

# Analysis & Design of Antenna Array Using Windowing Technique

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**Abstract-** In this paper a new class of adjustable window function is proposed using a combination of Tangent hyperbolic function and Blackman-Harris 4-term window function. To derive the Tangent Hyperbolic Window function, the authors used the scaled independent variable Tangent hyperbolic functions shifted in opposite directions. The proposed window has the advantage of having 4-shape parameters that have lot of flexibility to vary the shape of the window for the desired spectral characteristics. The performance is compared with Hamming, Hanning, Kaiser and Gaussian windows in terms of the First Null Beam Width, Main Lobe Beam Width, Ripple ratio and Sidelobe roll-off ratio for the same window length with other windows presented for comparison. Simulation results show that Tanh window combined with Blackman-Harris window provides better sidelobe roll off characteristics and other spectral metrics that may be useful for some applications such as filter design and beamforming. Moreover, the paper presents the application of the proposed window in the field of array synthesis, and the comparison is performed with Hamming, Hanning, Kaiser and Gaussian windows. The results show that the array design with Tanh-Blackman-Harrish window provides better results in terms of the spectral metrics such as First Null Beam Width, Main Lobe Beam Width, Ripple ratio and Sidelobe roll-off ratio.

**Keywords –** Tanh window; 4-term Blackman-Harris window; Kaiser window; Hamming window; Hanning window Kaiser window; Gaussian window functions; window shape parameters; Array design; First Null Beam Width; Main Lobe Beam Width; Ripple ratio; Sidelobe suppression.

## I. INTRODUCTION

Window functions (or simply as windows) are widely used in digital signal processing for the applications in signal analysis and estimation, digital filter design and other signal processing areas. In literature many windows have been proposed [1-4]. But since they are suboptimal solutions, the best window depends on the application it is used. The signal data abrupt truncation causes side lobes. The gradual tapering of the signal smooths the signal and reduces the amplitude of the sidelobes in signals like radar or antennas. Ideally, the window spectrum should have a narrow main-lobe and small side-lobes. However, there is an inherent trade-off between the width of the main-lobe and the side-lobe attenuation. A wide main lobe will average the adjacent frequency components and larger side lobes will introduce contamination (or spectral leakage) from other frequency regions. For rectangular windows, the main lobe is narrower than that of the Hamming window, while its side-lobes are higher. The important spectral characteristics of a window function are (i) main-lobe width (ii) ripple ratio and (iii) its side-lobe roll-off ratio.

Windows can be categorized as fixed or adjustable [4], both types of window functions are used in signal processing and data analysis. The fixed window function like Hanning and Hamming having one parameter “N” (length). Adjusting the length changes the main lobe width, but other spectral parameters are fixed for a given window type. Adjustable window functions like Kaiser and Dolph-Chebyshev have two or more parameters that allow for more control over spectral properties like sidelobe levels and main lobe width. Due to the flexible spectral properties of windows, in the recent past various windows based on the exponential and their combinations have been proposed [5-11] for many signal processing applications. Since the windows are sub-optimal solutions, every window is used according to the specifications required [11].

In this work the authors proposed an adjustable window function that is a combination of scaled and shifted versions of Tanh window and Blackman-Harris 4-parameter window [4] to get higher sidelobe roll-off ratio. The proposed window is compared with Hamming, Hanning, Kaiser and Gaussian windows in terms of the window spectral Metrics viz.(i) First-Null width (FNW) (ii) main-lobe width (MLW) (iii) ripple ratio

(R) and (iv) side-lobe roll-off ratio (S). This work also attempts to investigate the performance of the window when applied to antenna array synthesis.

This paper is organized in the following sequence. The overview of the windows is discussed in section I. Different window functions used in the synthesis are presented in section-II. The derivation of the proposed Tanh widow is presented in section-III. In section-IV window performance Analysis is presented. Antenna Array synthesis using the proposed window is presented in section-V. In section-VI results and discussions are presented. Conclusion is presented in section-VII.

## II. WINDOWS

An N-length window, denoted by  $w(nT)$ , is a time domain function which is nonzero for  $n \leq |(N-1)/2|$  and zero for otherwise. They are generally compared and classified in terms of their spectral characteristics [4]. Some of the most widely used windows are summarized below.

### A. Hamming window:

Hamming window is the most popular and most used window function. This window function also belongs to cosine window family. Hamming window function can be expressed as follows,

$$w(n) \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & \text{for } 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \quad \text{-----(1)}$$

where N is the window length, the starting and ending points of Hamming window do not touch the time-axis. This means that the coefficients of the Hamming window are always greater than zero.

### B. Hanning window:

Hanning window function also belongs to cosine window family. Hanning window function can be expressed as follows,

$$w(n) \begin{cases} 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right) & \text{for } 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \quad \text{-----(2)}$$

where N is the window length, the starting and ending points of Hanning window tapers smoothly to zero with the time-axis. This means that the coefficients of the Hanning window are always in between zero and one. The proposed window also touches time-axis symmetrically.

### C. Blackman-Harris Window:

Blackman-Harris [4] is a fixed window and is defined mathematically as

$$w_B(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) +$$

$$a_2 \cos\left(\frac{4\pi n}{N-1}\right) - a_3 \cos\left(\frac{6\pi n}{N-1}\right) \quad \text{for } 0 \leq n \leq N-1$$

$$0 \quad \text{Otherwise} \quad \text{-----(3)}$$

Table 1: Blackman-Harris 4-term constants

Blackman-Harris-4-Term (-92 dB SLL) Window constants			
$a_0$	$a_1$	$a_2$	$a_3$
0.35875	0.48829	0.14128	0.01168

This family of Blackman-Harris window with minimum 4-term can achieve a sidelobe level of -92 dB. The listed coefficients correspond to the minimum 4-term window is presented in Table-1.

### D. Kaiser Window:

The Kaiser window is an adjustable window with two parameters, N the length of the window and a shape parameter “ $\alpha_k$ ” to vary the spectral characters.

It is expressed mathematically as,

$$w(n) = \begin{cases} \frac{I_0\left(\pi\alpha_k \sqrt{1 - \left(\frac{2n}{N-1} - 1\right)^2}\right)}{I_0(\pi\alpha_k)} & \text{for } 0 \leq n \leq N-1 \\ 0 & \text{other wise} \end{cases} \quad \text{-----(4)}$$

Here “ $\alpha_k$ ” is an adjustable shape parameter, and  $I_0(x)$  is the modified Bessel function of the first kind of order zero and it is described by the power series expansion as

$$I_0(x) = 1 + \sum_{k=0}^{\infty} \left[ \frac{1}{k!} \left(\frac{x}{2}\right)^k \right]^2 \quad \text{-----(5)}$$

where  $I_0(\cdot)$  is the zeroth-order modified Bessel function of the first kind. As “ $\alpha_k$ ” increases, the main lobe width widens, and the side lobe attenuation increases. For “ $\alpha_k$ ” = 0, the Kaiser window is a rectangular window. For “ $\alpha_k$ ” = 5.3, the Kaiser window is close to a Hamming window. The two parameters, that are the sequence length “N” and the shape parameter “ $\alpha_k$ ” are useful to obtain the desired amplitude response pattern of the Kaiser window. For fixed “N”, as the shape parameter “ $\alpha_k$ ” increases, the side lobe level of the magnitude response decreases at the cost of main lobe width. In this work,  $\alpha_k = 4.5$  is used to generate the Kaiser window to study the performance analysis of the proposed window with Kaiser window.

### E. Gaussian Window:

The Gaussian Window [4] has also two parameters like Kaiser window, the length of the sequence “N” and a shape parameter “ $\alpha_g$ ”. The Gaussian window is defined by (6). Here, as the parameter “ $\alpha_g$ ” increases, the main lobe width widens, and the side lobe attenuation increases.

$$w(n) = \begin{cases} e^{-\frac{1}{2}[\alpha_g \left(\frac{2n}{N-1} - 1\right)]^2} & \text{for } 0 \leq n \leq N-1 \\ 0 & \text{other wise} \end{cases} \quad \text{-----(6)}$$

For  $\alpha_g = 3$ , the Gaussian Window shape almost exactly matches with the proposed window function shape as seen in the performance analysis.

**F. Spectral Properties of Windows:**

Window merits and demerits are generally compared and classified in terms of their spectral characteristics of the frequency response of the window function called frequency Spectrum. The frequency spectrum of any window function  $w(nT)$  can be given by (7).

$$W(ej\omega T) = |A(\omega)| e^{j\theta(\omega)} = w(0) 2 \sum_{n=1}^{(N-1)/2} w(nT) \cos(\omega nT) \quad (7)$$

where  $T$  is the sample period. A typical window has a normalized amplitude spectrum in dB as shown in Fig 1.

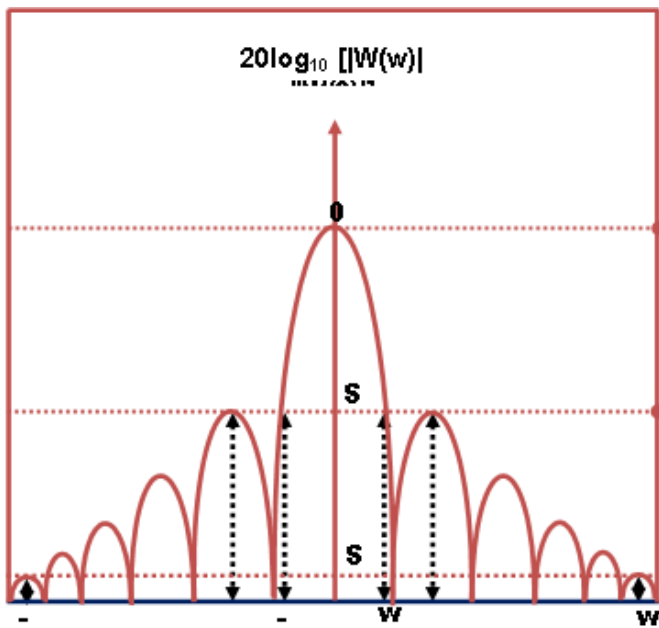


Fig.1: A Typical Window Function's Normalized Amplitude Spectrum in dB

The spectral characteristics [2] known as window's spectral Metrics that determine the window's performance are: (i) First-Null width (WN) (ii) main-lobe width (WM) (iii) ripple ratio (R) and (iv) side-lobe roll-off ratio (S). From the Fig (1) these Metrics can be defined as,

WN = Two times of the half of the main-lobe width up to First Null =  $2W1$ .

WM = Two times of the half of the main-lobe width =  $2WR$ .

$R = (\text{Maximum side-lobe amplitude in dB}) - (\text{Main-lobe amplitude in dB}) = S1$ .

$S = (\text{Maximum side-lobe amplitude in dB}) - (\text{Minimum side-lobe amplitude in dB}) = S1 - SN$ .

These spectral characteristics or metrics are important performance measures for windows. The width characteristics provide a resolution measure between adjacent signals while the ripple ratio determines the worst-case scenario for detecting weak signals in the presence of strong narrowband signals. The side-lobe roll-off ratio provides a simple description of the

distribution of energy throughout the side lobes, that can be important if prior knowledge of the location of an interfering signal is known [5-6].

**III. DERIVATION OF THE PROPOSED TANH-BLACKMAN-HARRIS WINDOW**

In this work, a new class of adjustable window function is proposed. The new window function is called Tangent Hyperbolic Window function, that is a combination of scaled and shifted in opposite directions of Hyperbolic Tangent functions defined in (8) with 4-term Blackman-Harris window defined in (3). The proposed window function has better spectral parameters than most of the windows commonly used in literature. The proposed window function is adjustable since the spectral characteristics of the window can be changed by varying simply with the window's controlling parameters " $\alpha$ ", " $\beta$ " and " $\gamma$ ". Here " $\alpha$ " is the shifted parameter and, " $\beta$ " is the scaling parameter. Due to the scaling parameter " $\beta$ " the frequency characteristics will be decided either expanded or contracted depending on whether " $\beta$ " is less than one or more than one. The shifting parameter " $\alpha$ " is responsible for contraction or expansion of the window function in time domain and therefore the reciprocal effect in the frequency domain. Hence, the spectral characteristic of the proposed window is studied, and its performance is compared with hamming, Hanning, Kaiser and Gaussian windows. Simulation results show that the proposed window yields better performance in terms of spectral characteristics.

**A. The Tangent Hyperbolic Window Function**

The Tangent Hyperbolic window function is a combination of scaled and shifted versions of Tangent hyperbolic functions given by (8).

$$wT(t) = \text{Tanh}(\alpha + \beta t) + \text{Tanh}(\alpha - \beta t) \quad \text{for } -\infty \leq t \leq \infty \quad (8)$$

Here " $\alpha$ " is shifting parameter and " $\beta$ " is the scaling parameter. The unnormalized shape of the resultant window of Eq.8 with fixed length " $N$ " for different " $\alpha$ " and " $\beta$ " values are shown in the Fig.2. Here " $\alpha$ " and " $\beta$ " are constants and will control the shape of the window.

The normalized window is obtained by dividing the Tangent Hyperbolic window function by its maximum value. The maximum value of the window function is obtained by differentiating the (8), w.r.t " $t$ " and equating to zero. Thus, the maximum value will occur at  $t=0$  and its maximum value is given by,

$$w_{Tmax}(t) = w_T(0) = \frac{2\sinh(2\alpha)}{[1+\cosh(2\alpha)]}$$

Using this maximum value and after some algebraic manipulations, we can find the normalized Tanh window function in its simplest form as,

$$w_T(t) = \frac{[1 + \cosh(2\alpha)]}{[\cosh(2\alpha) + \cosh(2\beta t)]} \quad \text{for } -0.5 \leq t \leq 0.5$$

The Continuous time Window is sampled with “N” samples to get Discrete time window as a function of window’s sample number “n” and is given by

$$w_T(n) = \frac{[1 + \cosh(2\alpha)]}{[\cosh(2\alpha) + \cosh\left(2\beta \frac{n - \frac{(N-1)}{2}}{(N-1)}\right)]} \quad \text{for } 0 \leq n \leq N-1 \text{ -----(9)}$$

It is seen from (9), the proposed window touches time-axis symmetrically like other windows reported in literature [1-4]. To meet better spectral requirements and additional fine details, the window function defined in (9) is multiplied with another fixed window a 4-term Blackman-Harris window [4] defined in (3), as it exploits larger side-lobe roll-off ratio. The product is raised to the power “γ” and formulated a new window function defined in Eq.10 as suggested in [5]. Here “γ” is a constant which also dictates the fine spectral characteristics of the resultant window. Now the proposed window w(n) is given by,

$$w(n) = \begin{cases} [w_T(n) * w_B(n)]^\gamma & \text{for } 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \text{ -----(10)}$$

Here “\*” indicates term by term multiplication. Now, the window shape depends on 4-parameters viz. “N” the length of the window, “α”, “β” and “γ” to vary the shape and thereby the frequency response characteristics of the proposed window. Here the constants “α” and “β” are used to control the coarse shape of the window and the constant “γ” is used for the fine adjustment to achieve the desired shape of the window and frequency response characteristics. Hence the three parameters “α”, “β” and “γ” are responsible for the desired window shape and its frequency characteristics for the fixed window length “N”. Therefore, these parameters are called window shape parameters. The different window shapes that are formulated with different shape parameters are shown in Fig. 3.

With  $\gamma = 0$ , the proposed window becomes rectangular shape. Keeping “α”, “β” are fixed and varying “γ” we get the shape of the window changes and nearing to any desired window shape, for example, when  $\gamma = 0.348$ , the proposed window shape almost becomes Hamming window shape with some better spectral metrics. Likewise, any shape of the window can be achieved by simply varying these shape parameters “α”, “β” and “γ”.

#### IV. WINDOW PERFORMANCE ANALYSIS

In this section, we compare the shape and the spectral characteristics of the proposed window with the following commonly used windows.

##### A. Hamming window:

Hamming window is the most popular and most used window function. This window function also belongs to cosine window

family and the starting and ending points of Hamming window do not touch the time-axis. This means that the coefficients of the Hamming window are always greater than zero. But the starting and ending points of the proposed window are less than the same values of hamming window above the time axis symmetrically.

From Fig.4, It has been observed that the main lobe width of the Proposed window is narrower than the Hamming window with its shape controlling parameters  $\alpha=0.01$ ,  $\beta=1.3$  and  $\gamma=0.360$  for the window length of  $N=51$ . The ripple ratio of the proposed window with is -35.39 dB which is slightly more than the Hamming window. The side-lobe roll-off ratio of the proposed window is 28.08 dB which is more than 20 dB better than Hamming window. Table-2 presents the numeric comparison between proposed window and Hamming window

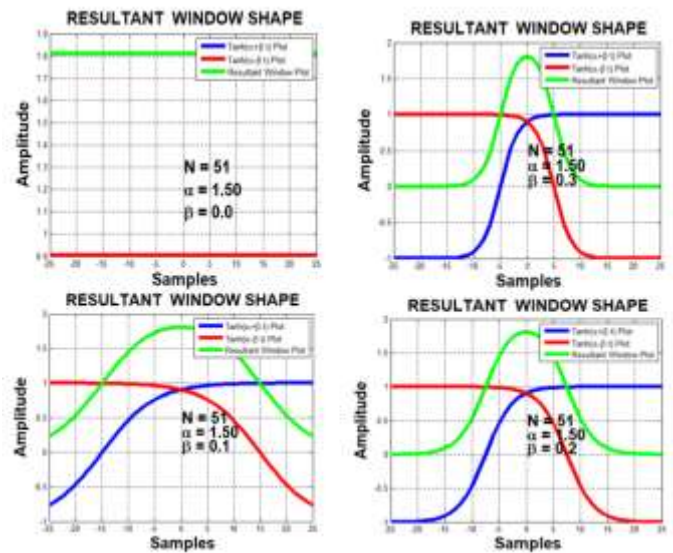


Fig.2: Different Resultant Window shapes obtained as a function of shape parameters “α” and “β” in Tanh Window

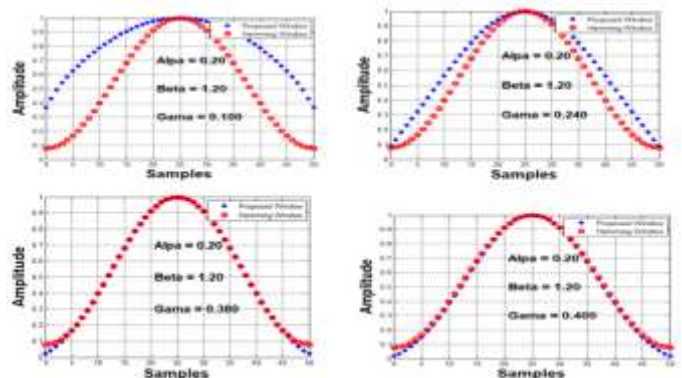


Fig.3: Hamming Window Shape generation using the proposed window with varying the values of α, β and γ

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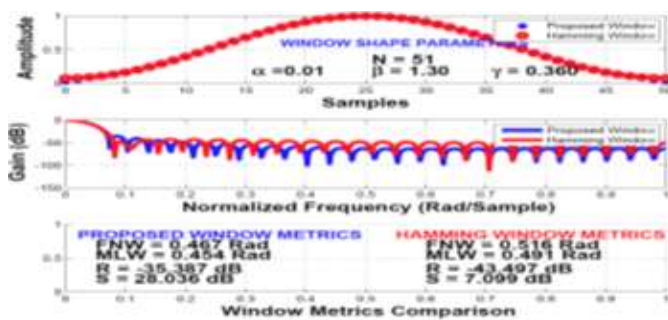


Fig.4: The comparison of the Proposed window with Hamming window in time and frequency domain representations

##### B. Hanning window:

Hanning window function also belongs to cosine window family. Here, the starting and ending points of Hanning window tapers smoothly to zero with the time-axis. This means that the coefficients of the Hanning window are always in between zero and one. The proposed window also touches time-axis symmetrically but not to zero unless the shape parameter " $\alpha$ " is zero. Therefore, the starting and ending points of the proposed window will be non -zero but more smoothing effect than abrupt truncation when compared to other windows. From Fig.5, It has been observed that main lobe width of the Proposed window is slightly more than the Hanning window with its shape controlling parameters  $\alpha=0.50$ ,  $\beta=2.5$  and

$\gamma=0.370$  for window length  $N=51$ . However, the main lobe width can be increased by adjusting the shape parameters of the proposed window, but it is possible only at the cost of other metrics of the frequency response. Here it is observed that, even when the proposed window takes on the almost the same shape of the Hanning window, the roll-off ratio is better in the case of Hanning window. The ripple ratio of the proposed window with is  $-42.53$  dB which is  $-11$  dB less than the Hanning window. The side-lobe roll-off ratio of the proposed window is  $24.03$  dB and is much inferior to Hanning window's Sidelobe roll-off ratio that is  $80.67$ . Table-2 presents the numerical comparison between proposed window and Hanning window.

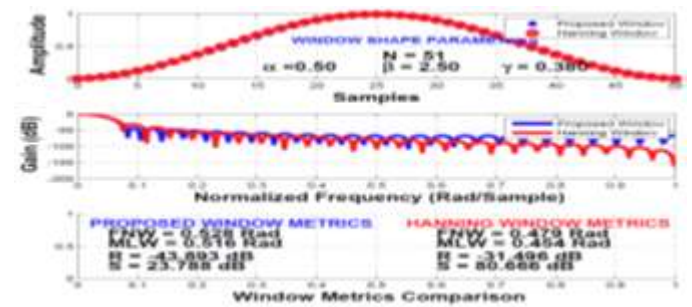


Fig.5: The comparison of the Proposed window with Hanning window in time and frequency domain representations

##### C. Kaiser Window:

The Kaiser window is an adjustable window with two parameters,  $N$  the length of the window and a shape parameter " $\alpha k$ " to vary the spectral characters. As " $\alpha k$ " increases, the main lobe width widens, and the side lobe attenuation increases. For " $\alpha k$ " = 0, the Kaiser window is a rectangular window. For " $\alpha k$ " = 5.3, the Kaiser window is close to a Hamming window. The two parameters are that are " $N$ " i.e. the sequence length and the shape parameter " $\alpha k$ " are useful to obtain the desired amplitude response pattern of the Kaiser window. For fixed " $N$ ", as the shape parameter " $\alpha k$ " increases, the side lobe level of the magnitude response decreases at the cost of main lobe width. For  $\alpha k = 4.5$ , the Kaiser Window frequency response is shown in fig.6. In the proposed window also when  $\gamma=0$ . The proposed window becomes rectangular window. From Fig.6, It has been observed that main lobe width of the Proposed window is same as with the Kaiser window with its shape parameter  $\alpha k = 4.5$  and for the proposed window controlling parameters of  $\alpha=0.50$ ,  $\beta=2.5$  and  $\gamma=0.280$  for the window length  $N=51$ . However, the main lobe width can be increased or decreased by adjusting the shape parameters of the proposed window but at the cost of other metrics of the frequency response.

Here it is observed that, even when the proposed window takes on almost the shape of the Kaiser window, the roll-off ratio is slightly more than the case of the proposed window. The side-lobe roll-off ratio of the proposed window is  $25.07$  dB and more

than the Sidelobe roll-off ratio 21.87 dB for the case of Kaiser window. Table-2 presents the numerical comparison between proposed window and Kaiser window.

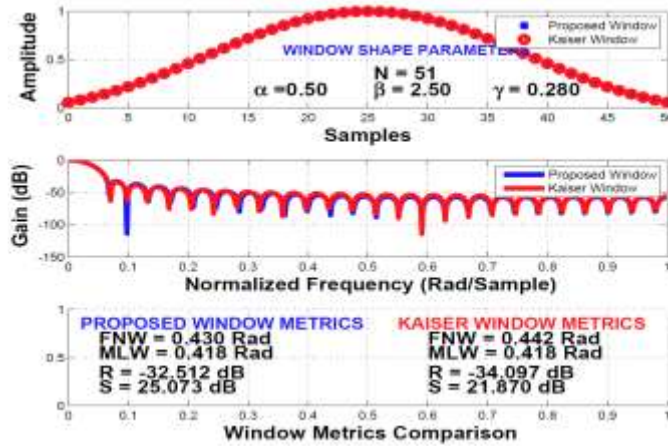


Fig.6: The comparison of the Proposed window with Kaiser window with shape parameter  $\alpha_k=4.5$  in time and frequency domain representations

**D. Gaussian Window:**

The Gaussian Window [4] has also two parameters like Kaiser window, the length of the sequence N and a shape parameter “ $\alpha_g$ ”. As the parameter increases the side lobe level of the frequency response decreases. As “ $\alpha_g$ ” increases, the main lobe width widens, and the side lobe attenuation increases. as the shape parameter “ $\alpha_g$ ” increases. For  $\alpha_g=3$ , the Gaussian Window shape almost exactly matches with the proposed window function shape as shown in fig.7. From the fig.7, It has been observed that the main lobe width of the Proposed window with its shape parameters  $\alpha=0.50, \beta=2.5$  and  $\gamma=0.280$  is smaller than Gaussian window with its shape parameter  $[\alpha]_g=3$  with window length,  $N=51$ .

However, the main lobe width can be increased or decreased by adjusting the shape parameters of the proposed window but at the cost of other metrics of the frequency response. An interesting observation is seen here, even when the proposed window with the help of shape parameters takes on almost the same shape of the Gaussian window, an advantage of more than 23dB in the case of Side lobe roll-off ratio metric for the proposed window. The proposed window, another metric Ripple ratio is less than -6.49 dB when compared to Gaussian window of almost same shape. Table-2 presents the numerical comparison between proposed window and Gaussian window that finds favor to the Proposed window than Gaussian window.

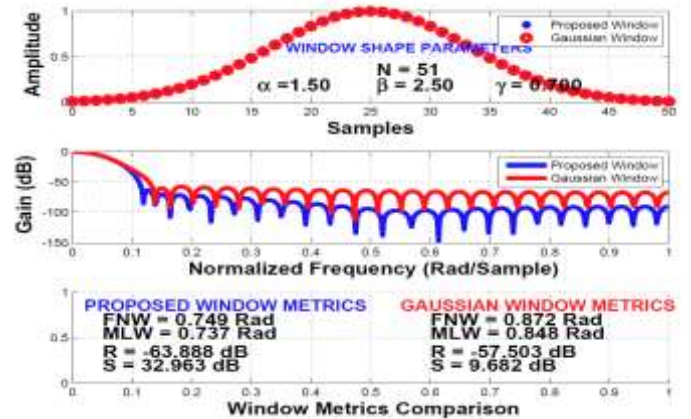


Fig.7: The comparison of the Proposed window with Gaussian window with shape parameter  $\alpha_g=3$  in time and frequency domain representations

Table.2: The comparison of the Proposed window Metrics with other windows

Window N=51	First Null Width (Rad/Sample )	Main-lobe Width (Rad/Sample )	Ripple Ratio (dB)	Side-lobe Roll-off Ratio (dB)
Proposed window $\alpha=0.01, \beta=1.3, \gamma=0.360$	0.467	0.454	-35.39	28.04
Hamming Window	0.516	0.491	-43.50	7.10
Hanning Window	0.479	0.454	-31.50	80.67
Kaiser Window ( $\alpha_k=4.5$ )	0.442	0.418	-34.10	21.87
Gaussian Window( $\alpha_g=3$ )	0.872	0.848	-57.50	9.68

The proposed window is very simple as it does not involve any Bessel function like the Kaiser window. From the comparison Table 2 shows the change in the sidelobe roll-off ratio for Tanh window in terms of the normalized width parameter for  $N=51$ . From the windows spectral parameters data in Table.2, one can infer that all forms of windows starting from rectangular to desired Window can be formulated by changing the window parameters “ $\alpha$ ”, “ $\beta$ ” and “ $\gamma$ ”. Hence, depending on the need and application different window shapes and their resultant frequency responses can be used. Therefore, the new window is yet another adjustable window with variable window parameters to include along with the existing adjustable windows in the literature [4]. When the window coarse parameters “ $\alpha$ ” and “ $\beta$ ” are fixed to get approximate desired

window shape and vary the fine parameter “ $\gamma$ ” we get almost the true shape of the desired window for the desired application. The window metrics obtained with the proposed Tanh window are better than the metrics obtained with Hamming, Hanning, Kaiser and Gaussian windows. When the proposed window is applied for array synthesis the desired radiation characteristics like narrower main lobe and side lobe suppression can be achieved.

## V. ANTENNA ARRAY SYNTHESIS USING THE PROPOSED WINDOW

Array synthesis using windows is a technique used in signal processing and antenna design to create a desired radiation or frequency pattern by manipulating the signal amplitudes in an array of elements [12-16]. The process involves multiplying the ideal, infinite impulse response of an array by a finite-length mathematical function, or "window," to minimize undesired effects like sidelobes in the resulting pattern. The primary goal is to shape the overall response of an array to meet specific performance requirements, such as Reduce side lobes to decrease the intensity of radiation in directions other than the main beam. A uniform, or rectangular, window has high side lobes due to abrupt edges, while tapered windows smooth these transitions to suppress side lobes and reduce wasted energy in side lobes. Hence, a higher proportion of the total radiated power is concentrated in the main beam, increasing the array's directivity. The low SLLs in antenna arrays can be achieved by varying the array elements number, the separation between the elements and window function coefficients [12].

### A. Array Pattern Synthesis

The Windowing techniques are mainly suitable for uniform linear arrays with isotropic elements. Since every window has its own feature, in this paper different window functions described in section-2 and section-3 are applied to linear antenna array configurations to produce minimum side lobe patterns [12,13]. In antenna arrays, getting the desired signal while filtering out the interfering signals and internal and external noise is known as array beamforming. Array beamforming can be divided into switched beamforming (array pattern synthesis) and adaptive beamforming [17,18]. Adaptive antenna array systems provide better signal reception but are costly due to the use of fast computing digital processing systems. Because of the simplicity of system design switched beam systems can be considered in some array synthesis problems as compared to fully adaptive antenna array systems. Switched beam (or array pattern) synthesis can be further classified into weight and geometry synthesis. (i) Array Weight Synthesis, (ii). Array geometry synthesis. In the array weight synthesis process the desired array pattern is obtained with fixed weights. These synthesis methods are most suitable for the design linear antenna arrays with isotropic antenna

elements. In array geometry synthesis process array element positions are adjusted to get required synthesized pattern. That is spacing between the elements is adjusted to achieve the optimized pattern.

### B. Linear Arrays

A linear antenna array in which identical antenna elements are placed on one side from the origin. The AF for N elements can be considered as follows [12]. where N is the number of antenna elements,  $a_i$ ,  $d_i$ ,  $\phi_i$ , and  $k$  are the excitation amplitude, the inter-element spacing, the excitation phase and the propagation constant for the  $i$ th element

Because the element “N” in an antenna array plays an important role in beam forming, beam steering, and interference reduction. The performance of an antenna array is also dependent on the distance between two consecutive elements.

$$AF(\theta) = \sum_{n=0}^{N-1} e^{jn(kd \cos \theta + \beta)} \text{-----(11)}$$

Excitation amplitude for each individual element, commonly known as a weight factor, also changes radiation characteristics of an array antenna. By changing its value for various elements of an array, one can change the overall array pattern. The element phasing in a Uniform Linear Array can be set such that the primary lobe points either perpendicular to the array axis, creating a broadside array, or along the array axis, creating an end-fire array. Therefore, the major lobe appears at  $\theta = 90^\circ$  in a broadside array and at  $\theta = 0^\circ$  or  $180^\circ$  in an end-fire array.

### C. Radiation Pattern

An antenna's radiation pattern explains how power is distributed spatially in various directions. Side lobes, which are undesirable radiation, and a main lobe, which indicates the direction of highest radiation, make up this structure. The main lobe's beamwidth, which is typically measured at the half-power (-3 dB) points, determines the energy's angular spread. Elevated sidelobe levels might decrease directivity and produce interference. In this work windowing techniques are used to reduce side lobes and enhance the overall radiation pattern in linear arrays by applying them to the excitations of the array elements. An antenna array's directional characteristics are described by the Array Factor (AF), which is calculated by adding up the contributions of each element with its corresponding amplitudes, phases, and placements. Constructive and destructive interference are used to shape the overall radiation pattern. The AF for an N-element linear array with spacing “d” and progressive phase shift “ $\beta$ ” is given by Eq.11. The overall radiation pattern  $F(\theta, \phi)$  of an array is the result of multiplying the array factor  $AF(\theta, \phi)$  by the element pattern  $E(\theta, \phi)$  is called Pattern Multiplication principle.

## VI. RESULTS AND DISCUSSION

In this paper, using the Proposed Tanh-Blackman window, the array radiation patterns in both polar and 3D views have been generated to show the suitability for the use of Array synthesis and for better results. The radiation patterns of linear arrays are generated using an algorithm developed in MATLAB. As the aim of the work is to reduce the side lobes to avoid the interference, the proposed window function performance is studied in terms of array antenna metrics and compare with other window functions like Hamming, Hanning, Kaiser and Gaussian window functions. To study the efficiency of the proposed window used in a linear array synthesis, the metrics chosen are Side lobe level (SLL), First Null Beam Width (FNBW), Half Power Beam Width (HPBW) and Directivity(D)

which are important to consider a particular array configuration is suitable for the chosen application or not.

For this, the proposed Tanh-Blackman window function array weight synthesis is carried out with different array element spacings  $d=0.25\lambda$ ,  $d=0.5\lambda$  and  $d=\lambda$  and with the different number of array elements of 7, 9 and 11 elements and with the window shape parameters of  $\alpha=1.5$ ,  $\beta=3$  and  $\gamma=0.5$  to reduce side lobe levels. The results are presented in Fig.8.1 and Fig.8.2. From Fig. 8.1 and Fig.8.2, it is observed that the proposed Tanh-Blackman window is very suitable for array synthesis to yield desired results in terms of the array metrics. A comparison of the Proposed Window with 10 element array of spacing  $0.5\lambda$  is made in terms of the array metrics with the selected windows. The results are tabulated in Table.3 and Table.4 for both End-Fire and Broad-Side arrays used respectively.

Table 3: Antenna Array Metrics Comparison with different window functions (End-Fire Array)

Table 3: Antenna Array Metrics Comparison with different window functions (End-Fire Array)				
Window Function	End-Fire Array, $\theta_0=0$ , $N=10$ , $\lambda=1$ , $d=0.5$			
	SLL (dB)	FNBW (Rad)	HPBW (Rad)	D (dB)
Hamming Window	-35.81	2.14	1.09	8.53
Hanning Window	-31.79	1.96	1.09	7.98
Kaiser window $\alpha_k = 4.5$	-36.23	1.86	1.09	8.65
Gaussian Window $\alpha_g = 3$	-62.82	2.77	1.23	7.45
Tanh-Blackman $\gamma=0.2$	-25.35	1.03	0.45	8.99
Tanh-Blackman $\gamma=0.3$	-30.93	1.18	0.51	8.46
Tanh-Blackman $\gamma=0.4$	-37.48	1.35	0.53	8.03
Tanh-Blackman $\gamma=0.5$	-45.92	1.52	0.59	7.66
Tanh-Blackman $\gamma=0.7$	-60.90	1.88	0.68	7.06
Tanh-Blackman $\gamma=0.9$	-72.28	2.18	0.75	6.59
Tanh-Blackman $\gamma=1$	-77.38	2.31	0.78	6.38

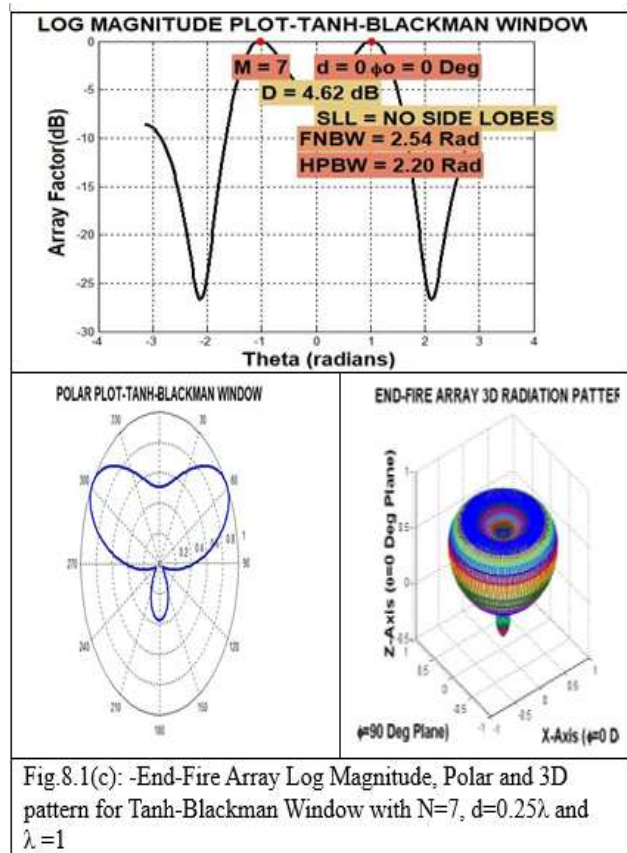
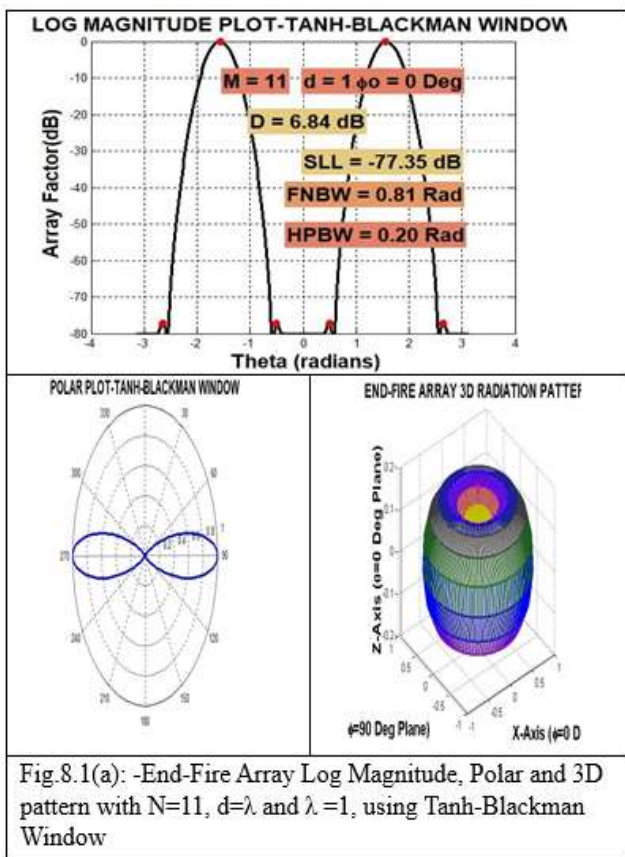
From Table.3 for End-Fire array, one can observe that in the case of Hamming window, all the metrics except the directivity are inferior to the metrics obtained with the proposed Tanh-Blackman window function. But only the directivity 8.53 dB for Hamming Window is slightly decreased to 8.03 dB in the case of the Proposed window. But directivity can be increased by adjusting the smoothing parameter “ $\gamma$ ” of the proposed window at the cost of other array metrics. Similar comparison of Hanning window with the proposed Tanh window with shape parameter  $\gamma=0.3$  yields far better results than Hanning window in all the metrics used. In the case of Kaiser window with  $\alpha_k = 4.5$  when compared with the metrics of the proposed

Tanh window with shape parameter  $\gamma=0.4$  gives a far better results as observed in the case of Hamming window. The comparison of Gaussian window with attenuation constant  $\alpha_g = 3$  and the proposed Tanh window with shape parameter  $\gamma=0.9$  also reveals that in all the metrics except the directivity, the proposed window has yielded far better results. Therefore, the superiority of the proposed window function is established in terms of Antenna array metrics like Side Lobe Level (SLL), First Null Beam Width (FNBW), Half Power Beam Width (HPBW) and Directivity (D). Hence the proposed Tanh-Blackman window can be used in Array antenna synthesis to achieve better results. From Table.4 it is also observed that a similar type results are yielded for the Broad-Side array.

Table 4: Antenna Array Metrics Comparison with different window functions (Broad-Side Array)

Table 4: Antenna Array Metrics Comparison with different window functions (Broad-Side Array)				
Window Function	Broad-Side Array, $\theta_0=900$ , $N=10$ , $\lambda=1$ , $d=0.5$			
	SLL	FNBW	HPBW	D

	(dB)	(Rad)	(Rad)	(dB)
Hamming Window	-35.81	1.09	0.28	8.53
Hanning Window	-31.76	0.91	0.32	7.98
Kaiser window $\alpha_k = 4.5$	-36.24	0.81	0.28	8.65
Gaussian Window $\alpha_g = 3$	-62.83	1.93	0.35	7.45
Tanh-Blackman $\gamma=0.2$	-25.68	0.63	0.27	8.99
Tanh-Blackman $\gamma =0.3$	-31.25	0.77	0.27	8.46
Tanh-Blackman $\gamma =0.4$	-37.75	0.90	0.30	8.03
Tanh-Blackman $\gamma =0.5$	-46.22	1.10	0.37	7.66
Tanh-Blackman $\gamma =0.7$	-61.01	1.49	0.40	7.06
Tanh-Blackman $\gamma =0.9$	-72.41	1.89	0.43	6.59
Tanh-Blackman $\gamma =1$	-77.50	2.15	0.43	6.38



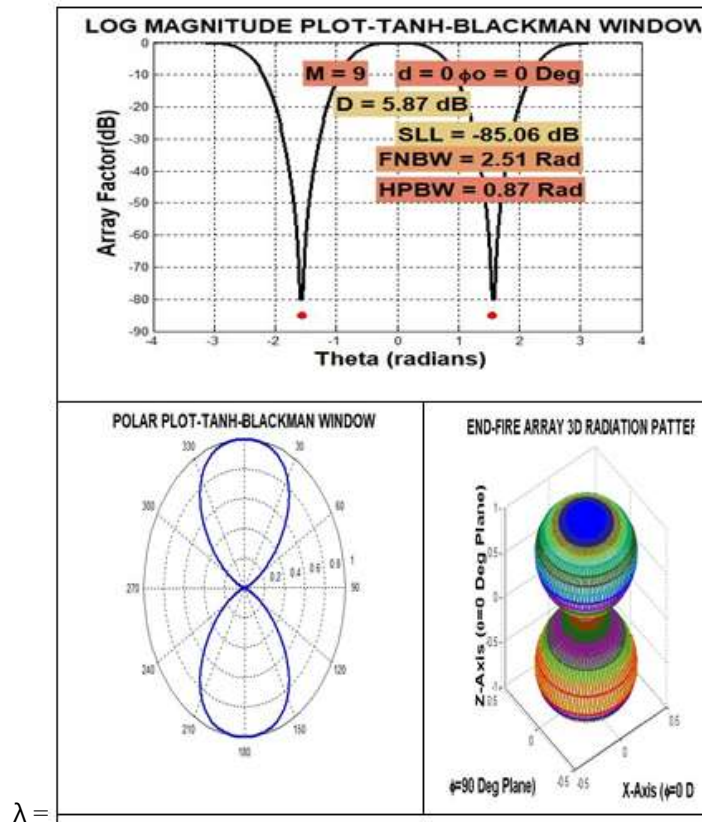


Fig.8.1(b): -End-Fire Array Log-Magnitude, Polar and 3D pattern with  $N=9$ ,  $d=0.5\lambda$  and  $\lambda=1$ , using Tanh-Blackman Window

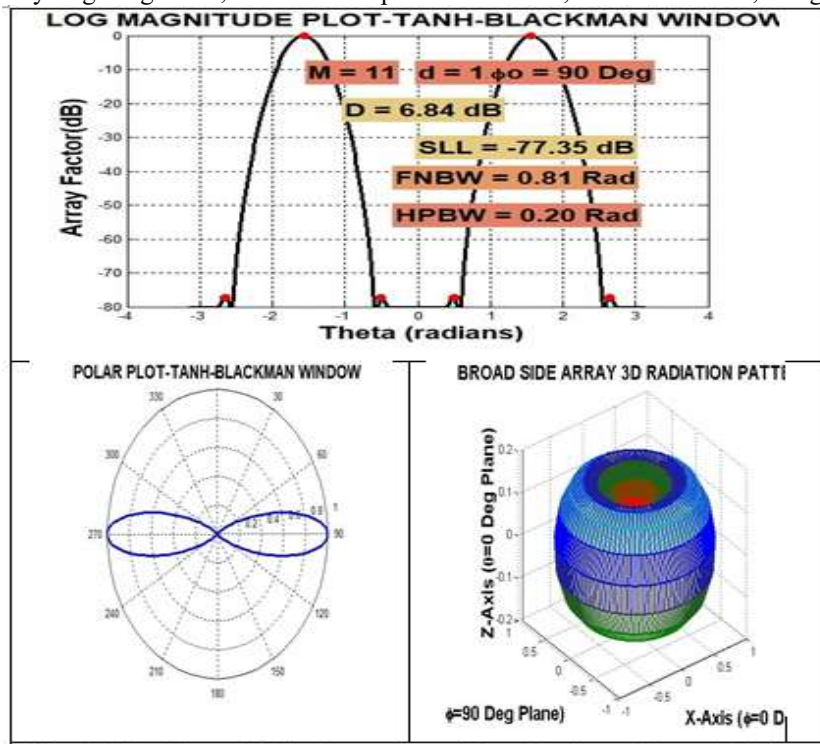


Fig.8.2(a): Broad-Side Array Log-Magnitude, polar and 3-D plot with  $N=11$ ,  $d=\lambda$ ,  $\lambda=1$

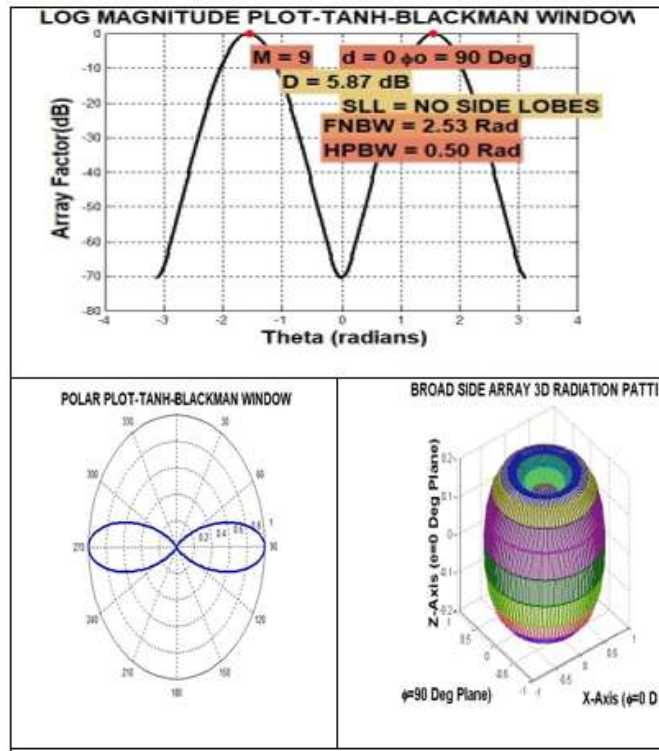


Fig.8.2(b): Broad-side Array Log-Magnitude, polar and 3-D plot with  $N=9$ ,  $d=0.5\lambda$ ,  $\lambda=1$

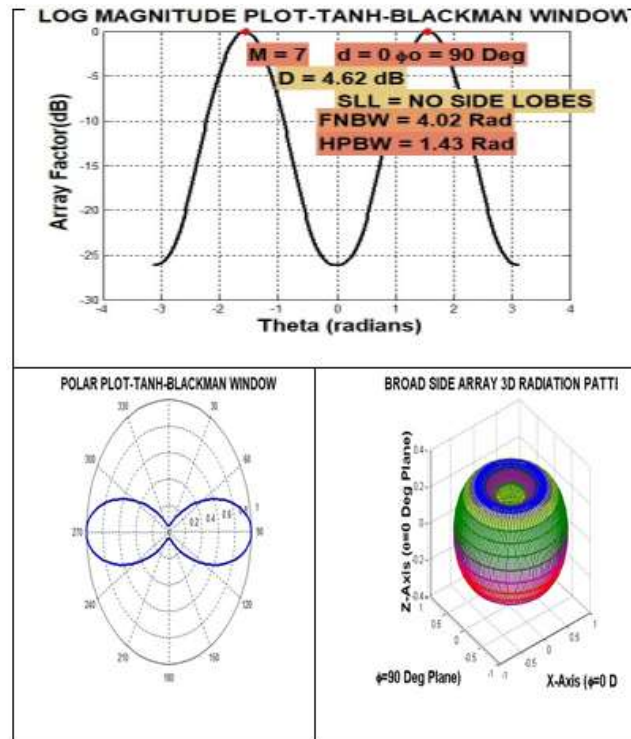


Fig.8.2(c): Broad-Side Array Log-Magnitude, polar and 3-D plot with  $N=7$ ,  $d=0.25\lambda$ ,  $\lambda=1$

## VII. CONCLUSION

In this paper a new adjustable window function called Tanh-Blackman window is proposed. The superiority of the newly proposed window function over the other selected windows has been established by changing the window shape using window shape parameters. The application of the newly proposed window has been tested by applying the window function for Antenna array synthesis. The results have shown the superiority of the proposed window over the other windows selected for this work in terms the Antenna array metrics like SLL, FNBW, HPBW, and D.

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