International Journal of Scientific Research & Engineering Trends
Volume 12, Issue 3, May-Jun-2026, ISSN (Online): 2395-566X

Analysis of Cosmological Constant in Bianchi Type-1 With Cosmological Model

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Abstract- This Paper analyzed the effects of the cosmological constantΛ in the context of the Bianchi Type I cosmological model. The Bianchi Type I model represents an anisotropic but spatially flat universe, where expansion rates can differ along three spatial directions.This study analyzes the effects of the spatial directions with respect to axes . The cosmological, plays a significant role in the universe expansion. This work aims to understand how influences the expansion, energy density, and anisotropy of the universe. Einstein's field equations with variable cosmological constant if considered in the presence of a perfect fluid for a Bianchi type I universe by assuming that the cosmological term is proportional to the square of the Hubble parameter. The variation law for vacuum density was recently proposed by many researches on the basis of the quantum field estimation in a curved expanding background. The cosmological term tends asymptotically to a genuine cosmological constant and the model tends to a de-Sitter universe. More obtained some new results by using a slightly different method from that of other researchers obtained the result that the present universe is accelerating with a large fraction of cosmological density in the form of a cosmological term.

Keywords- Bianchi type 1 cosmological model anisotropic and cosmological constant.

I. INTRODUCTION

In this paper explore the cosmological constant problem the most remarkable observation in recent time is the cosmological constant problem, which is very interesting to all researchers. The cosmological constant Λ was originally given by Einstein in his field equations. In an evolving universe, it appears no natural to look at this constant as a function of time term Λ anises naturally in general relativistic quantum field theory, where it is interpreted as the energy density of the vacuum [1-4]. Some author [5-3] argued for the dependence of Λ . Cosmological models with variable $G\ and\ \Lambda$ have been studied by a number of researchers [10-17] for a homogenous and isotropic FRW line element.

Also Bianchi type-I models are studied by using variable $G\ and\ \Lambda$ [18-24] Schutzhled [25,26] recently proposed that the vacuum energy density is $\propto$ Hubble parameter, which leads to vacuum energy density decaying as $\Lambda \approx$ as $m2H$, where $m \approx 150\ MeV$ if the energy scale of the Chiral phase transmission of QCD. Also, Borges and Cormenio [07] have considered an Isotropic and homogenous flute spore fielded with matter and a cosmological tem αH obeying the equation of state of the vacuum recently Tiwari and Divya Singh [20] investigated the anisotropic Bianchi type I model with a varying term.

Tiwari and Sonia [29] investigated the non-existence of shear in Bianchi type-III String cosmological models with bulk-viscosity and time dependent Λ. Also Tiwari and Sonia [30] investigated the Bianchi type-I string cosmological model with bulk viscosity and time dependent Λ term and Mukesh and Sbmir investigated A study of Bianchi Types I cosmological model with cosmological constant [ ] for studying the possible effects of anisotropy in the early universe on present day observations many researchers [3-35] have investigated Bianchi type I models from a different point of view. In the present paper, we investigate the homogenous and an anisotropic Bianchi type I space time variable Λ containing matter in the form of a project fluid. We obtain the solution of the Einstein field equation assuming that cosmological term is proportional to Hubble parameter for stiff matter.

II. MODEL AND FIELD EQUATIONS

The line element for spatially homogeneous and anisotropic Bianchi type I space-time is described by

$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2$ (2.1)
Where, A, B, C are functions of t only.
We assume that cosmic matter is represented by the energy momentum tensor fluid

$T_{ij} = (P + \rho)V_iV_j + Pg_{ij}$ (2)

where P and ρ are energy density and thermodynamic pressure, and Vi is the four velocity vector of the fluid satisfying the relation

V_iV^i=-1.

We assume that the matter content obeys an equation of state

$p = \omega\rho, 0 \le \omega \le 1$ (3)

The Einstein's equations with varying A in suitable units are

$R_{ij} - {}^1 Rg_{ij} = -T_{ij} + \mathsf{A}\, g_{ij}$ (4)

Spatial volume V as an average scale factor of the model (1) be defined as

$V = R^3 = ABC$ (5)

Hubble parameter H in anisotropic models may be defined as

$$H = \frac{R^{\cdot}}{R} = \frac{1}{3}\left(\frac{A^{\cdot}}{A} + \frac{B^{\cdot}}{B} + \frac{C^{\cdot}}{C}\right)$$ (6)

where a dot stands for ordinary time derivative of concerned quantity

$H = 1\ (H_1 + H_2 + H_3)$ (7)

. . .

Where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factory in the x, y and z directions, respectively.

For the metric (1) and energy moment tensor (2) in the) communing system of coordinate, the field equation (4) yields:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\rho + \Lambda \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -\rho + \Lambda \quad (9)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\rho + \Lambda \quad (10)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = -\rho + \Lambda \quad (11)$$

In view of the vanishing divergence of the Einstein tensor, we have:

$$\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] + \dot{\Lambda} = 0 \quad (12)$$

$\sigma_{ij} = u_{ij} + u_{j,I} - \frac{2}{3} g_{ij} u_k^k$ are obtained as:

$+ C^{\cdot})$ (13)

1 3 A

3 B C

$\sigma 2 = 4\ B^{\cdot}$

$-\ 2\ (C^{\cdot} + A^{\cdot})$ (14)

2 3 B

3 C A

$\sigma 3 = 4\ C^{\cdot} - 2\ (A^{\cdot} + B^{\cdot})$ (15)
3 3 C 3 A B

$\sigma 3 = 4\ C^{\cdot}$

$-\ 2\ (A^{\cdot} + B^{\cdot})$ (15)

$A^{\cdot\cdot}$

International Journal of Scientific Research & Engineering Trends
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3

$B^{\cdot\cdot}$

3 C

$A^{\cdot}B^{\cdot}$

3 A

$B^{\cdot\cdot}$

B

$C^{\cdot\cdot}$

$B^{\cdot}$ $C^{\cdot}$

$A + B + AB - (B + C + BC) = -\rho + A - (-\rho - A)$

$A^{\cdot} - C^{\cdot}$

$=$ $K2$

$= K2$

(22)

$A^{\cdot\cdot}$

A C

$B^{\cdot\cdot}$ $A^{\cdot}B^{\cdot}$

ABC

$A^{\cdot\cdot}$

$R3$

$C^{\cdot\cdot}$

$A^{\cdot}C^{\cdot}$

$A + B + AB - (A + C + AC) = -\rho + A - (-\rho - A)$

$B^{\cdot}$

B

$- C^{\cdot}$
C

$=$ $K3$
ABC

$K3$

$R3$

(23)

Where $K1, K2, K3$ are constants of integration. We now assume energy conservation equation $Tij = 0$ yields
$\rho + \rho(1 + \omega) (A^{\cdot} + B^{\cdot} + C^{\cdot}) = 0$ (24)
A B C

Using equation (5) and (24), we get
$K4$
$R3(Hw)$

where $K4$is the constant of integration. Again integration equation (21)-(23), we get

(25)

$A = m1\ exp\ exp\ (K1 \int 1\ dt)$ (26)

$A = m2\ exp\ exp\ (K2 \int 1\ dt)$ (27)

$B = m3\ exp\ exp\ (K2 \int 1\ dt)$ (28)

where $m1, m2, m3$ are integration constants from equation (20) we obtain

$3\sigma2 = 1 - 3\rho - 3\Lambda$

(29)

$\theta2$

$\theta2$

$\theta2$

implying $\Lambda \geq 0, 0 \leq \sigma2 \leq 1 , 0 < \rho - 1$

$\Theta2$ 3 $\Theta2$ 3

Thus the presence of positive A lowers the upper limit of anisotropy whereas a negative value of A given more room for anisotropy.

Equation (29) we can be written as

$3\sigma2 = 1 - \rho - \Lambda$

$3H2$
$= 1 - \rho$
ρc

$3H2$
$- \rho v$
ρc

$3H2$

(30)

where ρc=$3H2$ is the critical density and ρv =A is the vacuum density.

$d\Theta$

dt

$= 3\ (\rho - p) - 3\sigma2$ (31)
2

Showing that the rate of volume expansion decreases during time evolution and presence of positive A slows down the rate of decrease whereas a negative A would promote it. from equation
(19) (20).

$\Lambda = (2 - q)H2 - (1-\omega)\rho$
2

(32)

This implies $\Lambda \leq 0\ for\ q \geq 2$.

The system of Eq" and Eq" (8)-(11) are five independent eq" in six unknowns A, B, C, ρ, P and A. therefore, me extra condition is needed to solve the system completely. for this we take the cosmological term proportional to the Hubble parameter since many authors considered it as A decay, Schutzholde consider

variation law for vacuum density, Borges and Carnerio [27], R.K. Tiwari, Divya Singh and Sonia [28] have considered a cosmological term proportional to H. Thus we take the decaying vacuum energy density.
$\Lambda = \beta H2$ (33)
where, β is the positive constant.
let Ω =A/ρ be the ratio between the vacuum and matter densities from eq" (19) & (23) we get

$\beta =$ 3n 1+n

(1 − σ2

International Journal of Scientific Research & Engineering Trends
Volume 12, Issue 3, May-Jun-2026, ISSN (Online): 2395-566X

27θ

) (34)

Thus the value of β in an anisotropic background is smaller in comparison to its value in isotropic.
For stiff fluid $\omega = 1$, Eqn (18) (19) (33) lead to a differential equation:

$H^{\cdot} + (3 - \beta)H2 = 0$ (35)

Integrating we get:

$R = [(3 - \beta)(C1t + C2)]1/3-\beta$ (36) where $C1 and\ C2$ are the constant of integration.

$H = R^{\cdot}$

R

$H = C1[(3 - \beta)(C1t + C2).\ ]-1$ (37)

1

$A = [(3 - \beta)(C1t + C2)]3-\beta$

$exp\ exp\ [$ $2k1 + k2$

1]

$(6\{(3 - \beta)(C1t + C2)\}3-\beta$

1

$B = [(3 - \beta)(C1t + C2)]3-\beta$

$exp\ exp\ [$ $k2 - k1$

1]

$(3\{(3 - \beta)(C1t + C2)\}3-\beta$

1

$C = [(3 - \beta)(C1t + C2)]3-\beta$

$exp\ exp\ [$ $2k2 - k1$

3]

$2\{(3 - \beta)(C1t + C2)\}3-\beta$

1

$ds = -dt + [(3 - \beta)(C1t + C2)]3-\beta$

$exp\ [$ 2k1−k2

3] $dx2$

$3\{(3-\beta)(C1t+C2)\}3-\beta$

$+exp\ [$ 2k2−k1

3] $dy2 +$

$\{(3-\beta)(C1t+C2)\}3-\beta$

$exp\ [$ 2k2−k1

3] $dz2$

$\{(3-\beta)(C1t+C2)\}3-\beta$

For this model, the matter densityρ , Pressure P, cosmological terms Λ, shear scalar σ and expansion scalar θ are given by:

−6

$\rho = p = k4\{(3 - \beta)(\ (C1t + C2)\}3-\beta$

$A = \beta C2\{(C1t + C2)\}-2$

$\theta = H = C1\ \{(3-\beta)(C\ t + C$

$)\}-1$

3 3 1 2

$-3C2$
$\sigma = (C1t + C2)$ 1
$3-\beta\ C3$

vacuum and matter densities is given by

$\Omega = \rho$ 1

$\beta C2$
$=$ $k4$

$\{(3-\beta)(C1t + C2)\}2\beta/3-\beta$

the deceleration parameter q for this model is

$q = 2-\beta$

The vacuum energy density ρv and the critical density $\rho c$are given by

ρv

ρc

$= \beta C2\{(3-\beta)(C1t + C2)\}-1$

$= 3C2\{(3-\beta)(C1t + C2)\}-2$ (38)

Spatial volume

$V = R3 = \{(3-\beta)(C1t + C2)\}3/3-p$ (39)

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