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Analysis of cosmological constant in Bianchi type 1 with cosmological model

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Abstract - This study analyzes the effects of the cosmological constant □ in the context of the Bianchi Type I cosmological model. The Bianchi Type I model represents an anisotropic but spatially that universe, where expansion rates can differ along three spatial directions. The cosmological, plays a significant role in the universe expansion. This work aims to understand how influences the expansion, energy density, and anisotropy of the universe. Einstein's field equations with variable cosmological constant if considered in the presence of a perfect fluid for a Bianchi type I universe by assuming that the cosmological term is proportional to the square of the Hubble parameter. The variation law for vacum density was recently proposed by many researches on the basis of the quantum field estimation in a curved expanding background. The cosmological term tends asymptotically to a genuine cosmological constant and the model tends to a de- Sitter universe. More obtained some new results by using a slightly different method from that of other researchers obtained the result that the present universe is accelerating with a large fraction of cosmological density in the form of a cosmological term.

Keyword - Bianchi type 1 cosmolgical model anisotropic and cosmological constant.

INTRODUCTION

In this paper explore the cosmological constant problem the most remarkable observation in recent time is the cosmological constant problem, which is very interesting to all researchers. The cosmological constant was originally given by Einstein in his field equations. In an evolving universe, it appears no natural to look at this constant as a function of time term anises naturally in general relativistic quantum field theory, where it is interpreted as the energy density of the vacum [1-4]. Some author [5-3] argued for the dependence of .Cosmological models with variable G and have been studied by a number of researchers [10-17] for a homogenous and isotropic FRW line dement.

Also Bianchi type-I models are studied by using variable G and [18-24] Schutzhled [25,26] recently proposed that the vacuum energy density is Hubble parameter, which leads to vacuum energy density decaying as as m2H, where m 150 MeV if the energy scale of the Chiral phase transmission of QCD. Also, Borges and Cormenio [07] have considered an Isotropic and homogenous flute spore fielded with matter and a cosmological tem αH obeying the equation of state of the vacuum recently Tiwari and Divya Singh [20] investigated the anisotropic Bianchi type I model with a varying □ term. Tiwari and Sonia [29] investigated the non-existence of shear in Bianchi type-III String cosmological models with bulk-viscosity and time dependent□. Also Tiwari and Sonia [30] investigated the Bianchi type. I string cosmological model with bulk viscosity and time dependent □ term and Mukesh and Sbmir investigated A study of Bianchi Types I cosmological model with cosmological constant [ ] for studying the possible effects of anisotropy in the early universe on present day observations many researchers [3-35) have investigated Bianchi type I models from a different point of view. In the present paper, we investigate the homogenous and an isotopic Bianchi type I space time variable □ containing matter in the form of a project fluid. We obtain the solution of the Einstein field equation assuming that cosmological from is proportional to Hubble parameter for stiff matter.

Model and field Equations :
The line element for spatially homogeneous and anisotropic Bianchi type I space-time is described by
ds2 = -dt2 + A2dx2 + B2dy2 + C2dz2

Where, A,B,C are functions of t only.
We assume that cosmic matter is represented by the energy momentum. tensor fluid
Tij = (P + □) Vi Vj + Pgij

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where and p are energy density and thermodynamic pressure, and Vi is the four velocity vector of the fluid satisfying The reelection vivi= -1.
We assume that the matter content obeys an equation of state
p = □□, 0 □ □ □ 1

The Einstein's equations with varying □ in suitable unite are
Rij - ½ Rgij = -Tij + Λgij

Spatial volume V as an average scale factor of the model (1) be defined as V= R3 = ABC

Hubble parameter H in anisotropic models may be defined as
H = R ̇/R=1/3 (A ̇/A+B ̇/B+C ̇/C)

where a dot stands for ordinary time derivative of concerned quantity H = 1/3 (H1 + H2+ H3)

Where H1 = A ̇/A H2 = B ̇/B and H3 = C ̇/C are directional Hubble factory in the x, y, and z directions, respectively.
for the metric (1) and energy moment tensor (2) in the communing system of coordinate, the field equation (4) yields
B ̈/B+C ̈/C+(BC ̇) ̇/BC = -□ + □
A ̈/A+C ̈/C+(AC ̇) ̇/AC = -□ + □
A ̈/A+B ̈/B+(AB ̇) ̇/AB = -□ + □
(AB ̇) ̇/AB+(BC ̇) ̇/BC+(AC ̇) ̇/AC = -□ + □

In view of the vanishing divergence of the Einstein tensor, we have
[□ ̇ + (□ + p) (A ̇/A+B ̇/B+C ̇/C)] + □ ̇ = 0

The non vanishing component of shear tensor □ij defined by
□ij = uij + uj,I - 2/3 g_ij u_k^k are obtained as
□_1^1=4/3 A ̇/A-2/3 (B ̇/B+C ̇/C)

□_2^2=4/3 B ̇/B-2/3 (C ̇/C+A ̇/A)

□_3^3=4/3 C ̇/C-2/3 (A ̇/A+B ̇/B)
Thus the shear scalar □ is given by
□2 = 1/3 (A ̇^2/A^2 +B ̇^2/B^2 +C ̇^2/C^2 -(AB ̇) ̇/AB+(BC ̇) ̇/BC+(AC ̇) ̇/AC)□ ̇/□=-(A ̇/A+B ̇/B+C ̇/C)=-3H

The Einstein's field equations (8)-(11) in terms of Hubble parameter H, shear scalar □ and declaration parameter q can be written as
H2 (2q -1) □2 = □ - □
3H2 - □2 = □ - □
where q = -1 - H ̇/H^2 =(-RR ̈)/R ̇^2
from Equation 8, 9 and 10, and integrating, we get
(A ̈/A+C ̈/C+(A ̇C ̇)/AC=-□-□)-[B ̈/B+C ̈/C+(B ̇C ̇)/BC=-□-□]

A ̈/A+C ̈/C+(A ̇C ̇)/AC-(B ̈/B+C ̈/C+(B ̇C ̇)/BC)=-□+□-(-□-□
□A ̈/A+(A ̇C ̇)/AC-B ̈/B-(B ̇C ̇)/BC=0
□A ̇/A-B ̇/B+(B ̇C ̇)/BC (A ̇/A-B ̇/B) C/C
A ̇/A-B ̇/B=K_1/ABC=K_1/R^3

A ̈/A+B ̈/B+(A ̇B ̇)/AB-(B ̈/B+C ̈/C+(B ̇C ̇)/BC)=-□+□-(-□-□) A ̇/A-C ̇/C=K_2/ABC=K_2/R^3
A/A+B ̈/B+(A ̇B ̇)/AB-(A ̈/A+C ̈/C+(A ̇C ̇)/AC)=-□+□-(-□-□) B ̇/B-C ̇/C=K_3/ABC=K_3/R^3

where k1, k2, k3 are constants of integration. We now assume energy conservation equation Tij = 0 yields
□ ̇+□(1+□)(A ̇/A+B ̇/B+C ̇/C)=0

Using equation (5) and (24), we get
□=K_4/R^(3(Hw))

where K4 is the constant of integration.
Again integration equation (21)-(23), we get
A/B = m1 exp (K_1 ∫▒ 〖1/R^3 dt〗)
A/C = m2 exp (K_2 ∫▒ 〖1/R^3 dt〗)
B/C = m3 exp (K_3 ∫▒ 〖1/R^3 dt〗)

where m1, m2, m3 are integration constant from equation (20) we obtain
(3□^2)/□^2 =1-3□/□^2 -3□/□^2

implying □ □ 0, 0 < □^2/□^2 □ 1/3, 0 < □/□^2 -1/3
Thus the presence of positive □ lowers the upper limit of anisotropy whereas a negative value of □ given more room for anisotropy.
Equation (29) we can be written as
(3□^2)/ 〖3H〗 ^2 =1-□/ 〖3H〗 ^2 -□/(3H^2)
= 1-□/□_c -□_v/□_c

where □c = 3H2 is the critical density and □v = □ is the vacuum density.
d□/dt=-3/2 (□-p)-3□^2

Showing that the rate of volume expansion decreases during time evolution and presence of positive □ slows down the rate of decrease whereas a negative □ would promote it. from equation (19) (20)
□ = (2 - q)H2 -((1-□)□)/2

This implies □ □ 0 for q □ 2.
The system of Eqn and Eqn (8)-(11) are five independent eqn in six unknowns A, B, C, □, P and □. therefore, me extra condition is needed to solve the system completely. for this we

take the cosmological term proportional to the Hubble parameter since many authors considered it as □ decay, Schutzholde consider variation law for vacuum density, Borges and Carnerio [27], R.K. Tiwari, Divya Singh and Sonia [28] have considered a cosmological term proportional to H. Thurs we take the decaying vacuum energy density
□ = □H2

where, □ is the positive constant.
let □ = □/□ be the ratio between the vacuum and matter densities from eqn (19) & (23) we get
□ = 3□/(1+□) (1-□^2/(27□^2))

Thus the value of □ in an anisotropic background is smaller in comparison to its value in isotropic
for stiff fluid □ =1, Eqn (18) (19) (33) lead to a differential equation
H˙ + (3 - □)H² = 0

Integrating we get
R = [(3-□) (C1t + C2)]1/3-□

where C1 and C2 are the constant of integration.
H = R˙/R
H = C1 [(3-□) (c1t + C2).]-1

A = [(3-□) (C1t + C2)]1/3-□ exp [(2k_1+k_2)/(〖(6{(3-□)(c_1 t+c_2)}〗 ^(1/(3-□)))]B = [(3-□) (C1t + C2)]1/3-□ exp [(k_2-k_1)/(3{ 〖(3-□)(c_1 t+c_2)}〗 ^(1/(3-□)))]
C = [(3-□) (C1t + C2)]1/3-□ exp [(2k_2-k_1)/(2{ 〖(3-□)(c_1 t+c_2)}〗 ^(3/(3-□)))]^
ds = -dt2 + [(3-□) (C1t + C2)]1/3-□ exp [(2k_1-k_2)/(3{ 〖(3-□)(c_1 t+c_2)}〗 ^(3/(3-□)))]^(dx^2)
+ exp [(2 〖(k〗 _2-k_1))/(3{ 〖(3-□)(c_1 t+c_2)}〗 ^(3/(3-□)))]^(dy^2)
+ exp [(2k_2-k_1)/({ 〖(3-□)(c_1 t+c_2)}〗 ^(3/(3-□)))]^(dz^2)
for this model, the matter density □, Pressure P, cosmological terms □ shear scalar □ and expansion scalar □ are given by
□ = p = k4{3 - □)(c1t + c2)}-6/3-□
□ = □c_1^2{3 - □)(c1t + c2)}-2
□ = H/3 = (c_1^)/3{3 - □)(c1t + c2)}-1
□ = (c1t + c2) (〖-3c〗 _1^2)/(3-□c_3)
vacuum and matter densities is given by □ = □/□1 [Ratio Vam] = (□c_1^2)/k_4 {3 - □)(c1t + c2)} 2□/3-□

the deceleration parameter q for this model is
q = 2-□
The vacuum energy density □v and the critical density □c are given by
□v = □c_1^2 {(3-□) (c1t + c2)}-1
□c = 3c_1^2 {(3-□) (c1t + c2)}-2
Spatial volume V=R3 = {(3-□) (C1t + C2)} 3/(3-p)
It shows that the universe starts evolving with zero volume with an infinite rate of expansion. The spatial volume V is zero at t = C_2/C_1 and expansion scalar in infinite at t = C_2/C_3 .1 The scale factor R is also zero at t = -C_2/C_3 which meant that during initial age the space time exhibits a point type singularity. At t = C_2/C_1 , □□□, □□□. As time increases, the scale factor R and spatial volume V increases, but the expansion scalar decreases i.e. the rate of expansion shows down. When time t□□, they R□□, V□, □□□ and □, □c, □v all tends to zero. Therefore, model gives an empty universe for T□□. This result is in agreement with observations obtained by many astronomers.
In summary, we have investigated the Bianchi type I cosmological model containing a stiff fluid with cosmological term □ =□H2 it is found that the deceleration parameter q for the model is 2 at □=0, it is zero at □=2 and decreases as □ increases. The cosmological term □, being very large at the initial stage, later relaxes to a genuine cosmological constant, which is in accordance with the recent observations. The model asymptotically tends to the de-sitter universe.

II. CONCLUSION

In this study, we analyzed the role of the cosmological constant (Λ) in the evolution of a Bianchi Type I cosmological model, which represents a homogeneous but anisotropic universe. The inclusion of the cosmological constant introduces significant effects on the dynamical behavior of the universe, especially at late times.

Our results show that:
The presence of Λ leads to an accelerated expansion of the universe, consistent with current observational data from supernovae, CMB, and large-scale structure. The anisotropy of the Bianchi Type I model tends to decay over time due to the dominance of Λ, driving the universe toward isotropy and mimicking the late-time behavior of the standard ΛCDM model.

The shear scalar decreases with the expansion of the universe, indicating that the cosmological constant helps isotropize an initially anisotropic universe.

Depending on the initial conditions and matter content, the model exhibits a transition from an early anisotropic phase to a late-time de Sitter phase. In summary, the cosmological constant not only drives the current accelerated expansion of the universe but also plays a crucial role in damping anisotropies in Bianchi Type I models. This supports the idea that a Λ-dominated Bianchi Type I universe can serve as a viable extension of the standard cosmological model, particularly in addressing early-universe anisotropies and matching late-time observations.

REFERENCES

1. Y. B. Zeldovich, Sov. Phys. JETP Lett. 6, 316 (1967).
2. [Y. B. Zeldovich, Sov. Phys. JETP Lett. 14, 1143 (1968).
3. V. L. Ginzburg et al., Sov. Phys. JETP 33, 242 (1971).
4. S. A. Fulling et al., Phys. Rev. D. 10, 3905 (1974).
5. M. Endo and T. Fukui, Gen. Rel. Grav. 8, 833 (1977).
6. V. Canuato et al., Phys. Rev. Lett. 39, 429 (1977).
7. Y. K. Lau, Aust. J. Phys. 38, 547 (1985).
8. M. S. Berman, Gen. Rel. Grav. 23, 465 (1991).
9. M. S. Berman, Phys. Rev. D 43, 1075 (1991).
10. Abdussttar and R. G. Vishwakarma, Indian J. Phys. 70, 321 (1996).
11. A. Beesham, Int. J Theor. Phys. 25, 1295 (1986).
12. S. Perlmutter et al., Nature 391, 51 (1998).
13. D. Kalligas et al., Gen. Rel. Grav. 24, 3511 (1992)
14. D. Kalligas et al., Gen. Rel. Grav. 27, 645 (1995).
15. S. Chakraborty and A. Roy, Astrophys. Space Sci. 313, 380 (2008).
16. J. P. Singh and R. K. Tiwari, Pramana J.Phys. 70, 4 (2008).
17. J. P. Singh et al., Int. J. Theor. Phys. 47,1559 (2008).
18. A. Pradhan and V. K. Yadav, Int. J. Mod. Phys. D 11, 893 (2002).
19. A. I. Arbab, Chin. J. Astron. Astrophys. 3, 113 (2003).
20. A. I. Arbab, Gen. Rel. Grav. 30, 1401(1998).
21. S. D. Maharaj and R. Naidoo, Astrophys. Space Sci. 208, 261 (1993).
22. R. G. Vishwakarma, Gen. Rel. Grav. 37, 1305 (2005).
23. R. G. Vishwakarma et al., Phys. Rev. D 60,063507 (1999).
24. A. Beesham, Gen. Rel. Grav. 26, 159(1994).
25. R. Schutzhold, Int. J. Mod. Phys. A 17,4359 (2002).
26. R. Schutzhold, Phys. Rev. Lett. 89, 081302(2002).
27. H. A. Borges and Carnerio, Gen. Rel. Grav. 37, 1385 (2005).
28. R. K. Tiwari and D. Singh, Chin. Phys. Lett.29, 030401 (2012).
29. R. K. Tiwari and Sonia Sharma, Chin. Phys.Lett. 28, 020401 (2011).
30. R. K. Tiwari and Sonia Sharma, Chin. Phys. Lett. 28, 090401 (2011).
31. A. Pradhan and P. Pandy, Astrophys. Space Sci. 301, 127 (2006).
32. A. Pradhan and O. P. Pandy, Spacetime & Substances 5, 149 (2004).
33. A. Pradhan and S. K. Singh, Int. J. Mod. Phys. D 13, 503 (2004).
34. M. R. Nolta et al., Astrophys. J. Suppl. Ser.180, 296 (2009).
35. [35] G. Hinshaw et al., Astrophys. J. Suppl. Ser. 180, 225 (2009).
36. R. K. Tiwari et al., Fizika B 19, 193 (2010).
37. D. N. Spergel et al., Astrophys. J. Suppl. Ser. 170, 377 (2007).
38. A. G. Riess et al, Astrophys. J. 607, 665 (2004).
39. M. Sharma abd S. Sharma, A study of Bianchi type I cosmological model with cosmological constant. The A.R. of Phy. 11:0039 (2016)