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A Critical Review of Small-Strain Stiffness and Liquefaction Behavior of Sands: Insights from Bender Element and Local Small-Strain Measurements

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Abstract — The small-strain stiffness of sands is an important characteristic in geotechnical earthquake engineering because it governs the first response of soil deposits to dynamic loads. This measure also provides important information on the stress condition, liquefaction susceptibility, density and soil fabric. Conventional triaxial testing, relying on external displacement measurements, is often unable to discern the very modest strains associated with the initiation of the stress–strain response. The development of bender element testing and local strain measurement techniques has substantially improved the characterization of sand stiffness over very small to small strain ranges. Local strain transducers directly measure the stress–strain stiffness and its deterioration with increasing strain. Bender elements offer an indirect estimate of the maximum or very-small-strain shear modulus, Gmax or Gmin, by measuring shear-wave velocity. This review critically assesses the theoretical basis, experimental interpretation and limitations of the two alternative approaches, with special regard to their consequences for liquefaction behavior. The review has a significant impact on effective stress, void ratio, fabric, particle properties, stress anisotropy and saturation circumstances. A high small-strain stiffness does not necessarily imply a high liquefaction resistance, but provides useful insight into the initial condition of a sand and can be related to the cycle resistance, when the state variables are appropriately normalized. Recent work on silty sands has shown that normalized small-strain stiffness can be a useful link between laboratory shear-wave data and the assessment of liquefaction triggering. However, care must be taken to consider the uncertainties related with wave arrival-time interpretation, near-field effects, specimen boundary conditions, membrane compliance, bedding errors, and differences between dynamic and quasi-static strain amplitudes. A more reliable characterization is obtained by combining bender element measurements with local strain measurements and cyclic testing, rather than depending on one single parameter. The review continues with a discussion of research needs in the areas of state-dependent stiffness, fines effects, anisotropy, fabric evolution and the development of unified stiffness-liquefaction frameworks.

Keywords— small-strain stiffness; sand; liquefaction; bender elements; local strain measurement; shear-wave velocity; cyclic resistance; triaxial testing.

I. INTRODUCTION

The mechanical response of sands under seismic loading is very nonlinear and situation dependent. At small stresses the behavior of a sand deposit is almost elastic and it is mainly controlled by the stiffness of the interparticle connections and by the effective stress in the soil framework. The increasing strain causes significant contact slippage, particle rearrangement, changes in fabric and progressive softening. Under undrained cyclic loading these mechanisms can result in increased pore-water pressure and liquefaction in particularly susceptible sands. Thus the small-strain shear modulus is an important parameter in earthquake engineering, site-response evaluation, soil-structure interaction and liquefaction analysis. The early study by Hardin and Richart (1963) empirically showed that the wave velocity in granular soils is strongly controlled by the effective confining stress and the void ratio.
The results they obtained formed an important basis for the modern view of shear wave velocity as a measure of the stiffness of granular soils. Recent study has demonstrated that the elements such as stress state, density, fabric and particle properties have to be taken into account when evaluation is made (Hardin and Richart, 1963; Viggiani and Atkinson, 1995). A major experimental difficulty is that standard external

displacement measurements cannot accurately estimate stresses less than about . These measurements are affected by bedding errors, device compliance, membrane bulging and seating effects. Thus, local measuring procedures were developed for direct measurement of deformation in the central region of a specimen. Localized strain measurements are found to give much higher and more reliable stiffness estimates than global specimen deformation measurements, especially at low strain levels, according to studies. The auxiliary method is Bender element (BE) testing. In a traditional BE system, shear waves are generated and detected by piezoelectric elements at either extremity of a soil sample. The observed travel time provides an estimate of the shear-wave velocity, from which the small-strain shear modulus can be obtained as: where is the bulk density of the soil.

Bender elements are widely used because they provide a non-destructive measurement of stiffness at very low strain levels. The first research using the approach to soil analysis were by Dyvik and Madshus (1985). Later studies were focused on the interpretation of the signal and the variables affecting the accurate estimation of wave-velocity (Lee and Santamarina, 2005).This paper seeks to conduct a comprehensive analysis of the link between small-strain stiffness measurements and liquefaction features of sands, highlighting the extra information derived from bender elements and localized strain measurements.

II. FUNDAMENTAL CONCEPT OF SMALL-STRAIN STIFFNESS

Definition

The main shear stiffness of soil can be expressed in terms of shear modulus: Where is shear stress found? What is shear strain? Many sands show an almost elastic response at small strain amplitudes, often below around to . The associated modulus is usually referred to as the maximum shear modulus or the very-small-strain shear modulus. Derived from shear-wave velocity in the assessment of wave propagation: This relationship links the geophysical assessment directly to mechanical stiffness. The importance of is more than just its numerical value. It represents the stiffness of the inter-particle contact network and is therefore sensitive to the effective stress and soil structure. Hardin and Richart (1963) demonstrated that shear-wave velocity increases with confining pressure and is a function of void ratio. Modern study has verified that density, stress anisotropy, particle shape and grading can significantly impact.

Stress dependency

A well known result in granular soil mechanics is that the small strain stiffness increases with effective stress. The universal formulation is: where is a material constant, is a function of the void ratio, is the mean effective stress and is a stress dependent exponent. The original work of Hardin and Richart (1963) demonstrated a quarter power type association between wave velocity and confining pressure for the sands examined. As is proportional to , the corresponding modulus shows an enhanced dependence on effective stress. This dependency on stress has major consequences for liquefaction research. Two samples of the same sand can only have different values because they have been consolidated under different effective strains. A simple juxtaposition of unstandardized values can therefore be misleading.

Bender Element Testing

Principle

Bender elements are elongated piezoelectric elements embedded in the end platens of a triaxial or oedometer device. One part acts as transmitter and the other as receiver. When an electrical pulse is applied to the transmitter it bends and generates a shear wave which propagates through the specimen. The receiver picks the incoming wave. The basic calculation is: where is the distance travelled by the wave is the time taken for the travel. The final shear modulus is then determined as: Hence, Bender elements evaluate the soil stiffness at strain amplitude levels far lower than those attainable in normal mechanical tests. This method has been widely adopted because of its simplicity and compatibility with traditional triaxial apparatus. Presently, BE testing is still used for saturated sands, including those with varying fines content.

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Interpretation of wave arrival time

BE signals are conceptually simple. Uncertainty as to their interpretation is one of the main sources of uncertainty. Usually, the signal received is formed of several components, rather than just one pulse which is clearly identifiable. Frequency dependent propagation, cross-talk, reflected waves and near-field phenomena might make the first arrival hard to identify. Lee and Santamarina (2005) highlighted the dominant effect of the setup of the experiment and the interpretation of the signal on the estimated shear modulus and the computed transit time.

Local Small-Strain Measurements

Need for local measurements

In standard triaxial testing the axial displacement is normally measured exterior to the loading plates. For small stresses, this procedure may lead to large errors, as the measured deformation included not only the deformation of the soil but also bedding, seating, apparatus compliance and deformation outside the center region of the specimen. In this context, local strain measurements are used, which determine the deformation directly at the specimen, generally at mid-height. For this purpose local axial LVDTs, proximity sensors and various high-precision instruments are developed. Studies on the measurement of local lateral strain showed that localized equipment can be used to continuously monitor the modulus and Poisson's ratio throughout an exceptionally broad range of strain, from around to several percent.

New experimental research also shows that local strain measurements are much more accurate than global displacement measurements for the range of strains that matter for small-strain stiffness.

Advantages over external measurements

The main advantage of the local strain evaluation is the possibility to determine the whole deterioration stiffness curve not one single value. This difference is important. Bender elements can evaluate stiffness at relatively low strain levels, however the strains experienced by engineering structures are often larger than the BE strain limit. And then, as stress is ramped up, stiffness falls off. This allows to determine the location of the transition from the mostly elastic region to the nonlinear stage. For instance, the path-dependent small-strain rigidity and the stiffness degradation have been defined for various stress paths by local strain measures in the investigations on Toyoura sand.

Relationship Between Bender Element and Local-Strain Measurements

These two techniques are not mutually exclusive methodologies.

They rather present some parts of the soil reaction.

Characteristic

Component: Bendr

Regional deformation evaluation

Basic metric

Shear wave speed

Physical structural modification

Measurable deformation primary variable

tension amount

tiny

Minimal to moderate

Loading category...

Kinetic wave

Quasi-static / cyclic

Main advantage

Direct measurement of elastic wave

No stiffness

Simple ambiguity

Wave arrival interpretation

Instrument calibration and localization

Importance of liquefaction

Preliminary rigidity sign / condition

Periodic and non-linear reaction

Best used in interpersonal relationships

Recent comparative evaluation of liquefiable sands has suggested that shear-wave velocities obtained from bender-element and resonant-column tests can be different for which relative density could be an influencing factor. Crucially, these discrepancies can be reduced by employing an appropriate void-ratio function.

III. FACTORS CONTROLLING SMALL-STRAIN STIFFNESS OF SANDS

Void ratio and relative density

The major aspect affecting is the void ratio. More dense sand has a higher number of particle interactions, which often yields a more durable contact framework. However, the relative density is not sufficient to describe the state of the soil completely. Two sands with the same relative density might have quite different stiffnesses due to differences in their minimum and maximum void ratios, particle shapes, and gradation properties. Recent research on liquefiable sands have emphasized the need for state based normalization since raw relative density may not fully represent material variability.

Effective stress

An increase in the effective stress increases the normal contact forces of the particles and generally increases the velocity of the shear wave. This link is important for understanding the results of BE and for providing an explanation for the fluctuation of the field shear-wave velocity with depth. The comparison of laboratory and field measurements without considering the stress dependency may lead to wrong findings on liquefaction risk.

Fabric and anisotropy

Sand is really a function of texture. The stiffness depends on the orientation of the particles, on the distribution of contacts, and on the history of deposition. Stress anisotropy is of major importance, since natural deposits are rarely isotopically cemented. Laboratory studies have shown that this reaction is affected by alterations under anisotropic stress settings and particle shape and grading. ScienceDirect Thus, an isotropic lab sample may not reproduce the stiffness qualities of an in-situ sand formation.

Grain characteristics
The interparticle interaction stiffness is influenced by the particles angularity, their mineral composition, surface texture and gradation. Angular particles usually have more interlocking than rounded particles, however the relationship between particle form and interlocking is not always simple. Gradation can impact the packing density and contact coordination, and thus the stiffness. The relevance of these impacts is heightened when the laboratory results are extended from typical sands such as Toyoura or Ottawa sand to genuine sedimentary formations.

Fines content
It involves fines, which complicates things. Nonplastic silt can fill the spaces between the sand grains, alter the packing of the grains and affect the structure that supports the weight. If the fines content is modest they may fill the holes without taking over the force network. If the fines content is high the mechanical behavior may be similar to that of a fines-controlled structure. New investigations on sand-silt mixtures reveal that the amount of fines, the gradation of the host sand and the stress state consistently affect both the resistance to liquefaction and the. The study recommended that the cyclic resistance and the small-strain stiffness should be associated in the view of changes in the state of soil.

IV. SMALL-STRAIN STIFFNESS AND LIQUEFACTION

Conceptual relationship
Liquefaction occurs when the steady loading causes a large increase of the excess pore-water pressure, thus reducing the effective stress. A simplified form of an effective-stress correlation is: The effective stress is shown as σ' , the total stress is shown as σ and pore-water pressure is shown as u. The effective stress decreases as the excess pore pressure increases. It decreases because the small-strain stiffness is stress-dependent. Cyclic liquefaction can therefore be partly viewed as an evolving stiffness problem: As cyclic loading generates effective tension and soil structure. However, the initial value should not be seen as a direct indicator of resistance to liquefaction.

Why high stiffness does not automatically mean high liquefaction resistance
Generally, compact sands are more resistant to liquefaction than loose sands and there is a definite relationship between stiffness and liquefaction resistance.
However, the resistance to liquefaction depends on additional factors:

relative density or condition factor;

effective stress;

cyclic stress ratio ;

Seismic occurrences magnitude and length;

exhausting situations;

preliminary static shear stress;

Therefore, it has to be taken as a state-dependent measure, but not as an independent liquefaction criterion.

Linking to Cyclic Resistance
The cyclic stress ratio (CSR) and the cyclic resistance ratio (CRR) are two common indices for evaluating liquefaction triggering. In simple terms: denotes the initial vertical effective stress and the cyclic shear stress. The common factors associated with the liquefaction triggers include for a certain reference earthquake magnitude and number of loading cycles. SPT and CPT measurements are an important part of routine field operations. Commonly used CPT-based liquefaction triggering procedures were developed by Boulanger and Idriss, aided by a large case-history database and corrections for different variables such as size and fines content. Now we have plenty of accurate shear-wave measurements (ASCE Library +1) so that we can include stiffness-related parameters in liquefaction evaluations.

Based liquefaction assessment
The great advantage of shear-wave velocity is that it provides a direct measure of small-strain stiffness.

Field-based liquefaction correlations have been developed on the basis that susceptible loose deposits are generally characterized by a relatively low shear-wave velocity in terms of effective stress and soil condition. Laboratory analyses have supported this idea. Recent work on sand-silt composites showed a link between the normalized small-strain shear modulus and cyclic resistance, and proposed liquefaction triggering thresholds based on this association.

Stiffness Degradation During Cyclic Loading
The more strain, the more fluid the sand. The universal normalized stiffness equation can be written as:

- where the function depends on the soil condition, stress history and loading trajectory.
- Local strain assessments are particularly important as they allow for the experimental acquisition of this function.

The degradation process is further aggravated by cyclic loading:

- accumulation of excess pore pressure;
- re-organization of communication;
- of fabrics anisotropy;
- periodic motion; and
- possible stress concentration.

Therefore, the rigidity determined in the first cycles can be somewhat different than the rigidity measured when reaching liquefaction.

This indicates an important limitation of a single BE test performed before cyclic load. It indicates the initial stage, but not the complete evolution of soil stiffness during liquefaction.

Saturated Sands and Experimental Complications
Bender element tests are more difficult to perform on totally soaked sands than on dry or marginally saturated sands.

Water affects wave propagation, signal properties and the interaction between the solid skeleton and the pore fluid. Investigations on saturated sands have clearly identified differences in the stated small-strain modulus under saturated and dry conditions and have investigated wave dispersion using pyroclastic concepts.

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A reliable evaluation therefore calls for careful consideration of:

- degree of concentration;
- pressure residual;
- Skempton's -value.
- draining conditions;
- frequency of stimulation;
- measurements of the specimen;
- electrode configuration; and
- Distance of wave propagation.

Also the perceived wave velocity and hence the assessment of might be affected by insufficient saturation.

V. CRITICAL EVALUATION OF EXPERIMENTAL LIMITATIONS

Bender element limitations
Although BE testing has many advantages, it has its disadvantages too. Wave dispersion and near-field effects can lead to a discrepancy between the geometric tip-to-tip distance and the wave travel distance.

Secondly, the criterion used for the signal identification may influence the arrival time measurement.
Third, boundary reflections can taint the receiving signal.
Fourth, BE testing examines a dynamic reaction with a very modest strain, but many geotechnical problems entail much bigger forces.
Finally, the cyclic resistance is not quantified directly. Therefore, a stiffness value obtained via BE should not be regarded as an independent liquefaction resistance characteristic.

Local strain limitations
Local strain systems also have limitations.
The transducers must be carefully calibrated and positioned without disturbing the specimen. Radial strain, especially, could impact the data due to membrane contacts. In addition, local strain field may be inhomogeneous under severe loading condition. Despite these limitations, local strain measurement is still among the most effective methods for identifying the real stiffness degrading features of soil. Recent investigations show that localized assessments can provide stiffness values consistent with extremely low-strain elastic measurements, but also indicate the onset of nonlinear behavior. Springer

Integrated Testing Strategy
A broad experimental effort should include three synergistic measurements:

Phase 1: Bender Element Tests
Calculate and evaluate:
under varying effective stress situations.

Phase 2: Local strain measurement
Use cyclic loading with monotonic or small amplitude and measure local axial and radial deformations.

This gives:

Phase 3: cyclic liquefaction test
Perform undrained cyclic triaxial tests and measure:

cyclic stress ratio;

pore pressure ratio;

axial deformation;

shear deformation;

number of cycles to fire; and

Deterioration of stiffness.

Then the three data sets can be merged to find correlations between:

The comprehensive approach is more scientifically justifiable than attempting to infer liquefaction resistance from a single BE test.

Implications for Natural and Silty Sands
Application of these approaches to the natural soils should be done with special care.

Standard laboratory sands are often homogeneous and very clean, whereas natural deposits may contain particles, organic material, cementation, ageing effects and heterogeneous fabric. Silty sands deserve special attention since their behavior can be very different from pristine sands. Recent experimental evidence indicates that state normalization can boost considerably the consistency of correlations. Both small-strain stiffness and liquefaction resistance are affected by fines content and host-sand gradation.

VI. RESEARCH GAPS AND FUTURE DIRECTIONS

There are many important gaps in the research.

Cohesive model for stiffness liquefaction
The relationship between soil conditions and CRR is well known but no framework has been developed that can be used for a universal integration. Future studies need to combine laboratory measures with field case studies.

Textiles Development
Most BE assessments define the original construction. Further work is needed to understand how fluctuates under cyclic stress, and whether real-time wave assessments can detect the onset of liquefaction.

Consequences for Penalty
The behavior of sand-silt mixtures cannot be predicted using clean-sand correlations in a consistent way. Penalties that fill gaps have to be distinguished from penalties that fit into the load bearing framework.

Directional Dependence
Nature reserves are not isotropic. Future experimental efforts should incorporate BE assessments with controlled stress-path tests for estimating directional stiffness.

Sophisticated regional machinery
Improved local strain measurements, digital image techniques, and miniature sensors may be able to capture the entire stiffness deterioration process more accurately.

Hybrid and Standard Models for Machine Learning
Data-driven models can be significantly improved with large datasets containing density, stress conditions, fines content and cycle resistance. Such techniques should be bounded by soil mechanics, rather than supplanting physical interpretation.

VII. CONCLUSIONS

This comprehensive review demonstrates the significance of small-strain stiffness in the understanding of the behavior of sand under dynamic and cyclic loading. Although offering a lot of information on the non-linear loss of stiffness beyond the very small strain range, the local strain measurement is not as efficient and sensitive as bender element testing for determining shear wave velocity and maximum shear modulus. You can draw a few conclusions.

First, the stiffness of all sands is not solely characterized by density, but is strongly affected by effective stress and void ratio. Moreover, grading, stress anisotropy, fabric and particle morphology have to be considered.

Second, bender elements and local strain measurements do not provide contradicting data, but rather complementary data. Local measurements give the stiffness-strain relation over an extended range of engineering strains, BE testing describes the elastic response at extremely small strains.

Third, high should not be considered as an indication of liquefaction resistance. Rather, it is typically an indication of a stiffer state of soil. Cyclic loading, effective stress, density/state, pore-pressure generation and fabric development interactions are the controlling factors for liquefaction.

Fourth, current investigations indicate that normalized small-strain stiffness and cyclic resistance can be strongly associated, particularly when the effects of changes in fines content, host-sand gradation, effective stress and void ratio are accounted for.

Finally, for the most precise characterization of liquefiable sands, bender element measurements, local strain measurements and cyclic triaxial tests need to be included. This approach provides a continuum from the initial elastic response, to stiffness deterioration, pore-pressure accumulation and liquefaction. Future research should focus on the development of empirically confirmed state-dependent frameworks relating cyclic resistance, stiffness degradation and natural sands and sand-silt mixtures.

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