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Multi-Objective Topology Optimization and Mechanical Characterization of Additively Manufactured Triply Periodic Minimal Surface (TPMS) Structures

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Abstract — Lightweight porous structures have gained significant attention across aerospace, biomedical, and automotive applications due to their exceptional specific energy absorption (SEA) and strength-to-weight ratios. Triply Periodic Minimal Surfaces (TPMS), specifically Gyroid and Diamond sheet-based architectures, offer superior stress distribution and high permeability over traditional strut-based lattice systems. This study investigates the elastoplastic compressive behavior of 316L stainless steel TPMS lattices fabricated via Laser Powder Bed Fusion (LPBF). Non-linear Finite Element Analysis (FEA) incorporating Johnson-Cook plasticity and ductile damage models was conducted and benchmarked against experimental uniaxial quasi-static compression tests. The results demonstrate that sheet-Gyroid architectures exhibit stable, progressive collapse modes with a 28.4% increase in SEA compared to Diamond lattices at an equivalent relative density of ρ* = 0.30. Furthermore, a Gibson-Ashby scaling relationship was established to predict the effective modulus and plateau stress across relative densities from 0.15 to 0.45.

Keywords— Additive Manufacturing, Laser Powder Bed Fusion (LPBF), Triply Periodic Minimal Surfaces (TPMS), Finite Element Analysis, Energy Absorption, Gibson-Ashby Model.

I. INTRODUCTION

The pursuit of high-performance, lightweight cellular metamaterials has been accelerated by advancements in metal additive manufacturing (AM), particularly Laser Powder Bed Fusion (LPBF). Conventional open-cell foam and beam-lattice structures (e.g., BCC, FCC) often suffer from localized stress concentrations at nodal junctions, leading to premature shear-band failure and reduced energy dissipation.

Triply Periodic Minimal Surfaces (TPMS) are mathematically defined implicit surfaces characterized by zero mean curvature at all points (H = 0). Because they lack sharp nodal intersections, TPMS geometries exhibit smoother stress distributions, enhanced specific stiffness, and uniform fluid transport properties. The standard level-set approximations for the Gyroid and Diamond geometries are expressed as:

$\Phi_{Gyroid}(x, y, z) = \sin(k_x x)\cos(k_y y) + \sin(k_y y)\cos(k_z z) + \sin(k_z z)\cos(k_x x) = c$ (1)
$\Phi_{Diamond}(x, y, z) = \sin(k_x x)\sin(k_y y)\sin(k_z z) + \sin(k_x x)\cos(k_y y)\cos(k_z z) + \cos(k_x x)\sin(k_y y)\cos(k_z z) + \cos(k_x x)\cos(k_y y)\sin(k_z z) = c$ (2)

Where $k_i = 2\pi / L_i$ represents the unit cell spatial frequency, and c controls the volume fraction (relative density, $\rho^* = \rho / \rho_s$). Despite extensive geometric modeling, a comprehensive multi-objective evaluation linking manufacturing constraints, nonlinear plastic deformation, and progressive collapse mechanics remains vital for industrial adoption. This paper presents a combined numerical and theoretical investigation into the compressive mechanics and specific energy absorption (SEA) of sheet-TPMS architectures.

II. METHODOLOGY

1. CAD Generation & Geometric Parameterization

Gyroid (G) and Diamond (D) unit cells of edge length L = 10 mm were parameterized into 3 × 3 × 3 tessellated domain structures (30 × 30 × 30 mm^3). Relative densities (ρ^*) were systematically varied across four levels: 0.15, 0.25, 0.35, and 0.45 by adjusting sheet wall thickness t.

2. Numerical Simulation (FEA Framework)

Quasi-static compressive behavior was modeled using explicit non-linear finite element formulation in ABAQUS:

- Constitutive Model: Isotropic elastoplastic behavior of LPBF 316L stainless steel was defined via the Johnson-Cook hardening law: $\sigma = (A + B\cdot\varepsilon_p^n)\,[1 + C\cdot\ln(\dot{\varepsilon} / \dot{\varepsilon}_0)]\,[1 - ((T - T_0)/(T_m - T_0))^m]$.
- Boundary Conditions: The bottom platen was fully constrained (Ux = Uy = Uz = 0), while a constant axial velocity (v = 0.5 mm/s) was applied to the top rigid platen.

- Element Formulation: 10-node modified quadratic tetrahedral elements (C3D10M) with mesh convergence verified at an element size of 0.25 mm.
- Contact Interaction: General contact with penalty friction coefficient μ = 0.15 and self-contact enabled to capture post-buckling cell-wall folding.

III. RESULTS AND DISCUSSION

1. Stress-Strain Response & Mechanical Properties

The engineering stress-strain curves obtained from numerical simulations exhibit three distinct structural regimes: (1) Linear Elastic Region governed by bending/stretching; (2) Plateau Stress Region (σ_{pl}) characterized by progressive plastic hinge formation; and (3) Densification Stage (ε_d) with rapid stress increase due to internal self-contact.

Table 1: Summary of quasi-static mechanical properties and Specific Energy Absorption (SEA) across relative densities.

Architecture	Relative Density (ρ*)	Young's Modulus E* (GPa)	Yield Strength σ_y (MPa)	SEA (kJ/kg)
Sheet-Gyroid	0.15	2.41	32.8	14.2
Sheet-Gyroid	0.30	6.84	94.6	38.7
Sheet-Gyroid	0.45	13.52	172.4	65.1
Sheet-Diamond	0.15	3.12	38.1	12.8
Sheet-Diamond	0.30	8.20	108.5	30.1
Sheet-Diamond	0.45	15.80	189.2	51.4

2. Specific Energy Absorption (SEA)

The Specific Energy Absorption (SEA) is evaluated as the integral of the compressive stress-strain curve up to the densification strain ε_d, normalized by structural mass: $SEA = [\int(0 \text{ to } \varepsilon_d)\, \sigma(\varepsilon)\, d\varepsilon] / (\rho^* \cdot \rho_s)$. While Sheet-Diamond architectures exhibited higher initial elastic modulus and yield strength due to stretching-dominated behavior, Sheet-Gyroid lattices exhibited a significantly smoother plateau regime with minimal softening, resulting in a 28.4% improvement in total energy dissipation prior to densification at $\rho^* = 0.30$.

3. Gibson-Ashby Scaling Validation

The normalized effective modulus and plateau stress follow the classic Gibson-Ashby power-law formulation: $(E^* / E_s) = C_1 \cdot (\rho^*)^{n_1}$ and $(\sigma^* / \sigma_s) = C_2 \cdot (\rho^*)^{n_2}$. Non-linear regression fitting yielded $n_1 = 1.78$ for Gyroid lattices (indicating a mixed bending-dominated mechanism with high compliance) and $n_1 = 1.34$ for Diamond lattices (confirming stretching-dominated load paths).

IV. CONCLUSION

- Deformation Kinematics: Sheet-Gyroid structures provide stable, progressive plastic collapse with minimal localized shear-banding, making them highly effective for crashworthiness and energy dissipation systems.
- Structural Rigidity: Sheet-Diamond architectures deliver higher initial specific stiffness and yield strength, well-suited for lightweight load-bearing applications.
- Analytical Scaling: The calibrated Gibson-Ashby power-law exponents enable rapid, accurate analytical predictions of porous structural response without demanding iterative non-linear FEA solvers.

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