International Journal of Scientific Research & Engineering Trends
Volume 12, Issue 2, Mar-Apr-2026, ISSN (Online): 2395-566X

Formation of Dio-3 Tuples for Centered Hexagonal Number

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Abstract- A Diophantine triple is a set of three positive integer $\{a,b,c\}$ such that the product of any two distinct elements is added to one, is a perfect square .This article investigates the existence of a specific Diophantine triple involving Centered Hexagonal Number ensuring the product of any two members of the added to the property D(n).

Keywords – Diophantine triples, Centered Hexagonal Number.

I. INTRODUCTION

In Number Theory, a Diophantine equation is a polynomial equation in which only integer solution are evaluated or searched for, often with two or more unknowns. The term “Diophantine” relates to Diaphanotus of Alexandria, a Greek mathematician who examined comparable issues and was a pioneer in adding symbols to variable-based mathematics in the third century [1-3]. (Include about triple definition) Beardon AF, and Deshpande, M . N. [4]

Explored Diophantine triple .The expandability of diphantine paris $\{k-1_1 k+1\}$" was studied by Fujita, Y. in [6]. The Construction of the Diophantine triple Involving Pentatope number by Sangeetha, P and Janaki, G. in [7,8]. Janaki.G and Others [9-14] found various triples involving special numbers satisfying different properties.

H_n = Centered Hexagonal number = $3n^2 \square 3n \square 1$

In this paper, Centered Hexagonal Number is used to construct Dio 3-tuple under four cases.

Notations:

$CH_n \square 3n^2 \square 3n \square 1$

II. METHOD OF ANALYSIS

CASE 1. Construction of Dio-3 tuples for Centered Hexagonal number of n and n+1.

Let u = CHn

v $= CH_{n\square 1}$ be the Centered Hexagonal number of n and n □ 1 then,

$$uv+18n^3+18n^2+6n=\sigma^2$$

Let w be any non-zero integer such that,

$$uw+18n^3+18n^2+6n=\phi^2 \qquad (1)$$

$$vw+18n^3+18n^2+6n=\psi^2 \qquad (2)$$

Subtracting (1) and (2) we get

$$(u-v)w=\phi^2-\psi^2 \qquad (3)$$

Setting $\phi=u+\sigma$ and $\psi=v+\sigma$ (4)

Substituting equation (4) in (3) the values of w represented by

$$w=12n^2+6n+4$$

Hence, $\{3n^2-3n+1; 3n^2+3n+1; 12n^2+6n+4\}$ is a Diophantine triple under the property

$D(18n^3 + 18n^2 + 6n)$.

Table 1.Construction of Dio-3 tuple for Centered Hexagonal number of rank n and n+1.

n	Dio-3 Tuples	$D(18n^3 + 18n^2 + 6n)$
1	(1,7,22)	42
2	(7,19,64)	288
3	(19,37,130)	666
4	(37,61,220)	1464

CASE 2. Construction of Dio-3 tuples for Centered Hexagonal number of rank 2n and 2n+1.

Let $u_1 = CH_{2n}$ and $v_1 = CH_{2n+1}$ be the Centered Hexagonal number of rank 2n and 2n+1 then,

$$u_1 v_1 + 48n^3 + 40n^2 + 4n = \sigma_1^2$$

Let w_1 be any non-zero integer such that,

$$u_1 w_1 + 48n^3 + 40n^2 + 4n = \phi_1^2 \quad (5)$$

$$v_1 w_1 + 48n^3 + 40n^2 + 4n = \psi_1^2 \quad (6)$$

Solving (5) and (6) we get

$$(u_1 - v_1) w_1 = \phi_1^2 - \psi_1^2 \quad (7)$$

Setting $\phi_1 = u_1 + \sigma_1$ and $\psi_1 = v_1 + \sigma_1$ (8)

Applying equations (8) in (7) ,

$$w_1 = 48n^2 + 4n + 4$$

Hence, $\{12n^2 - 6n + 1 ; 12n^2 + 6n + 1 ; 48n^2 + 4n + 4\}$ is a Diophantine triple under the property $D(48n^3 + 40n^2 + 4n)$

Table 2. Construction of Dio-3 tuples for Centered Hexagonal number of rank 2n and 2n+1

n	Dio-3 Tuples	$D(48n^3 + 40n^2 + 4n)$
1	(7,19,56)	92
2	(37,61,204)	552
3	(91,127,448)	1668
4	(169,217,788)	3728

CASE 3. Construction of Dio-3 tuples for Centered Hexagonal number of rank 3n and 3n+1.

Let $u_2 = CH_{3n}$ and $v_2 = CH_{3n+1}$ be the Centered Hexagonal number of rank 3n and 3n+1 then,

$$u_2 v_2 + 378n^3 + 130n^2 + 4n = \sigma_2^2$$

Let w_2 be any non-zero integer such that,

$$u_2 w_2 + 378n^3 + 130n^2 + 4n = \phi_2^2 \quad (9)$$

$$v_2 w_2 + 378n^3 + 130n^2 + 4n = \psi_2^2 \quad (10)$$

Solving (9) and (10) we get

$$(u_2 - v_2) w_2 = \phi_2^2 - \psi_2^2 \quad (11)$$

Setting $\phi_2 = u_2 + \sigma_2$ and $\psi_2 = v_2 + \sigma_2$ (12)

Substituting (12) in (11), the values of w_2 is represented by

$$w_2 = 108n^2 + 14n + 4$$

Hence, $\{27n^2 - 9n + 1 ; 27n^2 + 9n + 1 ; 108n^2 + 14n + 4\}$ is a Diophantine triple under the property $D(378n^3 + 130n^2 + 4n)$.

Table 3. Construction of Dio-3 tuples for Centered Hexagonal number of rank 3n and 3n+1.

n	Dio-3 Tuples	$D(378n^3+130n^2+4n)$
1	(19,37,126)	512
2	(91,127,464)	3560
3	(217,271,1018)	11405
4	(397,469,1788)	26352

CASE 4. Constrution of Dio-3 tuples for Centered Hexagonal number of rank 4n and 4n+1.

Let $u_3 = CH_{4n}$ and $v_3 = CH_{4n+1}$ be the Centered Hexagonal number of rank 4n and 4n+1 then,

$$u_3v_3 + 768n^3 + 208n^2 + 16n = \sigma_3^2$$

Let w_3 be any non-zero integer such that,

$$u_3w_3 + 768n^3 + 208n^2 + 16n = \phi_3^2 \quad (13)$$

$$v_3w_3 + 768n^3 + 208n^2 + 16n = \psi_3^2 \quad (14)$$

Subtracting (13) and (14) we get

$$(u_3 - v_3)w_3 = \phi_3^2 - \psi_3^2 \quad (15)$$

Setting $\phi_3 = u_3 + \sigma_3$ and $\psi_3 = v_3 + \sigma_3$ (16)

Substituting equations (16) in (15) the values of w_3 represented by

$$w_3 = 192n^2 + 16n + 4$$

Hence $\{48n^2 - 12n + 1;\ 48n^2 + 12n + 1;\ 192n^2 + 16n + 4\}$ is a Diophantine triple under the property $D(768n^3 + 208n^2 + 16n)$.

Table 4. Construction of Dio-3 tuples for Centered Hexagonal number of rank 4n and 4n+1.

n	Dio-3 Tuples	$D(768n^3+208n^2+16n)$
1	(37,61,212)	992
2	(169,217,804)	7008
3	(397,496,1780)	22656
4	(721,8170,3140)	52544

III. CONCLUSION

In this paper, Dio-3 tuples involving Centered Hexagonal numbers, are constructed for differentrants and different properties.
It is observed that the triple involving several special numbers in such a way that the triple whose product of any two of its parts, added to the property is a perfect square. In this way, various triples and quadruples with different attributes can be found.

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