field is strongly influenced by the Prandtl and Grashof numbers. The results provide insight into the coupled effects of the magnetic field, porous-medium resistance, wall transpiration, and thermal transport in 3D MHD flows through porous configurations.

Keywords- Three-dimensional flow; magnetohydrodynamics; porous medium; suction and injection; heat transfer; Hartmann number; Prandtl number; Grashof number; finite difference method.

I. INTRODUCTION

Gersten and Gross (1974) studied on the three-dimensional convective flow and heat transfer through a porous medium, Therefore Singh (1991) worked on three dimensional MHD flow past a porous plate, and again Singh (1993) also worked in the same direction and studied the problem of three-dimensional viscous flow and heat transfer along a porous plate. I. pop (2009) discussed MHD stagnation point flows towards a stretching sheet. Statistical Mechanics and its Applications. (1985): Free convective through a porous medium bounded by a vertical porous plate with constant suction when the free steam velocity.

Ahmed and Sharma (1997) discussed about the three-dimensional free convective flow of an incompressible viscous fluid through a porous medium with uniform free steam velocity, while Singh (1999) again studied about a three-dimensional Couette flow with transpiration cooling by applying transverse sinusoidal injection velocity at the stationary plate velocity. Kim (2000) discussed the unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, and Kamel (2001) discussed about the unsteady MHD convection through porous medium with combined heat and mass transfer with heat source/sink. Tzer-Ming Jeng (2008) worked on a porous model for the square pin-fin heat sink situated in a rectangular channel with laminar side-by-pass flow.

Transporting electrically conducting fluids through porous media has received a high degree of attention from all domains of engineering, industry and technology. Fluid movement through porous structures is seen in geothermal energy extraction, petroleum engineering, groundwater transport, filtration, cooling of electronic devices, chemical processing, and thermal insulation. When the fluid is electrically conducting, the external magnetic field influences the flow of the fluid because the current is fed back to the fluid and the force of incoming current is different. This can introduce a Lorentz force which can affect the velocity field and hence the heat transfer behavior. The porous-medium resistance, magnetic forces, wall transpiration and thermal buoyancy are all important MHD transport problems.

The study of convective flow through porous media has a long history. Raptis [1] studied free convection through a porous medium bounded by an infinite vertical plate with oscillating plate temperature and constant suction. Raptis and Perdiker [2] investigated free convection through a porous medium bounded by a vertical porous plate under suction conditions. In these papers the wall transpiration and thermal effects played an important role in controlling the velocity and temperature field in porous structures. The inclusion of magnetic effects in these transport problems provides another means for controlling the fluid motion and thermal transport.

Three-dimensional flow is particularly relevant when the velocity field cannot be captured by a two-dimensional approximation. Gersten and Gross considered three-dimensional convective flow and heat transfer through porous media. They provided an initial setting for multidirectional transport in porous settings. Singh [3] investigated three-dimensional MHD flow through a porous plate, and Singh [4] investigated viscous flow and heat transfer along a porous plate. These investigations showed that suction, injection and magnetic effects can change the shape of the three-dimensional flow field.

The effect of wall transpiration is especially important in porous-wall flows. Suction can suppress fluid motion near a porous boundary and change the thickness of the hydrodynamic and thermal regions, while injection can increase fluid transport and change the near wall velocity distribution. A sinusoidally varying injection velocity adds additional spatial variation into the flow and provides a useful representation of nonuniform transpiration through a porous boundary. Such configurations are of interest in cooling and transport systems in which fluid is introduced through a porous surface in a nonuniform way.

Magnetic field effects have also been widely used in convective transport models. Kim [5] studied unsteady MHD convective heat transfer through a semi-infinite vertical porous moving plate with variable suction, and showed that the magnetic and suction effects are coupled to the transport properties. Kamel [6] considered unsteady MHD convection through a porous medium and combined heat and mass transfer in the presence of a heat source or sink. These studies suggest that the magnetic parameter is a powerful parameter for the momentum transport of electrically conducting fluids in porous environments.

More recently, MHD flows have been investigated in stretching surfaces, porous structures, and thermal effects. Ishak, Jafar, Nazar, and Pop [7] investigated MHD stagnation-point flow towards a stretching sheet, and Ishak, Nazar, and Pop [8] studied MHD mixed-convection boundary-layer flow to a stretching vertical surface at a constant temperature. Ishak, Nazar, and Pop [9] also studied MHD flow and heat transfer in a stretching cylinder. Barletta, Lazzari, and Magyari [10] studied mixed convection and heating effects in a vertical porous annulus under a radially varying magnetic field.

The thermal behaviour of porous materials is determined by a competition between momentum transport, thermal diffusion and buoyancy. The Prandtl number measures the relative importance of momentum and thermal diffusivities and the Grashof number measures the relative strength of the buoyancy-induced flow. Thus, changes in these parameters can result in large changes in both the velocity and temperature distributions. Additionally, the permeability of the porous medium determines the resistance of the fluid. A relatively permeable medium is less resistant to fluid motion whereas a relatively low permeable medium is more likely to suppress the flow. It is therefore crucial to consider permeability, magnetic damping and suction/injection in the description of transport through porous structures.

Despite the extensive literature on MHD flow over porous plates and through porous media, the combined effects of three-dimensional structure, transverse sinusoidal injection, constant suction, magnetic field, permeability of porous medium and thermal buoyancy are still of special interest. In this study we consider a three-dimensional incompressible viscous electrically conducting fluid in a porous configuration with two parallel plates maintained at different constant temperatures. The upper plate is subjected to constant suction and moves with a uniform speed in the longitudinal direction, whereas the lower stationary plate is subjected to a transverse sinusoidal injection. The plates are normal in magnetic field and the coupled momentum and energy equations are considered to be dimensionless.

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The resulting dimensionless system involves several important controlling parameters, including Hartmann number, suction/injection parameter, permeability parameter, Prandtl number, and Grashof number. The nonlinear governing equations are solved numerically using a finite difference approach. The magnetic field, wall suction/injection, porous-medium permeability, and thermal parameters have an important effect on the three-dimensional velocity components and temperature distribution.

The purpose of this study is as follows: we now calculate the coupled hydrodynamic and thermal behaviour of a three-dimensional MHD flow through a porous structure in the presence of nonuniform wall injection and constant suction. This study is intended to clarify how magnetic damping, porous resistance, wall transpiration and buoyancy affect fluid motion and heat transfer in porous systems. The results can be useful in understanding transport of electrically conducting fluids in porous systems where magnetic control and thermal management are at the same time of concern.

The main purpose of this work is to analyze the effects of electrically conducting three-dimensional, viscous incompressible fluid through a porous plate with the observation of the velocity and temperature distribution along with the effects of Hartmann number (M) and suction parameter (R).

II. MATHEMATICAL MODAL

A 3-D flow of a viscous incompressible fluid through a porous medium is considered here, which is bounded by a vertical infinite porous plate. The coordinate system with plate lying vertically on X*-Z* plane, where upper plate is at distant an apart such that X*- axis is taken along the plate in the direction of the plane and Y*- axis is perpendicular to the plane. The lower and upper plate's temperature considered to be T0* and T1* respectively with (T1*>T0*) and the upper plate is subjected to a constant velocity U along X*- axis, while the lower plate to a transverse sinusoidal injection velocity of the following forms

$$V^*(Z^*) = V^*\left(1+\varepsilon\cos\frac{\pi Z^*}{a}\right) \quad \dots (1)$$

Denoting the velocity component u*, v*, w* in the X*, Y* and Z* direction respectively and temperature by T*, then problem is governed by the following equations.

Continuity equation:

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \quad \dots (2)$$

The momentum equations are

$$v^*\frac{\partial u^*}{\partial y^*} + w^*\frac{\partial u^*}{\partial z^*} = \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) + g\beta(T_1^* - T_1^*) - \sigma\frac{\beta_0^2 u^*}{\rho} \quad (3)$$

$$v^*\frac{\partial v^*}{\partial y^*} + w^*\frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) - \frac{\nu}{k}v^* \quad (4)$$

$$v^*\frac{\partial w^*}{\partial y^*} + w^*\frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \nu\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\sigma\beta_0^2 w^*}{\rho} \quad (5)$$

And the energy equation is

$$v^*\frac{\partial T^*}{\partial y^*} + w^*\frac{\partial T^*}{\partial z^*} = \alpha\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) \quad (6)$$

While the boundary conditions of this modal are

$$\left.\begin{array}{ll} \text{at } y^* = 0,\ u^* = 0, & v^*(z^*) = v\left(1+\varepsilon\cos\frac{\pi z^*}{a}\right),\ w^* = 0,\ T^* = T_0 \\ \text{at } y^* = a,\ u^* = U,\ v^* = v, & w^* = 0,\ T^* = T_1^* \end{array}\right\} \quad (7)$$

Now we introduce following non-dimensional variables, which change the governing equations in the dimensionless form

$$y = \frac{y^*}{a},\quad z = \frac{z^*}{a},\quad u = \frac{u^*}{U},\quad v = \frac{v^*}{v}, w = \frac{w^*}{v},\quad p = \frac{P}{\rho v^2},\quad \theta = \frac{T^* - T_0^*}{T_1^* - T_0^*} \Bigg\} (8)$$

In this model we use some special numbers/ parameters, which are along with their notations as follows:

Suction parameter $(R) = \frac{av}{\nu}$, Hartmann number ,

$$(M) = \frac{\beta_0}{V}\sqrt{\frac{\sigma}{\mu}}$$

Prandtl number $(P_r) = \frac{\nu}{\alpha}$, Grashof number ,

$$(G_r) = \frac{g\beta\rho^2(T_0^* - T_1^*)}{\mu^2}a^3$$

- Magnetic field component along Y*- axis,

k- Permeability of the porous medium,

U- Velocity along X*-axis,
α- thermal diffusivity,
β- Volumetric coefficient of the thermal expansion,
ρ - Density of fluid,
ν - Kinematics viscosity,
μ - Viscosity,
θ - Temperature

Here in the light of (8) the boundary conditions (7) and solving the equations (2) to (4), becomes in the following form:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad \text{.... (9)}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{G_r\nu}{aRU}-\frac{M^2u}{R} \quad \text{.... (10)}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial P}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{a^2v}{Rk} \quad \text{.... (11)}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial P}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{M^2w}{R} \quad \text{.... (12)}$$

And temperature equation becomes:

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{RP_r}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right) \quad \text{.... (13)}$$

Then the boundary conditions (7) reduce in the dimensionless form:

III. NUMERICAL SOLUTION

In this model we apply the finite difference method to solve the governing non-linear equations. First we transformed the first and second order derivatives by using the forward and central differences formulas. Here i, j represent the moment along Y and Z direction respectively, and let, there is a square mesh ( ), then the equations (9) to (13) along with the boundary conditions are as:

$$u_{i,j+1}=\frac{1}{(Rhw_{i,j}-1)}\left[\begin{aligned}&(1-Rhv_{i,j})u_{i+1,j}+\{Rh(v_{i,j}+w_{i,j})-(4+M^2h^2)\}u_{i,j}\\&+(u_{i-1,j}+u_{i,j-1})+\frac{\nu hG_r}{aRU}\end{aligned}\right] \quad \text{.... (14)}$$

$$v_{i,j+1}=\frac{1}{(Rhw_{i,j}-1)}\left[\begin{aligned}&(1-Rhv_{i,j})v_{i+1,j}++\left\{Rh(v_{i,j}+w_{i,j})-\left(4+\frac{a^2h^2}{K}\right)\right\}v_{i,j}\\&+(v_{i-1,j}+v_{i,j-1})++Rh\Delta P\end{aligned}\right] \quad \text{.... (15)}$$

$$w_{i,j+1}=\frac{1}{(Rhw_{i,j}-1)}\left[\begin{aligned}&(1-Rhv_{i,j})w_{i+1,j}+\{Rh(v_{i,j}+w_{i,j})-(4+M^2h^2)\}w_{i,j}\\&+Rh\Delta P\end{aligned}\right] \quad \text{.... (16)}$$

And temperature equation

$$\theta_{i,j+1}=\frac{1}{(RhP_rw_{i,j}-1)}\left[\begin{aligned}&(1-RhP_rv_{i,j})\theta_{i+1,j}+\{RhP_r(v_{i,j}+w_{i,j})-4\}\theta_{i,j}\\&+(\theta_{i-1,j}+\theta_{i,j-1})\end{aligned}\right] \quad \text{.... (17)}$$

IV. RESULTS AND DISCUSSION

Numerical solutions of the dimensionless equations yield the velocity and temperature distributions of the three-dimensional MHD flow through the porous structure. The numerical results are especially sensitive to Hartmann number M, suction/injection parameter R, permeability of the porous medium \(k\), Prandtl number Pr, and Grashof number Gr. These parameters represent the relative contributions of magnetic damping, wall transpiration, porous resistance, thermal diffusion, and buoyancy, respectively. The other parameters are kept at their reference values while the parameter under consideration is varied.

1. Effect of the Hartmann number

The Hartmann number is an important parameter in the interaction between the applied magnetic field and the electrically conducting fluid. An increase in M corresponds to an increase in the magnetic influence on the flow. The numerical profiles show that the principal velocity component u decreases as the Hartmann number increases. This behaviour is explained by the Lorentz force that arises from the interaction of the induced electric current and the applied magnetic field. The Lorentz force acts in opposition to fluid motion and thus leads to the electromagnetic damping effect. As the magnetic field strength increases, the resistance to fluid motion becomes stronger and the velocity distribution decreases. The effect is particularly relevant in regions where the fluid motion is strongly influenced by the magnetic field.

The decrease in velocity with increasing M indicates that an external magnetic field can be used to control the motion of an electrically conducting fluid in a porous environment. This observation is consistent with the general behaviour reported for MHD flows in porous and boundary-layer configurations. The magnetic field also influences the transport of heat indirectly through its effect on the velocity field. Since convection is associated with fluid motion, suppression of the velocity modifies the convective contribution to the energy equation and consequently changes the temperature distribution. Therefore, the magnetic parameter affects both momentum and thermal transport, even though the magnetic field enters the momentum equations directly.

Effect of the suction/injection parameter

The wall suction/injection parameter R has an important impact on the velocity distribution. In the present model, the lower plate is subjected to a transverse sinusoidal injection velocity, while the upper plate is subjected to constant suction. Therefore, the amount of fluid that is moved through these porous boundaries depends on the wall suction parameter R. As shown in the numerical results, increasing the injection/suction parameter generally reduces the principal flow velocity in the configuration considered. This reduction can be explained in terms of enhanced wall-normal transport. Stronger suction removes fluid from the flow region and tends to suppress the longitudinal motion, whereas injection introduces fluid into the domain and modifies the momentum balance through the transverse velocity component. Since the injection is sinusoidal, its influence is spatially nonuniform and produces corresponding changes in the three-dimensional velocity field.

The suction effect also influences the redistribution of the thermal field. The removal of fluid from the vicinity of the wall modifies the thermal boundary region and affects the transport of thermal energy away from the heated or cooled surface. Consequently, wall transpiration represents an important mechanism for controlling both hydrodynamic and thermal characteristics. The combined presence of suction at the upper boundary and sinusoidal injection at the lower boundary makes the present configuration different from a conventional impermeable-wall channel. The resulting flow is inherently three-dimensional, and the transverse velocity component plays a crucial role in satisfying continuity and redistributing momentum within the porous region.

Effect of porous-medium permeability

The permeability parameter characterizes the ease with which the fluid can pass through the porous structure. The numerical results demonstrate that the velocity increases with increasing permeability of the porous medium. For a relatively low permeability, the porous matrix offers substantial resistance to fluid motion. Consequently, the fluid experiences greater drag and the velocity remains comparatively small. As the permeability increases, the resistance associated with the porous structure decreases, allowing the fluid to move more freely through the medium. This produces an enhancement of the velocity components. The effect of permeability is particularly important when combined with the magnetic field. The porous matrix tends to resist the motion mechanically, whereas the applied magnetic field provides an additional electromagnetic resistance. Hence, the velocity distribution is determined by the combined action of porous drag and Lorentz-force damping. An increase in permeability reduces the former, while an increase in Hartmann number strengthens the latter. The results therefore demonstrate that permeability and magnetic-field strength provide two competing mechanisms for controlling the flow. A highly permeable medium favours fluid motion, whereas a stronger magnetic field suppresses it.

Combined influence of permeability and suction/injection

The interaction between permeability and wall transpiration is also of physical significance. For a low-permeability medium, the fluid experiences strong resistance from the porous matrix, and the influence of suction/injection is superimposed on this resistance. As the permeability increases, the resistance of the porous structure decreases and the velocity becomes more responsive to the imposed wall-transpiration conditions.

The computed velocity profiles for different permeability levels show that increasing \(R\) suppresses the flow velocity, while increasing \(k\) enhances it. Thus, permeability and suction/injection have opposing influences on the magnitude of the longitudinal flow in the present configuration. This observation is useful for applications in which flow control is achieved simultaneously through the properties of the porous material and through controlled wall transpiration. Appropriate selection of permeability and suction/injection conditions can therefore substantially modify the velocity field without changing the basic geometry of the system.

Effect of the Prandtl number

The Prandtl number Pr measures the ratio of momentum diffusivity to thermal diffusivity. It therefore plays a central role in determining the temperature distribution. Fluids with relatively small Prandtl numbers possess comparatively high thermal diffusivity, whereas fluids with larger Prandtl numbers exhibit weaker thermal diffusion relative to momentum diffusion.

The numerical temperature profiles demonstrate that variation of Pr significantly modifies the thermal field. Increasing Pr reduces the ability of thermal energy to diffuse through the fluid, resulting in a more confined thermal region near the wall. Conversely, a lower Prandtl number permits heat to diffuse more rapidly into the fluid domain and consequently produces a broader temperature distribution.

The influence of Pr is also coupled with the velocity field. Since heat is transported through both diffusion and convection, modifications in the velocity distribution caused by the magnetic field, porous resistance, or wall transpiration can alter the thermal response associated with a particular Prandtl number.

Effect of the Grashof number

The Grashof number Gr represents how important the buoyancy forces are compared with the viscous ones. Since the plates are kept at different temperatures, the density fluctuations caused by temperature variations produce a buoyancy force in the fluid.

The incease of Gr strengthens the buoyancy component of momentum equations, so that the thermal field can induce more fluid motion and change the velocity distribution. The above behaviour is the thermal and hydrodynamic transport coupling in the present problem. For larger values of Gr buoyancy becomes more important than viscous resistance. The heating that is generated by thermally induced motion might compete with the damping effect of the magnetic field and porous medium. So, the flow behaviour is a balance between the buoyancy-induced acceleration, electromagnetic damping, porous resistance, and wall-transpiration effect.

This coupling is particularly important in free- and mixed-convection systems in which the temperature difference between the boundaries is sufficiently large to generate appreciable buoyancy forces.

Temperature distribution and heat-transfer behaviour

The temperature distribution is determined by the energy equation and by the thermal boundary conditions at the two plates. Since the lower and upper boundaries are kept at different constant temperatures, a thermal gradient is established in the flow region.

The numerical results show that the temperature varies continuously between the prescribed wall temperatures and is strongly affected by the Prandtl number and the flow parameters. The velocity field changes the temperature through convective transport, and thermal diffusion smooths the temperature gradient.

The magnetic field influences the temperature distribution indirectly through the suppression of fluid motion. As M increases, the Lorentz force reduces the velocity and thus changes the convective transport of thermal energy. Likewise, suction and injection affect the temperature field by transporting fluid across the boundaries.

The permeability of the porous medium also affects heat transfer because it affects the fluid velocity and hence the convective part of energy transport. Thus, the thermal response cannot be interpreted independently of the hydrodynamic parameters.

Heat-transfer coefficient

The rate of heat transfer at the stationary porous plate is an important quantity to evaluate the thermal performance of the system. It depends on the temperature gradient at the wall and hence reflects the combined influence of thermal diffusion and fluid motion.

The results indicate that the heat transfer characteristics depend significantly on the Prandtl number. Fluids with relatively high thermal diffusivity distribute thermal energy more readily throughout the flow region, while fluids with lower thermal diffusivity maintain stronger temperature gradients near the wall.

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So, a comparison of representative fluids shows that the rate of heat transfer is sensitive to the thermophysical properties of the working fluid. In our case, the Prandtl number is a convenient dimensionless measure to describe this dependence.

Physical interpretation of the combined parameters

The numerical results show that the flow and heat transfer characteristics are affected by several competing mechanisms. The Hartmann number introduces electromagnetic damping, the permeability of the porous medium determines the resistance to flow through the matrix, suction/injection determines wall-normal mass transport, and the Grashof number controls buoyancy-induced motion. The Prandtl number determines the importance of momentum and thermal diffusion.

The overall behaviour can therefore be understood through the following physical competition:

The higher the M, the stronger the magnetic damping effect and the lower the fluid motion. Increasing the R, the stronger the wall suction/injection effect and the transverse velocity distribution. Increasing k, the porous resistance is reduced and fluid motion is higher. Increasing the Pr, the relative thermal diffusion is reduced and the thermal field is stronger near the wall. Increasing the Gr, the buoyancy effect is increased and the velocity field gets driven by the thermal force.

The interaction of these parameters is important since there is no single mechanism that determines the flow behaviour. For example, buoyancy can enhance fluid motion, but the magnetic field slows it down. On the other hand, increasing permeability promotes more motion and wall suction can impede the longitudinal flow. The numerical solution therefore provides a useful framework for exploring the balance of electromagnetic, porous, viscous, buoyancy, and thermal effects. The main flow velocity component u, applied through the porous plate at rest, due to the transverse sinusoidal injection velocity is obtained from equation (14). We described the from the equation (14), which shows the flow component u decreases with the increases of Hartmann number M, injection parameter R and permeability k of the porous medium.

We described velocity profile for the small and large value of permeability of the porous medium through the velocity decrease with the increase of the injection suction parameter R, it is also observed that increase in the permeability of the porous lead to an increase in the flow velocities. The rate of heat transfer coefficient at the stationary porous plate. In this case the values of Prandtl number Pr are chosen as 0.4 approximately, which represents air and water at . Here it is found that the rate of heat transfer is much lower in the case of water than in air.

V. CONCLUSION

In this work, we investigated the mathematical and numerical behavior of fractional diffusion equations, and the nonlocal and memory-dependent properties of fractional differentiation. In contrast to classical diffusion equations, fractional diffusion equations can provide a more general model for anomalous transport and diffusion processes, where the current state is not only about instantaneous but also the history of the system. We have developed a fractional diffusion model and investigated its basic mathematical properties. We have demonstrated how the fractional order parameter changes the dynamics of the solution and demonstrated that fractional diffusion can lead to much different diffusion behavior from classical models.

Furthermore, the nonlocal nature of the fractional operator introduces additional computational difficulties, particularly in the approximation of the solution over long time intervals. In order to overcome this problem, we considered a high-order numerical approximation approach for the fractional diffusion problem. Our approach allows for a good representation of the solution while retaining the essential memory of the fractional operator. We have seen numerically that our numerical solution can be achieved for a variety of fractional orders and problems. The solutions are consistent with the qualitative behavior of fractional diffusion as expected and the solution goes to the classical diffusion regime as fractional order increases to unity. Numerical analysis also shows that the fractional order is a significant factor in the rate and geographic distribution of diffusion.

Lower fractional order is generally associated with higher memory effects and slower temporal evolution while values approaching one then recover the behavior of the classical diffusion equation. These observations underline the need to choose the right fractional order when modeling the transport conditions or long-range temporal dependence of processes. The results show that fractional diffusion equations provide a useful mathematical model for describing diffusion processes that cannot be adequately described by standard integer-order equations. Mathematical analysis with numerical approximation in this paper is also available to study more complicated fractional diffusion systems.

There are a few other possible extensions that can be considered in the future, such as variable-order fractional operators, nonlinear diffusion coefficients, fractional diffusion-reaction systems, space-time fractional derivatives, nonlocal boundary conditions, etc. We also have a good numerical framework and efficient algorithms to reduce the computational cost of the history-dependent terms. We can also extend our framework to fractional operators with nonsingular kernels like Caputo–Fabrizio and Atangana–Baleanu derivatives to compare the different memory mechanisms in a single diffusion framework.

REFERENCES

1. A. A. Raptis (1983): Free convective flow through porous medium bounded by an infinite vertical plate with oscillating plate temperature and constant suction. Int. J. Eng. Sci., Vol. 21, Pp. 345-348.
2. A. A. Raptis and C. P. Perdiker (1985): Free convective through a porous medium bounded by a vertical porous plate with constant suction when the free steam velocity. Int. J. Eng. Sci., Vol. 31, Pp. 51-55.
3. A. Barletta, S. Lazzari and E. Magyari(2008): Mixed convection with heating effects in a vertical porous annulus with a radially varying magnetic field. Int. Journal of Heat and Mass Transfer, Vol. 51, Pp. 5777-5784.
4. A. Ishak, K. Jafar, R. Nazar and I. pop (2009): MHD stagnation point flows towards a stretching sheet. Statistical Mechanics and its Applications, Vol. 388, Pp. 3377-3383.
5. A. Ishak, R. Nazar and I. Pop (2010): MHD mixed convection boundary layer flow towards a stretching vertical surface with constant wall temperature. International Journal of Heat and Mass Transfer, Vol. 53, Pp. 1905-1916.
6. A. Ishak, R. Nazar and I. pop (2008): Magneto hydrodynamic (MHD) flow and heat transfer due to a stretching cylinder. Energy Conversion and Management, Vol. 49, Pp. 3265-3269.