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Pseudo Irregular Fuzzy Soft Graphs

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Abstract - This paper deals with pseudo irregular fuzzy soft graphs. The definition of pseudo irregular graphs is introduced with some properties. The pseudo edge irregular fuzzy soft graphs are illustrated with examples. The properties of the defined graphs are studied. Highly, neighbourly, strongly pseudo irregular graphs are explained with examples. Also some pseudo edge irregular fuzzy soft graphs are illustrated. The relation between strongly pseudo irregular fuzzy soft graphs with highly and neighbourly pseudo irregular FSG is given. Results on total pseudo irregular FSG and total pseudo edge irregular FSG is examined.

Keywords - pseudo irregular, pseudo edge irregular, total pseudo irregular, strongly pseudo irregular, fuzzy soft graphs, highly pseudo irregular Mathematics Subject Classification. 05C12, 03E72, 05C72.

INTRODUCTION

Results on soft set theory were presented in [5].[4] dealt with soft sets in decision-making, which proved to be more effective. A study on fuzzy soft graphs was done by [1]. Regular fuzzy graphs were discussed in [2]. Santhi Maheswari and Sekar worked on a pseudo degree in fuzzy graphs and their properties in [6]. [7], [10] deal with various properties of pseudo irregular bipolar fuzzy and irregular intuitionistic fuzzy graphs.[3],[9] are about pseudo edge regular fuzzy graphs and edge pseudo regular fuzzy graphs respectively.
Here we extend the idea of pseudo degree and pseudo edge degree in fuzzy soft graphs. And bring out some of its characteristics and also provide some illustrations to prove them.

Preliminaries
Definition 2.1. The pair (F, A) is soft set over the universal set, where A ⊆ E and
F : a → P(U). That is a soft set over U is parametered collection of subsets of U.
Definition 2.2. [1] An FSG G˜ is a 4-tuple, such that:

- G∗ is crisp graph.
- A is the parameter set.
- (F˜, A) is fuzzy soft set over vertex set V.
- (K˜, A) is fuzzy soft set over edge set E.

Then (F(a), K(a)) is fuzzy (sub)graph of G∗, ∀ a ∈ A and can be denoted as H(a).
The membership value of the edge in an FSG is given as,
K˜(a)(xy) ≤ min F˜(a)(x), F˜(a)(y) .

Definition 2.3. [1] If G˜ is an FSG, then the vetex degree is
G
ai∈A u≠v
Definition 2.4. The support (2 − degree) of a vertex u in an FSG G is the addition of degrees of its adjacent vertices and denoted as sG˜(u), whereas its total support is given as
tsG˜(u) = sG˜(u) + ∑ai∈A F˜(ai)(u).
Definition 2.5. The support (2 − degree) of an edge in a FSG is the sum of edge degrees of edges which are adjacent to given edge and can be defined as
sG˜(uv) = ei∈N (uv),ai∈A K˜(ai)(uv).
Definition 2.6. Let G˜ be a FSG, then pseudo degree of the vertex is
sG˜ (x)
PdG˜(x) = DG∗ (x) , where sG˜ is support of x and DG∗ (x) is number of edges incident to x.
Definition 2.7. Let G˜ be a FSG, then total pseudo degree of vertex is
TPdG˜(x) = PdG˜(x) +
ai∈A
F (ai)(x).
Definition 2.8. Let G˜ be a FSG. Then the pseudo degree of the edge is defined as PEdG˜(vivj) =
sG˜ (vivj)
Ed ∗ (vivj) . Here sG˜(vivj) is support of the edge and EdG∗ (vivj) is the number of edges in-
cident with vivj.
Definition 2.9. Let G˜ be a FSG, then the total pseudo edge degree is given by TPEdG˜(vivj) =
PEdG˜(vivj) +
ai∈A,uv∈E

K˜ (ai)(uv).

Pseudo Irregular FSG

Definition 3.1. A FSG, G˜ is pseudo irregular, if ∃ atleast one vertex, whose adjacent
vertices are with distinct pseudo degrees.

Definition 3.2. A FSG, G˜ is total pseudo irregular, if ∃ atleast one vertex, whose adjacent vertices are with distinct total pseudo degree.

Example 3.3. Demonstration of above definitions.

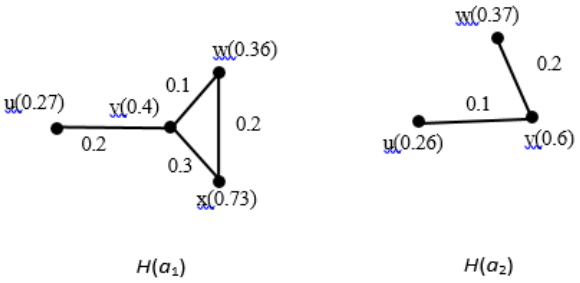

figure 3.3

Here the pseudo degree of all vertices are pdG˜(u) = 0.9, pdG˜(v) = 0.433, pdG˜(w) = 0.7, pdG˜(x) = 0.7. Here vertex x is adjacent to vertices v and w, which have distinct pseudo degrees.

Example 3.4. Consider the below graph.

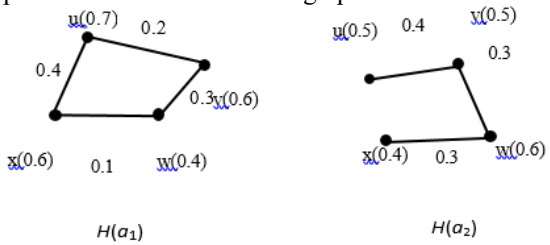

figure 3.4

The total pseudo degree of all vertices are tpdG˜(u) = 2.2, tpdG˜(v) = 2.1, tpdG˜(w) = 2.0,
tpdG˜(x) = 2.0. The vertex u adjacent to vertices v and x with distict total pseudo degrees.

Remark 3.5. A pesudo irregular FSG, need not be total pseudo irregular and vice versa.

Example 3.6. Consider the Example 3.3, in which the total pseudo degree of all vertices are same.

While considering Example 3.4, G˜ is total pseudo irregular but pseudo regular. Hence the
remark

Definition 3.7. A FSG, G˜ is neighbourly pseudo irregular, if no two adjacent vertices have same pseudo degrees.

Example 3.8. Let us consider below fsg G˜.

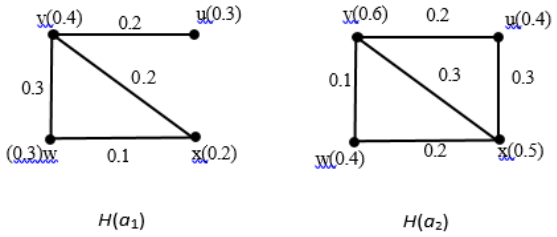

figure 3.8

The pseudo degree of the vertices are PdG˜(u) = 1.2, PdG˜(v) = 0.833, PdG˜(w) = 1.2, PdG˜(u) = 0.9. Here no pair of adjacent vertices have same pseudo degree, thus it is neighbourly pseudo irregular fsg. Here no pair of adjacent vertices have same pseudo degree, thus it is neighbourly pseudo irregular fsg.

Definition 3.9. A FSG, G˜ is highly pseudo irregular, if every vertex in the graph is adjacent to vertices with distinct pseudo degrees.

Example 3.10. Following is a FSG.

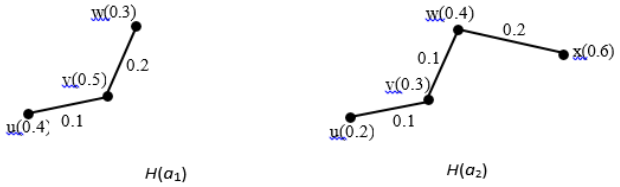

figure 3.10

Here PdG˜(u) = 0.5, PdG˜(v) = 0.35, PdG˜(w) = 0.35, PdG˜(x) = 0.5 and so the following is highly pseudo irregular FSG, since every vertex is adjacent to vertices with different pseudo degrees.

Definition 3.11. A FSG, G˜ is strongly pseudo irregular, if every pair of vertices have distict pseudo degrees.

Example 3.12. Consider the below graph.

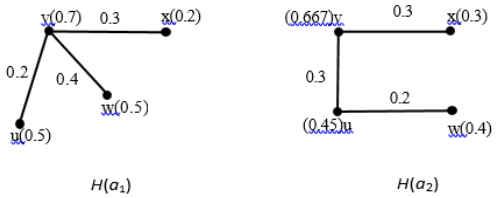

figure 3.12

The pseudo degree of all vertices PdG˜(u) = 0.35, PdG˜(v) = 0.5, PdG˜(w) = 0.3,
PdG˜(x) = 0.6, which are all distinct, ⇒ G˜ is strongly pseudo irregular.

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Remark 3.13. A strongly pseudo irregular FSG, is both neighbourly and highly pseudo irregular.
Remark 3.14. A highly pseudo irregular may be neighbourly irregular, but the converse not necessarily be true.
Example 3.15. In figure 3.12, which is strongly pseudo irregular, it can be found that no pair of adjacent vertices have same pdG˜ ⇒ it is neighbourly irregular, also no adjacent edges have same pdG˜ and hence it is highly pseudo irregular.
Definition 3.16. An FSG is highly totally pseudo irregular, if every vertex is adjacent to vertices with distinct total pseudo degree.

Example 3.17. The below is an example.

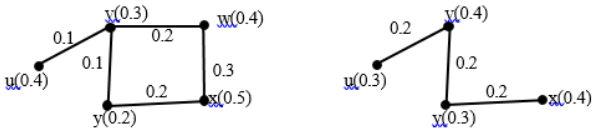

figure 3.17

Here TPdG˜(u) = 1.5, TPdG˜(v) = 1.2, TPdG˜(w) = 1.15, TPdG˜(x) = 1.5, TPdG˜(y) = 1.25
and it is highly totally pseudo irregular fsg.
Definition 3.18. We call an FSG as neighbourly totally pseudo irregular, if no two adjacent vertices have same total pseudo degree.

Example 3.19. Consider the following

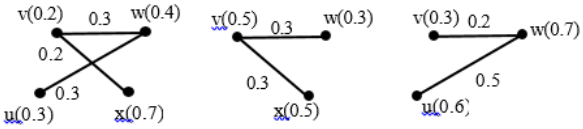

figure 3.19

The pseudo degrees are TPdG˜(u) = 2.5, TPdG˜(v) = 2.05, TPdG˜(w) = 1.45, TPdG˜(x) = 2.5
and it is totally neighbourly pseudo irregular fsg.
Definition 3.20. An FSG is strongly totally pseudo irregular, if the total pseudo degree of all vertices are distinct.

Example 3.21. Example for the above definition.

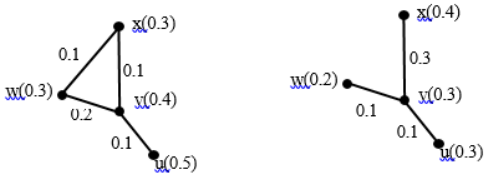

figure 3.21

Here, TPdG˜(u) = 1.7, TPdG˜(v) = 1.06, TPdG˜(w) = 1.2, TPdG˜(x) = 1.35 and all are distict and so it is strongly totally pseudo irregular fsg.

Pseudo Edge Irregular FSG
Definition 4.1. A FSG, G˜ is pseudo edge irregular, if ∃ atleast one edge whose adjacent edges are with distinct pseudo edge degree.

Definition 4.2. A FSG, G˜ is total pseudo edge irregular, if ∃ atleast one edge with adjacent edges having distinct total pseudo edge degree.

Example 4.3. Consider the example.

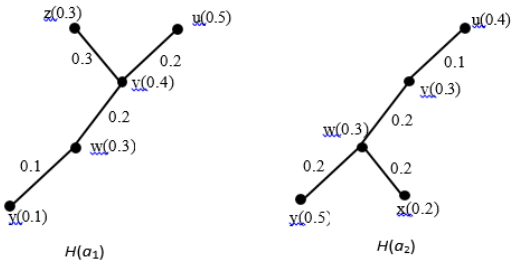

figure 4.3

There exist atleast one edge uv whose adjacent edges vzandvw are with different pseudo edge degrees. Thus it is pseudo edge irregular. Also the total pseudo edge degree of adjacent edges of atleast one edge is distict, hence it is total pseudo edge irregular FSG.
Definition 4.4. A FSG, G˜ is neighbourly pseudo edge irregular, if no pair of adjacent edges are with same pseudo edge degrees.

Example 4.5. The following is an example.

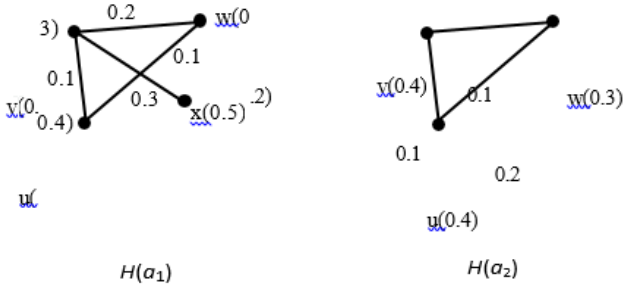

figure 4.5

Here PEdG˜(uv) = 0.6, PEdG˜(vw) = 0.633, PEdG˜(uw) = 0.85, PEdG˜(vx) = 0.85 No pair of adjacent edges have same pseudo edge degree ⇒ given G˜ is neighbourly pseudo edge irregular. eudo edge degree ⇒ givenG˜ is neighbourly pseudo edge irregular.

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Definition 4.6. A FSG, G˜ is highly pseudo edge irregular, if every edge of the graph is
adjacent to edges with different pseudo edge degrees.

Example 4.7. Illustration of above definition.

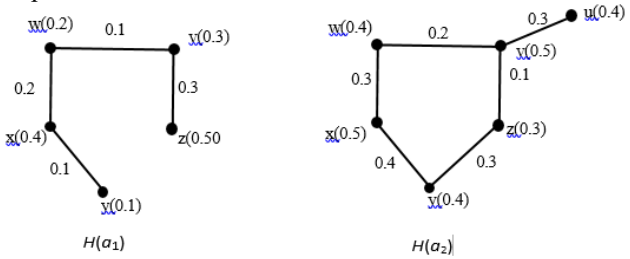

figure 4.7

Now, PEdG˜(uv) = 1.05, PEdG˜(vw) = 0.8, PEdG˜(wx) = 1.0, PEdG˜(xy) = 0.85,
PEdG˜(yz) = 0.85, PEdG˜(zv) = 0.933 and no edge in this example is such, that its adjacent edges have same pseudo edge degrees, thus highly pseudo edge irregular.
Definition 4.8. A FSG is strongly pseudo edge irregular only when the pseudo edge degree of all edges are distinct.
Example 4.9. Illustration of above definition.

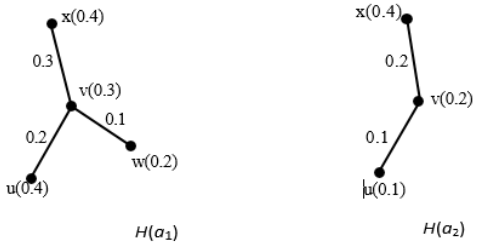

figure 4.9

The pseudo edge degree of the edges are PEdG˜(uv) = 0.6, PEdG˜(vw) = 0.5, PEdG˜(vx) = 0.7, ⇒ G˜ is strongly pseudo edge irregular FSG.

Remark 4.10. If a FSG G˜ is strongly pseudo edge irregular, then it is both highly and
neighbourly pseudo edge irregular.
Definition 4.11. An FSG is highly totally pseudo irregular, if the total pseudo edge degrees of adjacent edges of each edge in the graph are not alike.

Example 4.12. Consider the below graph.

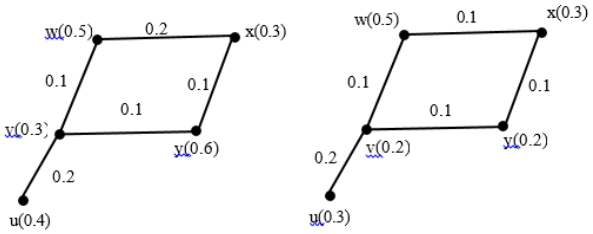

figure 4.12

Now, TPEdG˜(uv) = 1.25, TPEdG˜(vw) = 0.733, TPEdG˜(wx) = 1.0, TPEdG˜(xy) = 0.8,
TPEdG˜(vy) = 0.8 and no edge is adjacent to edges with same total pseudo edge degree.
Definition 4.13. We call an FSG as neighbourly totally pseudo edge irregular, if no two adjacent edges in G˜ have same total pseudo edge degree.
Definition 4.14. An FSG is strongly totally pseudo edge irregular, if the total pseudo edge degree of all edges are distinct.

Example 4.15. The above is demonstrated using this example.

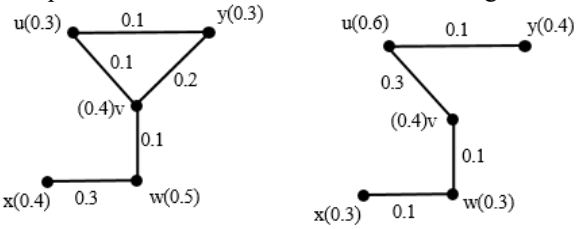

figure 4.15

Here, TPEdG˜(uv) = 1.2, TPEdG˜(vw) = 0.733, TPEdG˜(wx) = 1.4, TPEdG˜(vy) = 0.933,
TPEdG˜(uy) = 0.9, all of them have distinct total pseudo edge degrees ⇒ strongly totally pseudo edge irregular FSG.
Remark 4.16. A pseudo irregular FSG not necessarily be pseudo edge irregular and vice versa.

Example 4.17. The following is an example for the remark.

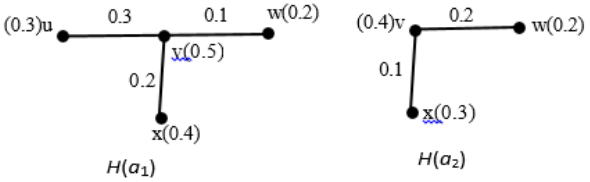

figure 4.17

Here PdG˜(u) = 0.9, PdG˜(v) = 0.3, PdG˜(w) = 0.9, PdG˜(x) = 0.9 ⇒ pseudo irregular. While, PEdG˜(uv) = 0.6, PEdG˜(vw) = 0.6, PEdG˜(vx) = 0.6 and hence not pseudo edge irregular.
Example 4.18. The below is pseudo edge irregular FSG but not pseudo irregular.

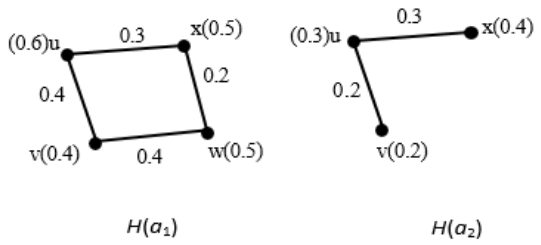

figure 4.18

Here, PdG˜(u) = 0.9, PdG˜(v) = 0.9, PdG˜(w) = 0.9, PdG˜(x) = 0.9. And PEdG˜(uv) = 0.8, PEdG˜(vw) = 1.0, PEdG˜(wx) = 0.8, PEdG˜(xu) = 1.0, thus it is pseudo edge irregular but not pseudo irregular FSG.

Result 4.19. The following results have been found using the definitions defined here.
A highly pseudo irregular may not be highly pseudo edge irregular FSG and other way around.

Example 4.20. Consider the figure 4.7 which is highly pseudo edge irregular. Ob- serving pseudo degree of its vertices PdG˜(u) = 1.0, PdG˜(v) = 0.6, PdG˜(w) = 1.0, PdG˜(x) = 0.8, PdG˜(y) = 0.85, PdG˜(z) = 0.9, the vertex v is adjacent to vertices with same pseudo degree and hence it is not highly pseudo irregular.
The figure 3.9 is highly pseudo irregular. But PEdG˜(uv) = 0.4, PEdG˜(vw) = 0.3,
PEdG˜(wx) = 0.4 ⇒ not highly pseudo edge irregular FSG.
A strongly pseudo irregular FSG may be strongly pseudo edge irregular and vice versa.
Example 4.21. Figure 3.4 is strongly pseudo irregular. And PEdG˜(uv) = 1.033, PEdG˜(vw) = 1.0, PEdG˜(vx) = 1.25, PEdG˜(wu) = 1.25, hence not strongly pseudo edge irregular.
Consider figure 4.5 which is strongly pseudo edge irregular. But PdG˜(u) = 0.9, PdG˜(v) = 0.3, PdG˜(w) = 0.9, PdG˜(x) = 0.9, implies it is not strongly pseudo ir- regular.
Neighbourly pseudo irregular FSG need not be neighbourly pseudo edge irregular FSG and vice versa.
Example 4.22. Examine figure 3.8 which is neighbourly pseudo irregular. But we have, PEdG˜(uv) = 1.266, PEdG˜(vw) = 1.266, PEdG˜(wx) = 1.266, PEdG˜(xu) = 1.266, PEdG˜(xv) = 1.2, ⇒ it is not neighbourly pseudo edge irregular FSG.

Look the below figure

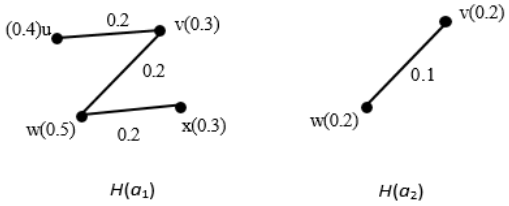

figure 4.22

Here, PEdG˜(uv) = 0.4, PEdG˜(vw) = 0.3, PEdG˜(wx) = 0.4 and here no pair of adjacent edges have same pseudo edge degree,⇒ it is neighbourly pseudo edge irreg- ular. While PdG˜(u) = 0.5, PdG˜(v) = 0.35, PdG˜(w) = 0.35, PdG˜(x) = 0.5, ⇒ not neighbourly pseudo irregular FSG.
Result 4.23. The following are observed.
A strongly pseudo edge irregular FSG need not be strongly totally pseudo edge irregular and vice versa.

Example 4.24. An example is given below.

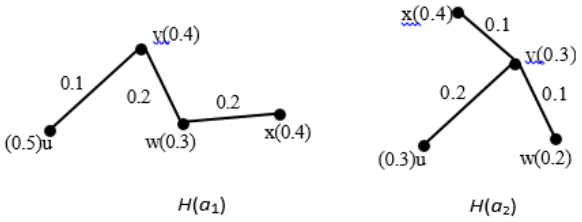

figure 4.24

Here PEdG˜(uv) = 0.6, PEdG˜(vw) = 0.433, PEdG˜(wx) = 0.6, PEdG˜(vy) = 0.5 and
TPEdG˜(uv) = 0.9, TPEdG˜(vw) = 0.733, TPEdG˜(wx) = 0.8, TPEdG˜(vy) = 0.6.
Thus we conclude that it is strongly totally pseudo edge irregular and not strongly
pseudo edge irregular.

A highly pseudo edge irregular not necessarily be highly totally pseudo edge irregular FSG and vice versa.
Example 4.25. The below is an example.

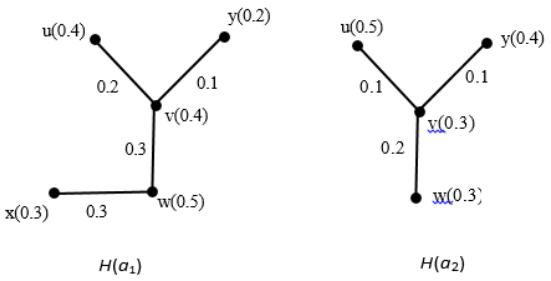

figure 4.25

Neighbourly pseudo edge irregular FSG not be neighbourly totally pseudo edge irregular FSG and vice versa.

Example 4.26. Below is an example

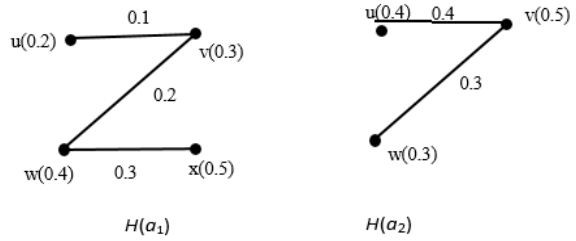

figure 4.26

Here PEdG˜(uv) = 0.8, PEdG˜(vw) = 0.5, PEdG˜(wx) = 0.8 and TPEdG˜(uv) = 1.3, TPEdG˜(vw) = 1.0, PEdG˜(wx) = 1.1. Thus it is totally neighbourly pseudo edge irregular and not neighbourly pseudo edge irregular.

A strongly pseudo irregular need not be strongly totally pseudo irregular and vice versa.

A highly totally pseudo irregular not necessarily be highly pseudo irregular FSG and vice versa.

A FSG which is neighbourly pseudo irregular need not be neighbourly totally pseudo irregular and vice versa.

II. CONCLUSION

Pseudo Irregular Fuzzy Soft Graphs form a flexible and enriched mathematical framework that extends classical graph theory to handle uncertainty, vagueness, and parameter-dependent information. By integrating the concepts of fuzziness, soft sets, and pseudo irregularity, this model provides a powerful tool for representing systems in which vertex and edge connections are not fixed but vary with respect to parameters and degrees of membership.

The structure allows analysts to capture irregular behaviors—such as asymmetric interactions or nonuniform connectivity—while still preserving the interpretability of soft-set-based parameterization. As a result, Pseudo Irregular Fuzzy Soft Graphs have strong potential in real-world applications including decision-making, social network analysis, pattern recognition, and complex system modeling, where uncertainty and irregularity frequently coexist.

Future research may explore algorithmic approaches, characterization theorems, operations on such graphs, and their practical implementation in dynamic and data-driven environments.

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