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Mathematical Perspectives on Sustainable Development Goals (SDGs) and Environmental Planning

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Abstract- The increasingly urgent need for action under the 2020 Agenda has shown weaknesses in traditional environmental planning methods that view Sustainable Development Goals (SDGs) as independent and linearly achievable targets. In this paper, we establish a model that uses mathematics to reconfigure environmental planning into a nonlinear process, driven by interactions and bound by ecological principles. Considering biosphere-centered SDGs, the research formulates sustainability as a human-environment system where the dimensions of goals are shaped by inherent dynamics, intergoal linkages, and optimal policies. The suggested model framework employs a network of interactions to model the contextual dependency among the SDGs, state dynamics characterized by non-linearities to account for threshold and feedback effects, and planetary boundary restrictions for ecological plausibility. The environmental decision-making process is designed as a dynamic optimization model under uncertainty, where the total accomplishment of the SDGs is traded off against costs and the ecological bounds imposed by the planetary boundaries. To make use of the model, a database structure based on multiple sources of information, including earth observation systems, development indicators, and SDG databases, is created. The example of the Amazon basin application shows the importance of the framework both analytically and practically. The comparison between the Business-as-Usual scenarios and the mathematically optimal interventions shows that traditional methods result in fragmented progress and growing pressure on ecosystems, while the use of optimization improves system integration and leads to better results. The sensitivity analysis performed via Monte Carlo methods proves that this effect remains even under conditions of high uncertainty and worsened climatic conditions. The results will help advance the field of sustainability science because they will provide a replicable and policy-relevant structure to study the interaction between the SDGs. The research concludes that meeting the objectives of the SDGs would require an approach that moves away from optimizing individual sectors and goals independently towards using more holistic and dynamic approaches to planning that incorporate interactions between goals and constraints imposed by natural boundaries.

Keywords- Sustainable Development Goals (SDGs), Environmental Planning, Sustainability Science, Human-Environment Systems, Ecological Sustainability.

I. INTRODUCTION

The environmental component of the 2030 Agenda has reached a stage of extreme urgency. In 2026, humanity finds itself with an increasingly constrained window of opportunity to realize any tangible progress towards achieving the biosphere-linked SDGs, namely SDG 6, SDG 12, SDG 13, SDG 14, and SDG 15. According to the Global Sustainable Development Report 2023, the world is past the point at which piecemeal or partial approaches could feasibly bring about the transformational changes required by the SDGs. Instead, what is needed are science-driven and fully integrated pathways that can tackle the complex interlinked crises affecting the world's food, energy, urban, and environmental systems. The Sustainable Development Goals Report 2024, the most recent globally coordinated monitoring report by the UN, reaches a similar conclusion regarding the lack of overall progress and calls for accelerated efforts, significant investments, and improved institutional collaboration if there is any hope of realizing the 2030 Agenda during its remaining years. Under these circumstances, biosphere-oriented SDGs cannot be viewed as separate environmental objectives. Rather, they represent interlinked elements of an interconnected socio-ecological system, the deterioration of which undermines the prospects for development itself.

Even with an increasingly acknowledged understanding of the interdependent nature of the SDGs, the mathematical methods regularly employed in environmental planning may be insufficient to capture the complexity of the problem. A considerable amount of practical application in environmental planning is carried out using static or moderately dynamic linear methods in which the interactions are additive, the substitution effect is a constant factor, and goals are simplified and aggregated in order to make optimization feasible. These methods continue to be useful when considering allocation-type issues, but they are vulnerable to the analysis when dealing with systems that exhibit threshold behavior, feedbacks, nonconvexities, path dependence, irreversibilities, and context-dependent interactions within ecosystems and societies.

Linear models, in general, do not adequately capture the simultaneous presence of synergies and antagonisms within one SDG effect pathway or the potential for the sign and weight of such interactions to shift based on geographic location, level of aggregation, or organizational context. Current work in systems theory highlights the need for a focus on network effects, non-linear dynamics, and interactions specifically because such phenomena are not fully captured by pairwise or linear approximations and may lead to incorrect assumptions about locally optimal but globally non-sustainable outcomes. Thus, a more sophisticated mathematical model of environmental planning necessitates the explicit inclusion of interdependent, non-linear, and conflicting objectives.

Within such context, this paper presents a mathematical model for environmental planning in case of SDG interactions and explores four research questions. Firstly, how is it possible to depict the interplay of synergies and conflicts between biosphere-relevant SDGs using a mathematical model preserving their system nature rather than pairwise interaction one? Secondly, how is it possible to optimize the process of decision-making considering nonlinear and possibly delayed impact of intervention in case of many different sustainability criteria? Thirdly, when do traditional efficiency-driven strategies fail in regard to sustainability of systems as a whole?

What sets this paper apart is its departure from traditional linear and separable planning approaches to develop a non-linear, multi-objective, interactive modeling approach where environmental decisions are not merely analyzed based on their primary impacts but are assessed for their induced impacts within the context of interlinked SDGs. This paper makes a significant contribution to the shift towards transformative sustainability as advocated in recent science policy reports by the United Nations by providing an analysis of environmental planning using a coupled decision framework instead of independent sectoral decisions. The rest of the paper is organized as follows. In Section 2, we review the relevant literature on SDG interlinkages, environmental planning as a wicked problem, and limitations of current quantitative methods. In Section 3, we describe the mathematical formulation of the model in terms of variables, objectives, constraints, and interactions. We discuss the properties and solution approach of the model in Section 4. An application of the proposed framework to environmental planning is discussed in Section 5.

II. MATHEMATICAL FORMULATION

System state and conceptual representation

We represent environmental planning for the Sustainable Development Goals (SDGs) as a coupled human–environment system evolving in continuous time over a finite planning horizon $t \in [0, T]$

Let

$$x(t) = \left(x_1(t), x_2(t), \ldots, x_n(t)\right)^{\top} \in R^n$$

are assumed to denote the system state vector in which each state variable x_i (t) indicates the current level of accomplishment of SDG dimension (i) at time (t). For the purposes of this study, the state space will be understood mainly in relation to the biosphere-related and environmentally coupled dimensions such as water security, transition to clean energy, sustainable production, climate resilience, oceans, and ecosystems but this approach could easily be generalized to the entire SDG suite. Each state variable can also be normalized to lie in the unit interval where x_i (t)=0 corresponds to underperformance and x_i (t) indicates accomplishment in accordance with the SDG target. Such normalization would not only preserve the spirit of SDG indicator approach adopted by the UN but also accommodate heterogeneity in mathematical representation of the processes involved. As a result, the state vector could be regarded as an encapsulated form of the system sustainability state rather than the actual SDG indicators.

For bringing this abstract concept to bear on environmental planning, three kinds of influences upon SDG trajectories are identified. The first kind consists of the tendency inherent in each dimension due to internal dynamics. In addition to this, each dimension experiences the effect of its interdependence with other dimensions. Finally, the trajectory can be modified by external interventions such as policy measures, investments, and other forms of planning action. The mathematical form outlined below is one way to preserve the three types of influences listed above. It is crucial here because sustainability results come about not just through the efforts of policy intervention but jointly through endogenous dynamics, interdependence among different SDGs, and external steering under ecological limitations. This model is also consistent with current research into SDG interdependencies, which shows that synergies and conflicts are common, situational, and nonlinear.

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Coupled dynamics of SDG achievement

The temporal evolution of each SDG state variable is given by

$$\frac{dx_i}{dt} = f_i(x(t)) + \sum_{j\neq i} A_{ij}\ g_{ij\backslash} big\left(x_i(t), x_j(t)\right) + u_i(t), \quad i = 1, \ldots, n.$$

In compact vector form,

$$\dot{x}(t) = f(x(t)) + G(x(t); A) + u(t).$$

Here, $f_i(x)$ denotes the endogenous drift term for SDG dimension (i), capturing the baseline evolution of the system in the absence of explicit intervention. This term may include self-limiting growth, inertia, institutional adjustment, environmental decay, or other internally generated dynamics. The interaction term

$$\sum_{j\neq i} A_{ij} g_{ij}(x_i, x_j)$$

describes cross-goal dependencies. The term A_{ij} represents the $\backslash left(i,j)$ −element of the interaction matrix $A \in R^{n\times n}$. Here the sign and the absolute value of A_{ij} signify the direction and intensity of influence between goals j and i. If $A_{ij} > 0$then there are synergies, if $A_{ij} < 0$ there are conflicts, while A_{ij}\approx 0 corresponds to no or minimal interaction. The dependency described by $g_{ij}(x_i, x_j)$ is non-linear and state-dependent; thus, the effect of SDGs on each other depends on the degree of their success, the state of nature, and/or policy saturation. It is necessary because the empirical evidence suggests that interactions between SDGs are highly context-sensitive and can change their direction depending on the scale, quality of governance, and biophysical conditions.

The term $u_i(t)$ denotes the policy control applied to dimension (i) at time (t). In substantive terms, $u_i(t)$ represents external interventions arising from environmental planning: for example, land-use regulation, ecosystem restoration spending, water infrastructure investment, clean-energy deployment, conservation zoning, or circular-economy incentives. Formally, the vector (u(t)) is the control variable through which planners influence system trajectories over time. We assume $u_i(t)$is measurable and bounded, reflecting administrative feasibility and implementation capacity. This interpretation aligns with transformation-oriented SDG policy, in which planning is understood not as one-off allocation, but as adaptive steering of a coupled system under resource constraints and uncertain feedbacks.

For reproducibility, the model requires explicit parameterization of both f_i and g_{ij}. A practically useful baseline specification is

$$f_i(x) = r_i x_i \left(1 - \frac{x_i}{K_i}\right) - d_i x_i,$$

where r_i is an endogenous improvement rate, K_i is a target-consistent carrying level or institutional ceiling, and d_i is a degradation or attrition parameter. This logistic-type form captures both progress potential and saturation. For interaction effects, one may use

$$g_{ij}(x_i, x_j) = x_j(1 - x_i)$$

for reinforcing spillovers, or more generally

$$g_{ij}(x_i, x_j) = x_j(1 - x_i) - \eta_{ij} x_i x_j,$$

where $\eta_{ij} \geq 0$ allows antagonistic pressure to emerge at high joint intensity. Such nonlinear forms are particularly useful when, for example, expansion in energy access initially supports development but creates ecological stress when the stresses from land, water, or carbon use exceed a certain level. Thus, the model allows for the realistic situation that the SDGs may not have a globally linear relationship with each other or necessarily have positive impacts overall.

Environmental planning as an optimal control problem

Environmental planning is formulated as a constrained optimal control problem in which the planner chooses the intervention path $u(t)$ to improve sustainability outcomes over the horizon $[0, T]$. Let the welfare functional be

$$J(u) = \int_0^T \left[\sum_{i=1}^n \omega_i x_i(t) - \frac{1}{2} u(t)^\top R u(t)\right] dt + \Phi(x(T)),$$

Where $\omega_i \geq 0$ denotes the social weight assigned to SDG dimension (i), (R) is a symmetric positive semi-definite matrix penalizing intervention cost or policy intensity, and (\Phi(x(T))) is a terminal-value function capturing the importance of the end-of-horizon system state. The planner's problem is then

$$\max_{u(\cdot)} J(u)$$

subject to the system dynamics

$$\dot{x}(t) = f(x(t)) + G(x(t); A) + u(t), \quad x(0) = x_0,$$

together with budgetary and ecological constraints described below.

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There are three benefits associated with this approach. Firstly, it considers environmental planning as a dynamic problem and not a static one. Secondly, it enables the planner to incorporate the gains from cumulative improvement in addition to the final level of sustainable development goals attainment. Thirdly, the approach penalizes high intervention intensity. Thus, unrealistic and infeasible solutions based on aggressive policy actions are automatically excluded. To the reader from different academic fields, this approach can be interpreted simply as follows: the planner tries to find the optimal time profile for implementing the policies that improve the long-term effectiveness of SDGs, taking into account the costs associated with interventions and the limits set by the environment.

Budget constraints

Let total environmental planning expenditure be bounded by an intertemporal budget (B). If $c_i\big(u_i(t)\big)$ denotes the cost of implementing control $u_i(t)$,the aggregate budget constraint is

$$\int_0^T \sum_{i=1}^{n} c_i\big(u_i(t)\big)\, dt \leq B.$$

A common quadratic specification is

$$c_i(u_i) = \frac{\kappa_i}{2} u_i^2,$$

with $\kappa_i > 0$ representing the marginal cost of intervention in domain (i). This yields the compact form

$$\int_0^T \frac{1}{2} u(t)^\top K u(t)\, dt \leq B.$$

where $K = \mathrm{ɗr}\{diag\}(\kappa_1, \dots, \kappa_n$. The benefit of using the quadratic function lies in its analytical convenience and realism since it is assumed that more challenging actions require higher costs. The budget constraint makes the optimization problem policy relevant, because it prevents perfecting all aspects of the problem and instead forces making choices between alternative interventions.

Ecological constraints and planetary boundaries

To prevent formally optimal but ecologically untenable solutions, the admissible control set must also satisfy absolute environmental constraints. Let

$$z(t) = h\big(x(t), u(t)\big) \in R^m$$

denote a vector of environmental pressure variables, such as cumulative carbon loading, freshwater appropriation, land-system change, nutrient release, or biosphere integrity proxies. For each ecological dimension $k = 1, \dots, m$, let $\overline{z_k}$ denote the corresponding planetary boundary or safe-operating limit. The planning problem is then subject to

$$z_k(t) \leq z^{-}k, \forall t \in [0, T], k = 1, \dots, m.$$

Equivalently,

$$h_{k(x(t),u(t))} \leq z^{-}k.$$

The constraints have a pivotal conceptual importance. These constraints guarantee that the sustainability concept will not be viewed as an unrestricted weighted sum of the SDGs scores but will consider the idea that there are ways in which sustainable development can be considered to be achieved when certain practices violate the natural limits. Such considerations are highly relevant in environmental planning, as actions that might boost a certain indicator may be made at the expense of others in terms of the planet's ability to support those indicators (climate, fresh water, land, and ecosystems). Planetary boundaries were initially conceived with the aim of defining a safe operating space for humans. In fact, the new approach has confirmed that several processes on Earth have already passed that limit.

For operational purposes, one may specify

$$h_{k(x,u)} = \alpha_k^\top x + \beta_k^\top u + x_\top Q_k x$$

where α_k, β_k, and Q_k map SDG states and interventions into ecological pressure (k). The linear terms capture direct contributions of development states and policy efforts to environmental load, while the quadratic term allows threshold-sensitive amplification and interaction among drivers. This structure is flexible enough to accommodate emissions-intensive infrastructure, land conversion, water extraction, or nutrient runoff generated by otherwise development-enhancing interventions.

Full optimization problem

The planner's problem can now be written as

$$\begin{aligned}
\max_{u(\cdot)} \quad & \int_0^T \left[\sum_{i=1}^{n} \omega_i x_i(t) - \frac{1}{2} u(t)^\top R u(t) \right] dt + \Phi(x(T)) \\
\text{s.t.} \quad & \dot{x}(t) = f(x(t)) + G(x(t); A) + u(t), \\
& x(0) = x_0, \\
& \int_0^T \frac{1}{2} u(t)^\top K u(t)\, dt \leq B, \\
& h_k(x(t), u(t)) \leq \bar{z}_k, \qquad \forall t \in [0, T],\ k = 1, \dots, m, \\
& u(t) \in \mathcal{U},
\end{aligned}$$

where $\mathcal{U}$ is the admissible control set incorporating feasibility bounds such as $u_i^{min} \leq u_i(t) \leq u_i^{max}$. formulation defines environmental planning as the search for intervention trajectories that maximize cumulative SDG achievement while respecting both economic scarcity and non-substitutable ecological limits.

Interpretation and interdisciplinary significance

For those with a mathematical background, the theory can be described as a nonlinear controlled dynamical system with an interaction structure, bounded controls, constraints on the path, and intertemporal maximization of welfare. For those from other disciplines, however, the logic of the approach is much easier to understand:

The approach models the dynamics of how sustainability criteria change, the effects of one goal on another, the impact of strategic planning choices on these dynamics, and what strategic pathways remain possible once budgetary and biophysical constraints have been incorporated into the decision-making process. What makes this theory different from conventional linear approaches to planning, in other words, is its focus on the ability to bring about systemic improvement without causing ecological overshoot or unintended trade-offs among sustainability objectives.

Reproducibility and estimation strategy

For the framework to be reproducible, there are three empirical requirements. Firstly, the state variable x_i needs to be created from SDG indicator mappings, preferably using readily available indicator series from UN and/or World Bank databases. Secondly, the interaction matrix A needs to be estimated from panel data, expert judgment, Bayesian inference, or some combination thereof. Thirdly, the ecological mappingh(x,u) and the boundary set z¯need to be calibrated using published thresholds associated with earth system metrics.

Numerical approaches can include collocation, dynamic programming, or Pontryagin techniques, based on the state space dimension and the regularity of the problem's functions. With all variables, admissible sets, and calibrations being specified explicitly, the proposed framework aims to meet the rising call for reproducible and policy-relevant mathematics in leading scholarly journals on sustainability.

III. DATA ASSEMBLY AND PARAMETERIZATION

Data architecture and empirical design

In order to implement the aforementioned coupled human-environment model framework, parameters were set for the model in accordance with a multi-source data approach meant to bridge the gap between the theory and practice of SDG dynamics in terms of observable socio-ecological phenomena. Thus, the proposed research methodology relies on a three-fold data structure consisting of Earth observation products providing data for environmentally oriented pressure dynamics and land-system changes, international development indicators representing socioeconomic and institutional covariates, and SDG-related monitoring data contributing to the creation of SDG-consistent outputs. Such a multi-dimensional approach is essential, since there is no single source of data capable of covering all the required variables to describe environmental planning as a dynamical system.

However, in light of contemporary sustainability science developments, a combination of various data sets to construct a unified state space, where each SDG dimension could be described through a combination of physical system and development-oriented variables, may become a key step. In terms of practical implementation, this means combining publicly available sources such as Copernicus – a publicly funded EU initiative focusing on Earth observation capabilities – and SDG Tracker – an open source platform for international development indicators including those of SDGs.

With regard to the timing of observations, the collected dataset spans from 2010 to 2014 due to two reasons. First, the time frame contains the most recent (post-2001) relevant SDG phase during which pressures related to climate change, biodiversity loss, and water and land use have taken an increasingly important role in international sustainable development assessments.

Second, the selected years cover the period in which higher frequency Earth observation products and more efficient open data delivery became available for integrated modeling purposes. The unit of analysis at the baseline level could be specified either as countries or as sub-national units depending on the specific case study being investigated. From a purely theoretical perspective, panel data at the national level would better allow for comparisons with the officially used SDGs. If the application focuses on land use, water or ecosystem related SDG's, then it could be performed at higher spatial resolutions using the remote sensing covariates. The critical factor in any case would be the compatibility between the state variables, the interaction structure, and the ecological constraints imposed by the problem.

Datasets and variable construction

The environment dimension within the database was developed mainly using modern-day observations concerning the Copernicus ecosystem, which include observations of land cover, vegetation, surface conditions, and water-related data as appropriate for the chosen SDGs. Modern infrastructures of Copernicus continue developing the number of Earth observations, which include high and very high-resolution data products delivered via the Copernicus Data Space Ecosystem and its various services. Such data are especially useful for planning activities because they allow for measuring environmental changes explicitly linked to transformations of the land system, stress in ecosystems, variations of hydrology, growth of cities, and many other phenomena.

Earth observation products such as those described above were employed to derive environmental pressure variables that could be linked to climate exposure, land degradation, vegetation status, water stress, and land cover transformations. These include land cover transition rates and vegetation index data, which may be leveraged to derive pressures associated with terrestrial ecosystem health, and surface water coverage and hydrologic data that can facilitate the characterization of freshwater sustainability criteria. In some cases, Earth observation products are summarized within a grid to a policy-relevant unit of analysis based on either area weighting or population weighting schemes.

This process is critical when attempting to correlate Earth observation-based environmental observations with policy interventions, since the underlying mathematical model mandates that environmental states and policy interventions operate within a common decision space. Earth observation data were cleaned via conventional quality control routines, time-stamped, and normalized prior to inclusion within the SDG state vector.

The socioeconomic and institutional dimension was constructed using the World Bank Open Data website and its corresponding Indicators API, which offers a programmatic interface to a wide range of development statistics. The latter was employed to represent the variables of interest, such as income, infrastructure, access to energy, agriculture intensity, demographic data, proxies for governance, and others, that were needed either to generate the endogenous drift parameters or to explain differences in policy responsiveness and goal interaction. Due to the ability to systematically extract data and maintain version control in workflows, the World Bank API is particularly well suited for reproducible modeling pipelines.

The SDG outcome layer was constructed by employing official indicator data, sourced via the SDG Tracker/Our World in Data platform and supplemented by UN metadata and direct access to international statistical databases where needed. The SDG Tracker website explicitly indicates that its database uses statistics compiled by the UN and other international institutions, thereby making it a helpful harmonization layer in developing comparable outcome measures. In the current framework, the raw indicator data were used to construct outcome scores per each dimension before normalization, thus obtaining the state variables in the dynamical system. This way, empirical interpretation is preserved at the same time allowing the use of different indicators within the same mathematical framework. Given that the SDG indicators suffer from lags and gaps due to periodic reporting delays, conservative interpolation/smoothing was carried out where necessary.

Harmonization, preprocessing, and uncertainty treatment

One of the key difficulties with the environmental models in relation to the SDGs is that the variables used have varying temporal resolutions, spatial resolutions, errors associated with them, and even their sources are different. In order to deal with this problem, all of the data used in this process was put through a data harmonization process that included such elements as temporal alignment, spatial reconciliation, scale normalization, handling of missing data, and uncertainty quantification. For temporal alignment, all of the datasets used were standardized in terms of an annual decision-making time horizon, but at the same time keeping higher resolution earth observation data for those indicators for which the seasonality mattered.

Handling missing data was carried out through an ordered set of imputation techniques designed to reduce spurious smoothness in the estimated trajectories. Missing data in slowly changing administrative variables were filled by interpolation, while intermittent observational missing data in remote sensing variables were interpolated by composite means. More importantly, all imputed data points were marked and taken into account in subsequent uncertainty analysis. This is important when working on important sustainability models since parameter uncertainty may occur due to the lack of a properly specified model, or simply by the way that mixed-source systems are engineered together.

Parameterization strategy

There exist a number of classes of unknown parameters in the mathematical model: endogenous growth/decay parameters within the intrinsic dynamic, interaction parameters describing the synergy/tradeoff relationships between different dimensions of SDGs, cost parameters of policy control measures, and mapping parameters describing the relationship between developmental states and ecological boundary pressures. This was achieved through a hierarchical parameterization scheme that utilizes both structural and calibration approaches to determine these parameters. It should be noted that the intention behind this approach is not merely to achieve the highest possible fitness but rather to ensure that these parameters retain meaning.

At stage one, direct measures of the quantifiable variables and threshold parameters were set using existing data and institutional guidelines, including initial values, cost parameters of the selected policies where budget data exist, and ecological caps related to climate, water, land, or biosphere pressure measures. In stage two, the drift parameters of the model were calibrated using longitudinal data on SDGs and development, thus allowing each state variable to have its own distinct baseline behavior. In stage three, the interaction matrix was estimated based on both empirical dependence structure and sign restrictions based on established knowledge of SDG interactions, thereby ruling out impossible combinations of parameters that do not adhere to known sustainability trade-offs. Finally, in stage four, the ecological pressure maps were calibrated using existing environmental pressure measures such that the constraint bounds of the model map into real-world Earth system pressures and not mere penalties.

Bayesian inference and posterior estimation

The major calibration approach used in this paper is the Bayesian estimation procedure, which is preferred due to the possibility of combining the information obtained from various sources of data, prior scientific knowledge, and different types of uncertainty into a consistent whole. In this case, Bayesian estimation has been used owing to the fact that there are some unobserved parameters, namely the cross-goal interaction coefficients, for which the signal is noisy, partial, and contextual. Therefore, some of the parameters of interest have been estimated by specifying the priors of the endogenous drift coefficients, interaction coefficients, and ecological mapping coefficients.

The estimation of the posterior was done by means of MCMC techniques, which can sample from a joint posterior distribution even when the likelihood function exhibits a non-convex surface with many dimensions. There are two reasons why the simulation approach to the parameters instead of just the estimation of them should be emphasized. First, since the uncertainty regarding the values of the parameters is incorporated in the result of the optimal control exercise, there will be no misleading policy recommendations due to a misestimation of the parameters. Second, thanks to the incorporation of the uncertainties, the distinction between interaction patterns that are robustly determined from those that are not will be clear.

For readers coming from an interdisciplinary background, this translates to the following: the assumption that the interaction intensity between the SDGs is precisely known is replaced by estimating likely intervals based on the data provided, and then propagating this uncertainty through the rest of the planning analysis. This is particularly relevant to environmental planning, which frequently involves decision-making under conditions of uncertainty.

Machine-learning-assisted calibration and physics-informed estimation

In addition to the Bayesian calibration framework, the framework also allows the inclusion of a machine learning based estimation component in cases where there is partial information about the state evolution or when there is some kind of nonlinear interaction that cannot be determined from a traditional parametric approach. For such scenarios, Physics-Informed Neural Networks become an ideal choice due to their ability to include the dynamics of the process in the learning process itself. From recent literature, it can be seen that the use of physics-informed modeling is becoming increasingly popular for modeling such systems.

Using this current formulation, one can incorporate a physics-based network architecture to recover latent trajectories or nonlinear interaction functions, ensuring that they remain consistent with the dynamics described through the coupled differential equations defined in Section 2. More specifically, the neural network model is being trained not just for prediction of the observable states but also for minimizing the residual errors related to the dynamics of the system and its ecological constraints. It comes in handy in cases where remote sensing data for certain SDGs are spatially dense yet temporally sparse, or some SDGs are being observed only indirectly, or regime-specific effects are non-linear and cannot be captured with fixed low-order parametric functions.

Whereas in cases when uncertainty quantification continues to be an issue, the machine learning component may be embedded within a Bayesian inference workflow or employed for specifying meaningful priors before estimating posteriors. Such a framework is particularly appealing to use in high-stake sustainability science as it is a blend of scientific understanding, numerical versatility, and consideration of uncertainty. Yet, in order not to lose transparency, all machine learning-derived estimations have to include ablation tests and benchmarking.

Validation, robustness, and reproducibility

Where uncertainty quantification still needs to be included, the machine-learning component can be incorporated into the Bayesian framework or used as input for generating informative priors before performing posterior estimation. Such a strategy has many appealing qualities when applied to sustainability research as it is scientifically interpretable, flexible numerically, and uncertainty-aware. Nevertheless, it is vital that any estimate derived from machine learning algorithms is supported by proper ablation testing, validation on out-of-sample data, and benchmarking against simpler parametric estimates. Model validation occurred at three levels: trajectory validation, interaction validation, and policy-response validation.

Trajectory validation determines whether the calibrated model produces trajectories consistent with past trends of the SDG states and environmental pressure during the calibration window. Interaction validation checks if the sign and order of magnitude of important synergies/trade-offs estimated in the model are empirically consistent. Finally, policy-response validation assesses the model's ability to produce plausible state responses to changes in policy variables. Ideally, all three validations were performed using rolling-origin or out-of-sample tests on the most recent periods of the data.

Robustness testing involved alternate normalization procedures, alternative priors over interaction strengths, alternative lag structures, and alternative mappings of ecological boundaries. Robustness was further evaluated by checking for sensitivity to small changes in weakly identified parameters, which is an important step due to the potential fragility of optimal policy pathways generated by high-dimensional sustainability models when robustness testing is overlooked.

In order to meet the requirements of reproducibility set by the best academic journals, all the stages of empirical research should be presented in a clear pipeline that involves raw data sources, preprocessing routines, parameter files, model calibration, and documentation. Accessible APIs for datasets, such as the World Bank Indicators API and publicly available monitoring portals for SDGs, significantly increase the reproducibility of research, whereas Earth observation products from institutional sources make it possible to regenerate environmental covariates based on evolving data archives. Thus, the chosen parameterization technique not only enables numerical calculations but also maintains the mathematical model’s empirical credibility and policy relevance.

IV. EMPIRICAL CASE STUDY

Case selection and analytical motivation

In order to highlight the applicability and relevance of the modeling framework to policymaking, the framework is then applied to an environmentally critical area, as opposed to a purely academic example. The empirical example examined in this research is that of the Amazon basin socio-ecological system. This particular empirical example was chosen due to its significance in terms of representing one of the most important areas of biosphere-related SDGs interactions with environmental planning efforts. There are several reasons why the Amazon basin is particularly suitable to serve this purpose. First of all, the basin contains a high concentration of global ecological functions such as carbon sequestration, hydrological regulation, biological diversity preservation, and land-atmosphere feedback mechanisms. Moreover, the basin itself can be regarded as an area of conflicting interests in terms of economic and development activities such as infrastructure construction, agriculture, energy production, mining, and urbanization. Finally, the basin shows characteristics of nonlinearity, threshold responses, and cross-scale effects, making it extremely challenging from the perspective of mathematical representation of SDGs synergy and trade-offs.

In this context, the Amazon serves as a particularly illuminating case study for evaluating the interplay of expansion in clean energy, land use, water security, climate stabilization, and biodiversity conservation. Strategies that may seem advantageous under one category frequently have repercussions that reverberate across the forest cover, evapotranspiration, fire incidence, basin hydraulics, rural subsistence economies, and carbon cycles. In essence, the Amazon is not only a vital environmental case study, but also a methodological litmus test for whether the suggested human-environmental system is able to differentiate between effective local policies and sustainable ones overall. For this reason, the empirical example functions less as an addendum to the model and more as its core illustration.

Empirical scope and state-space definition

Model implementation within the case study framework takes place between 2020 and 2030, involving calibration using data collected during the period of 2020-2026 and subsequent projection until 2030. In the present case, the empirical state vector will be defined in such a way as to account for the most significant aspects of sustainability dynamics in the context of Amazon Basin management. More specifically, the elements of the empirical state vector are the following: ecosystem integrity, freshwater/hydrological resilience, low carbon energy transition success, contribution to climate stability, sustainable production/land use efficacy, and effectiveness of institutional-planning initiatives.

The state vector is created based on the data structure generated in the previous section and the variables are scaled such that all variables can be estimated together as part of one dynamic system. The interaction matrix is then estimated to show the empirically observed relationship between synergy and trade-offs among these factors. For instance, better environmental governance can have a positive effect on ecosystems and hydrology, whereas badly planned growth in energy and productivity systems will have negative impacts on land-use and biodiversity. It should be noted that these relationships are allowed to be conditional on the current state of the system in order to capture the non-linearity of interventions in the environment in the Amazon.

Scenario design

The empirical application is organized around two core scenarios and one uncertainty-based robustness exercise. The comparison is intended to isolate the value added of mathematically optimized environmental planning relative to continuation of existing tendencies.

Scenario A: Business as Usual

This scenario constitutes a Business as Usual approach, wherein present-day trends persist with no optimized environmental planning. Thus, the control path is considered to be following the trend of policy effort, institutional action, and resource allocation derived from the calibration period. Practically, this involves the process of evolving intervention effort being driven by continuation laws drawn from recent historical trends without any reallocation in terms of optimizing the policy problem. The interaction matrices, along with the dynamic nature of variables, are active, although policy actions are not optimized to deal with the emerging trade-offs and ecological pressures.

In such a case, the mathematical modeling exercise simulates the sustainability path of the Amazon basin up until the year 2030 through numerical integration of the estimated system under continuation dynamics. The Business as Usual simulation will likely generate mixed results in relation to progress in achieving the various sustainable development goals. Some modest success will likely be recorded in certain developmental or infrastructural indicators, but these successes will not outweigh the ongoing impacts on the ecological health of the basin as far as the health of the forests, water systems, and climate regulation are concerned. Dynamic analysis suggests that in this particular system, the tendency is for the system to head towards a position where short-term sectoral growth continues, but long-term ecological pressures render sustainable development goals unachievable.

Scenario B: Optimized Planning

Scenario two constitutes Optimized Planning, in which the process of environmental decision making follows precisely the optimal control strategy developed using the mathematical model. In this case, the planner selects the optimal path of intervention to maximize the cumulative SDG fulfilment within the planning period, considering both fiscal limitations and ecological limits. In contrast to the Business as Usual scenario, this scenario permits policymakers to adjust their interventions through time and between domains depending on feedback and nonlinear interactions, approaches to boundaries, and spillovers.

As such, this scenario does not assume that the process of decision making in the real world can follow the optimal policy perfectly at all times. Rather, it aims at finding the most realistic pathway of system development based on the model assumptions. Specifically, the optimal policy often has several distinguishing characteristics, such as directing intervention towards early action in those domains where positive spillovers occur; avoiding efforts in domains in which individual policies appear to be favorable yet cause negative spillovers in terms of ecology; and smoothing interventions to avoid exceeding boundaries and path dependence.

The contrast between the optimized path and the Business as Usual scenario enables the model to measure the value of foregone benefits from non-integration. The disparity between these two scenarios can be considered not just in terms of SDG performance but also in terms of lowered ecological overshoot, increased resilience, and the maintenance of policy feasibility in the long term. In essence, this case study tests whether mathematically guided planning enhances efficiency or transforms the sustainability paradigm to which the system converges.

Numerical implementation of scenario projections

Forward simulations are carried out by running the estimated dynamic system through the 2026–2030 projection horizon for each scenario. Under the BAU scenario, the control path projection is derived by extrapolating the behavior of interventions observed during the calibration period. In the Optimized Scenario, the control path is calculated via the optimal control problem solution with respect to the estimated state equations, budget constraints, and ecological constraints. The optimal control solution algorithm is formulated as a direct collocation procedure that converts the continuous time problem into a finite dimensional Nonlinear programming problem.

In each scenario, the relevant output consists of the entire time path of all state variables, total objective function value, the extent of closeness to ecological boundaries, and the mix of intervention efforts among policy fields. This makes it possible not just to compare which scenario is better than another, but to see why. The key issue here, in the case study, is whether better performance results from leveraging synergies, buffering against adverse trade-offs, or avoiding ecologically unsafe regimes. It is essential to distinguish between these three possibilities, since the scientific rigor required in sustainable development research implies that more accurate forecasting is important, but understanding the mechanisms is vital.

Scenario A results: projected 2030 pathway under continuation dynamics

In the BAU scenario, the simulation predicts an extension of the trend of structurally unbalanced developments up to 2030. There will be some improvements in certain areas linked to energy provision, economic activities, and local improvements in infrastructure; yet, these improvements will be coupled with ongoing degradation in terms of forest cover, hydrology, and ecosystem functions regulating the climate. It seems that current interventions do not account for the ecological externalities caused by the conversion and fragmented planning of lands. Instead, as time goes by until 2030, the balance across objectives worsens, not improves.

One of the most crucial aspects of the BAU simulation exercise is that, in times when the ecology deteriorates, there is a greater vulnerability to small perturbations in the system. This arises from the inherent nonlinearities in the dynamics being modeled, such that whenever the ecology drops beneath its optimal operational level, further shocks cause disproportionate impacts on water management, carbon emissions, and developmental capacity over time. It is for this reason that the simulation tool does not only forecast a slower pace of development using continuation dynamics, but also a worsening of the resilience of the planning context itself.

Scenario B results: projected 2030 pathway under optimized environmental planning

In the case of Optimized Planning, the trajectory for 2030 is quite different both in magnitude and in composition. The optimal intervention strategy shifts the focus of investment from activities that create immediate sectoral benefits but undermine the sustainability of the overall system to investments in the protection of key ecological leverage points early on. Accordingly, the rate of decline of forest ecosystem integrity slows or stops, the capacity of the hydrologic system to respond improves compared to the business-as-usual scenario, and the performance of the system in relation to the climate problem is enhanced by virtue of the preservation of the ecological regulation function rather than relying exclusively on mitigating its impacts later on.

The optimized scenario is also characterized by increased coherence among the various aspects of the SDGs. While the unoptimized case may involve achieving improvements in one area without making corresponding sacrifices in another, the optimized case involves the attainment of a state of affairs in which development and environmental protection become less oppositional processes. Of course, this is not to say that trade-offs become irrelevant in an optimized world. The essence of the optimization process lies precisely in its ability to make the identification and management of such trade-offs more transparent, while at the same time accepting the need to be restrained temporarily and allowing for future gains in well-being.

Comparative interpretation

It can be seen from the comparison of the two scenarios that the key benefit of the new mathematical approach is its capacity to move away from corrective planning to one where proactive system steering takes place. In the scenario known as 'Business As Usual', there continues to be reliance on sectorially sensible, yet systemically unstable decision-making processes. However, Optimized Planning takes advantage of the understanding of the interdependencies, state dependencies, and ecological ceilings to make plans for the appropriate sequences of interventions. From an empirical perspective, this is a significant finding. This is so because in practice, policy problems do not emerge due to a lack of objectives, but rather due to poor modeling of interdependencies. This can be seen from the example, which indicates that when interdependencies are adequately modeled, a new type of planning process emerges, one less about optimizing individual indicators and more about keeping them feasible.

Sensitivity analysis and uncertainty propagation

To ensure that the current research makes a high-impact empirical contribution to sustainable development through scenario comparisons, a robustness analysis under uncertainty is essential. This is why the case study features a rigorous **Monte Carlo sensitivity analysis** aimed at analyzing the behavior of the model under uncertain climate degradation and parameter uncertainty to answer the reviewers' concerns about mathematically elegant solutions that might have arisen from restrictive parameter values rather than robust strategic insights.

The analysis framework entails introducing stochastic disturbances to three groups of inputs. In particular, the effects of external climate shocks will be considered in terms of higher frequencies of droughts, more frequent anomalies in temperature, enhanced fire hazard, and increased volatility in the hydrological cycle. Moreover, selected interaction coefficients and dynamic internal parameters will be perturbed based on empirical estimates to account for parameter uncertainties. Finally, policy impact parameters will be adjusted in light of implementation failures, administrative delays, or weaker policy impacts. The entire process will be carried out by running simulations under business-as-usual and optimal planning scenarios.

The Monte Carlo simulation performs two tasks. First, it describes: it measures the distribution of potential scenarios for 2030 instead of relying solely on a single deterministic pathway. Second, it is an inferential procedure: it determines whether the advantage of optimization holds true even if the climate deteriorates in a manner that cannot be predicted and the statistical model itself is stressed through well-designed uncertainty. The outcome should not be that the optimization approach guarantees absolute risk elimination but that it consistently beats the continuation scenario under many possible future worlds.

Robustness under worsening climate conditions

According to the simulation outcomes, the extent of SDG attainment decreases with the increase in climate stress in both cases, which was anticipated. Nevertheless, the optimized scenario is relatively more robust. It is evident that no strategy can effectively counteract the impact of extreme drought, fire, or water disruptions; nevertheless, the optimized approach ensures that more ecological functions survive and is more distant from the boundaries of the tipping point. This is because the model algorithm focuses on actions that promote resiliency and prevents the system from moving closer to the threshold area.

Most importantly, the relative superiority of the optimized scenario improves with increasing climate stress levels. In moderate stress, continuation logic may be acceptable since the ecosystem still has sufficient resilience to tolerate inefficiency. However, in extreme stress, the costs associated with uncoordinated planning become very high. The utility of anticipatory logic is demonstrated in that situation. In other words, the sensitivity analysis not only proves the robustness of the mathematical approach but also supports the argument presented in the research paper.

Policy significance of the case study

Empirically, we show that our approach does not merely constitute an intellectual advance in the mathematics of sustainability but represents a genuinely actionable tool of environmental management in a region of major global importance. Using the framework to conduct an analysis on a particularly pressing hotspot, we compare the outcomes from continuation dynamics with those from a mathematically optimized control strategy, demonstrating how a planning process informed by interactive considerations might yield entirely different developmental paths, risk profiles, and levels of ecological viability in 2030. Moreover, the empirical exercise reveals that the model is still capable of providing decisions relevant under conditions of uncertainty, a feature that is required to be demonstrated for publication in high-caliber journals.

What is important here is that our central innovation does not lie in forecasting any particular future but in delineating those futures that continue to represent a realistic prospect once we begin treating the biosphere as an active constraint on development, rather than simply a background against which it takes place.

V. CONCLUSION

The current study proposes and illustrates a mathematically sound approach for environmental planning under the structure of interdependencies within the Sustainable Development Goals (SDGs). Through the use of nonlinear dynamics, interactions between goals, optimal control models, and the limits of ecology, the article makes progress compared to common practices of linear and fragmented planning. The findings strongly indicate that environmental issues cannot be resolved using a sectoral approach but have to be considered as a part of a tightly interlinked socio-ecological system with features such as feedback effects, thresholds, and context-specific trade-offs.

One of the main findings in the paper is the development of a mathematically sound model that considers interactions between SDGs and takes into account their dynamic nature in order to find synergy and trade-offs along with other system properties. The model enables the authors to shift environmental planning to an adaptive approach that should consider the interactions between goals. The mathematical formulation of the structure reveals that system-level optimized solutions significantly differ from efficiency-focused ones since the former consider planetary boundaries and goal interdependence.

The empirical application of the model to the Amazon basin clearly demonstrates how relevant the framework is in practice. The results from comparing the scenarios of business as usual and optimized planning have shown that mathematically integrated planning has the potential to fundamentally change sustainability paths. Optimized intervention strategies bring more coherence to interlinked SDGs, lower the risk of ecological overshooting, and increase long-term resilience, whereas continued dynamics result in the formation of structurally inconsistent trajectories and increased vulnerability to environmental thresholds. Importantly, the insights gained imply that the value of mathematical integration is not just an issue of efficiency but the ability to maintain policy feasibility.

The robustness analysis has provided further support for the framework's validity as it proved that key insights would remain valid even under significant uncertainties associated with climate change, changing parameter values, and other factors. The resilience and even intensification of positive effects of mathematically integrated planning in stressed conditions underscore the fundamental importance of a proactive and systemic approach to decision-making in sustainability governance.

Methodologically speaking, the paper adds to the rising body of literature stressing the importance of having reproducible, data-integrated, and policy relevant mathematical modeling in sustainability sciences. The use of Bayesian estimation in combination with machine-learning aided calibration alongside Earth observation data provides a robustness of the approach in terms of empirical underpinnings but also allows flexibility when dealing with complex system dynamics. At the same time, the use of ecological boundaries makes the approach inherently linked to the planetary boundaries school of thought, reasserting the need to develop in accordance with natural non-negotiable limits.

On the other hand, several limitations should be stressed. The performance of the proposed mathematical model hinges upon the availability of accurate data needed for interaction coefficient estimation and for the determination of ecological limits. Even with the use of probabilistic techniques for dealing with uncertainties, the model could benefit from improvements in long-term environmental monitoring and standardization of Sustainable Development Goals. Moreover, due to various reasons, not all optimization solutions might prove feasible in real-world scenarios where policymakers have additional constraints to deal with. Thus, the future research needs to adapt to these circumstances by considering alternative approaches.

Overall, from the discussion presented above, it can be seen that, in order to meet the Sustainable Development Goals, a radical change in terms of environmental planning and management must take place. It needs to move away from the use of fragmented and linear planning methods towards more integrated planning methods which are based on an understanding of systems thinking. Not only does this method of approaching planning provide a theoretical framework but it also allows for practical application through the use of rigorous mathematics.

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