International Journal of Scientific Research & Engineering Trends
Volume 8, Issue 5, Sep-Oct-2022, ISSN (Online): 2395-566X

Mhd Flow Through Vertical Porous Plate With Heat Transfer

Dr. Satish Kumar
Department of Mathematics Government Girls Degree College, Budaun, Uttar Pradesh.

Abstract- This study investigates the unsteady magneto hydrodynamic free convective flow of a viscous, incompressible, and electrically conducting fluid past an infinite vertical porous plate with porous medium and applied uniform magnetic field in the direction of the flow. The effect of injection/suction velocity and the magnetic field on the flow field, skin friction and heat transfer are reported and discussed in detail. The Hartmann number and porosity parameter influence the flow velocity, while the Prandtl and Grashof number govern the heat transfer characteristics. The governing partial differential equations for momentum and energy are transformed into a dimensionless form using appropriate similarity variables.

Keywords – Magnetohydrodynamics (MHD), Free convection flow, Porous medium, Vertical porous plate.

I. INTRODUCTION

The purpose of the paper is to study the hydromantic effects of electrically conducting 3-D flow of viscous fluid through a porous medium, which is bounded by an infinite vertical porous plate with constant temperature.

Attia (2006) study the Unsteady MHD Couette flow and heat transfer of dusty fluid with variable physical properties. Dutta (1985) discussed the Temperature field in the flow over stretching surface with uniform heat flux. Yong (2000) study the Unstudy MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Yang et al (2001) discussed Numerical solution of thermal fluid instability between two horizontal parallel plates. Ahamad et al (2003) discussed the MHD effects on free convection and mass transfer flow through porous media between vertical wavy wall and a parallel flat wall. Muhammad (2005) discussed the Effects of Hall current and heat transfer on flow due to a pull of eccentric rotating. Ling Tian(2021)discussed the Stability of the 3D incompressible MHD equations with horizontal dissipation in periodic domain.

II. MODELING EQUATION

In this modal we consider 3-D flow viscous incompressible fluid through a porous medium, which is bounded by a vertical infinite porous plate. A coordinates system with plate lying vertically on x-z plane such that x-axis is taken along the plate in the direction of flow and y-axis perpendicular to the plane of the plate and direction in to the fluid which is flowing with free stream velocity U and lower plate is to have a transverse sinusoidal injection velocity of the form

$$v^1(Z') = v\left(1 \pm \varepsilon \cos\frac{\pi Z'}{c}\right) \quad \ldots (1)$$

Where ε <1. The distance c between the plates is taken equal to the wavelength of the injection velocity. The plates are assumed to at constant temperatures T_0 and T_1, respectively, with $T_1 > T_0$.

The problem is governed by the following non-dimensional equations:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \ldots (2)$$

The momentum equations are

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{\lambda}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{M}{\lambda}u \quad \ldots (3)$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{\lambda}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{1}{k'}v \cdots (4)$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{\lambda}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{M}{\lambda}w \quad \ldots (5)$$

And the energy equation is

$$v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{1}{\lambda p_r}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad (6)$$

The non-dimensional variables as:

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$$y=\frac{y'}{c},\quad z=\frac{z'}{c},\quad u=\frac{u'}{U},\quad v=\frac{v'}{v},w=\frac{w'}{v},\quad p=\frac{P'}{\rho v^{2,}},\quad \theta=\frac{T'-T_0}{T_1-T_0},\ k'=\frac{c\nu}{vk}\Bigg\}$$

The boundary conditions to this problem in dimensionless form are as:

$$\left.\begin{array}{l} u=0,\quad v(z)=1\pm\varepsilon\cos\pi z,\ w=0,\quad T=0,\quad for\ y=0. \\ u=1,\quad v=1,\quad w=0,\quad T=1,\quad for\ y=1. \end{array}\right\}$$

… (8)

III. MATHEMATICAL FORMULATION

As we know that the amplitude of injection velocity ε is very smalls so can assume the following form the solutions

$$f(y,z)=f_0(y)+\varepsilon f_1(y,z)+\varepsilon^2 f_2(y,z)+.........$$

… (9)

f stands for any of u, v, w, p, and T function. When ε=0, then the solution of two-dimensional is

$$u_0(y)=\frac{e^{s_1 y}-e^{s_2 y}}{e^{s_1}-e^{s_2}},\qquad w=0,\quad \ldots (10)$$

$$v_0(y)=\frac{1}{e^{t_1}-e^{t_2}}\left[(e^{t_1 y}-e^{t_2 y})+e^{t_1+t_2 y}-e^{t_2+t_1 y}\right] \ldots (11)$$

$$T_0(y)=\frac{e^{\lambda P_r y}-1}{e^{\lambda P_r}-1}\quad \ldots (12)$$

Where

$$s=\left[\lambda\pm\sqrt{\lambda^2+4M^2}\right],\quad t=\frac{\lambda k'\pm\sqrt{\lambda^2 k'^2+4k'\lambda}}{2k'}$$

When $\varepsilon\neq 0$, then equation (9) becomes in forms,

$$\left.\begin{array}{l} u(y,z)=u_0(y)+\varepsilon u_1(y,z) \\ v(y,z)=v_0(y)+\varepsilon v_1(y,z) \\ w(y,z)=w_0(y)+\varepsilon w_1(y,z) \\ p(y,z)=p_0(y)+\varepsilon p_1(y,z) \\ T(y,z)=T_0(y)+\varepsilon T_1(y,z) \end{array}\right\}$$

… (13)

Substituting the equation (13) in to the equation (2) to (6) and comparing the coefficient of identical power ofε, and neglecting the coefficient ε^2, ε^3 etc. we get

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0\quad \ldots (14)$$

The momentum equations as

$$v_1\frac{\partial u_0}{\partial y}+w\frac{\partial u_1}{\partial z}=\frac{1}{\lambda}\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right)-\frac{M^2}{\lambda}u_1$$

… (15)

$$\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{\lambda}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right)-\frac{1}{k'}v_1\quad (16)$$

…

$$\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{\lambda}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)-\frac{M^2}{\lambda}w_1 \ldots\quad (17)$$

And the energy equation is

$$v_1\frac{\partial T_0}{\partial y}+\frac{\partial T_1}{\partial y}=\frac{1}{\lambda p_r}\left(\frac{\partial^2 T_1}{\partial y^2}+\frac{\partial^2 T_1}{\partial z^2}\right) \ldots (18)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} u_1=0,\quad v_1=\cos\pi z,\ w_1=0,\quad T_1=0,\quad for\ y=0. \\ u_1=1,\quad v_1=1,\quad w_1=0,\quad T_1=1,\quad for\ y=1. \end{array}\right\}$$

… (19)

Cross Flow solution:

For the cross flow solution we assume the following form of v_1, w_1 and p_1:

$$\left.\begin{array}{l} v_1(y,z)=v\ (y)\cos\pi z \\ w_1(y,z)=-\frac{1}{\pi}v_1'(y)\sin\pi z \\ p_1(y,z)=p_1(y)\cos\pi z \end{array}\right\}\quad \ldots (20)$$

Where 1 denote the differentiation with respect to y. substituting equation (20) in equations (16) and (17). We get the following ordinary differential equations:

$$v_1''-\lambda v_1'-\alpha^2 v_1=\lambda p_1'$$

… (21)

$$v_1'''-\lambda v_1''-(\pi^2+M^2)v_1'=\lambda\pi^2 p_1\quad \ldots (22)$$

Now using the transformed boundary conditions the equations (21) and (22) obtained in the following form

$$v_1(y,z)=\frac{1}{D}\left(\sum_{i=1}^{4}D_i e^{r_i y}\right)\cos\alpha z\quad \ldots (23)$$

$$w_1(y,z) = -\frac{1}{\pi D}\left(\sum_{i=1}^{4} D_i r_i e^{r_i y}\right)\sin \pi z \qquad \ldots (24)$$

$$p_1(y,z) = \frac{1}{\lambda \pi^2 D}\left[\sum_{i=1}^{4} D_i\left[\left\{r_i^3 - \lambda r_i^2 - \left(\pi^2 + M^2\right) r_i\right\} e^{r_i y}\right]\right]\cos \pi z \qquad \ldots (25)$$

Where

$$r_i = \frac{1}{2}\left[p_i + \sqrt{p_i^2 + 4\pi^2}\right]$$

Main flow Equation

We consider the equations of the main flow component $u_1(y, z)$ and temperature field $T_1(y,z)$ in the following form:

$$u_1(y,z) = u_1(y)\cos \pi z \qquad \ldots (26)$$

$$T_1(y,z) = T_1(y)\cos \pi z \qquad \ldots (27)$$

Substituting these equations in equations (15) and (18) respectively

$$u_1'' - \lambda u_1' - (\pi^2 + M^2)u_1 = \lambda v_1 u_0' \qquad \ldots (28)$$

$$T_1'' - \lambda p_r T_1' - \pi^2 T_1 = \lambda p_r v_1 T_0' \qquad \ldots (29)$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} u_1 = 0,\ T_1 = 0,\ \ for\ \ y = 0. \\ u_1 = 0,\ T_1 = 0,\ \ for\ \ y = 1. \end{aligned}\right\} \qquad \ldots (30)$$

Using the boundary condition (30) and equation (26) and (27) in the equations (28) and (29), we get

$$u_1(y,z) = \left[\sum_{i=1}^{2} K_i e^{n_i y} + \frac{\lambda}{D(e^{m_1} - e^{m_2})}\left\{\sum_{i=1}^{2}\frac{m_1 D_i e^{(m_1+r_i)y}}{r_i(3m_1 - \lambda)} + \sum_{i=3}^{4}\frac{D_i e^{(m_1+r_i)y}}{r_i} - \sum_{i=1}^{2}\frac{D_i e^{(m_2+r_i)y}}{r_i} - \sum_{i=3}^{4}\frac{m_2 D_i e^{(m_2+r_i)y}}{r_i(3m_2 - \lambda)}\right\}\right]\cos \pi z \qquad (31)$$

Where

$$A = \frac{\lambda}{D(e^{m_1} - e^{m_2})(e^{n_1} - e^{n_2})}, \qquad B = \frac{\lambda^2 p_r^2}{D(e^{\lambda p_r^2} - 1)(e^{s_1} - e^{s_2})},$$

$$n_1 = \frac{1}{2}\left[\lambda + \sqrt{\lambda^2 + 4(\pi^2 + M^2)}\right], \qquad n_1 = \frac{1}{2}\left[\lambda - \sqrt{\lambda^2 + 4(\pi^2 + M^2)}\right],$$

$$s_1 = \frac{1}{2}\left[\lambda p_r + \sqrt{\lambda^2 p_r^2 + 4\pi^2}\right], \qquad s_2 = \frac{1}{2}\left[\lambda p_r - \sqrt{\lambda^2 p_r^2 + 4\pi^2}\right],$$

IV. RESULTS AND DISCUSSION

Fig-1 it is clear that the main flow velocity decreases with increases Hartmann number M, and injection parameterλWhile Cross flow velocity component w due to the transfer sinusoidal injection velocity distribution applied through out the porous plate at rest. The cross flow velocity profile is shown through the fig-2.

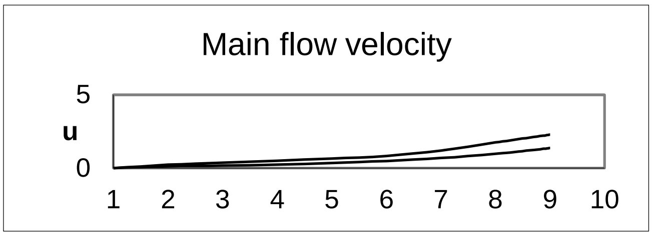

fig-1

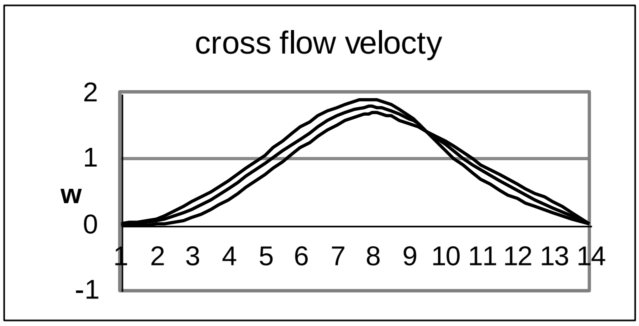

fig-2

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