International Journal of Scientific Research & Engineering Trends
Volume 11, Issue 2, Mar-Apr-2025, ISSN (Online): 2395-566X

MATLAB Based Analysis of Synthetic Chirp Signals Using FFT and CWT

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Abstract- This paper compares two popular methods for time-frequency analysis: the Fast Fourier Transform (FFT) and the Continuous Wavelet Transform (CWT) for a synthetic chirp signal wave. We apply these techniques to a chirp signal, which is a non-stationary signal with a frequency that changes over time. The results demonstrate the strengths and limitations of each method, highlighting the FFT's ability to provide global frequency information and the CWT's superior time-frequency localization. FFT renders the representation of signal only in the frequency domain. The CWT, using the morse wavelet, on other hand provides a more compact visualisation of the signal in both time and frequency domain visualising or representing the frequency domain of the signal simultaneously. The performance of these techniques is compared visually and computationally.

Index Terms- MATLAB, Fast Fourier Transform (FFT), Continuous Wavelet Transform (CWT), Chirp Signal, Morse Wavelet, Non-Stationary Signals, DSP TOOLBOX, eigen scalograms

I. INTRODUCTION

The ability to study signals with frequencies that change over time is crucial in fields like communications, radar, and biomedical signal processing [1]. One example of such a signal is the chirp signal, where the frequency varies as time progresses. Traditional Fourier analysis techniques, such as the Fast Fourier Transform (FFT), fall short in capturing the frequency changes over time, making it challenging to analyse such signals effectively. The FFT serves as a bridge between the time and the frequency domain [2].

The FFT provides a global frequency visualization of the selected signal. For the continuous signals, FFT is computed:

$$X(f) = \int_{-\infty}^{+\infty} x(t) \times e^{-j\omega t} dt$$

For the FFT of a Sampled Signal:

$$X[k] = \sum_{n=0}^{N-1} X[n] \times e^{-j \times \left(\frac{2 \times \pi}{N}\right) \times kn}$$

Where,

- X[k] is the frequency-domain representation of the signal,
- k is the frequency index,
- N is number of samples of the signal

However, the Continuous Wavelet Transform (CWT) addresses this limitation by offering a time-frequency representation, which is better suited for analysing signals that are non-stationary. Wavelets refer to a broad category of basis functions that have an integral value of zero[3].

$$CWT(a,b) = \frac{1}{\sqrt{a}} \times \int_{-\infty}^{+\infty} x(t) \times \psi^{*}(t) \times \left(\frac{t-b}{a}\right) dt$$

Where,

- x(t) is the signal,
- ψ is the mother wavelet,
- a is the scale factor that controls the width of the wavelet,
- b is the translation factor (shifting the wavelet over time),
- ψ* is the complex conjugate of the wavelet

CWT provides an easily interpretable visual representation of both stationary and non-stationary(non-periodic) signals[4].

To examine the advantages and drawbacks of each method, a synthetic linear chirp signal was created using MATLAB's

International Journal of Scientific Research & Engineering Trends
Volume 11, Issue 2, Mar-Apr-2025, ISSN (Online): 2395-566X

DSP toolbox. This signal starts at 0 Hz and increases linearly to 10 Hz over 100 seconds.

Linear Chirp Signal is represented by the below equation:

$$x(t) = A \times \sin\left(2\pi\left(f_0 t + \frac{k}{2}t^2\right)\right)$$

Where,

- x(t) is the Chirp signal as a function of time,
- A is the Amplitude of the signal
- f_0 Initial frequency at time
- k is the Chirp rate (Hz/s)
- t is the time in seconds

This paper primarily seeks to compare the time-frequency resolution capabilities of FFT and CWT when applied to a non-stationary chirp signal. The aim is to illustrate that, while FFT offers valuable insights into the frequency domain, it falls short in pinpointing frequency variations over time—a limitation effectively addressed by the CWT, which provides insights into both time and frequency dynamics[5] .
Morse wavelets are the eigenfunction wavelets appropriate for time-varying spectrum estimation through the averaging of time-scale eigenscalograms[6].

In other words, analytic wavelets are inherently organized into pairs: one resembling even or cosine-like functions and the other resembling odd or sine-like functions. This pairing helps then capture the phase variability[7].

II. METHODOLOGY

This research utilized a computational method in MATLAB to investigate the time-frequency properties of a linear chirp signal. The process was carried out in three key phases: generating the signal, conducting frequency-domain analysis via the Fast Fourier Transform (FFT), and performing time-frequency analysis using the Continuous Wavelet Transform (CWT).

1. Signal Generation

A linear chirp signal was synthesized using MATLAB's dsp.Chirp system object, which allows precise control over signal parameters such as frequency range, sweep duration, and sampling characteristics. Using MATLAB's `dsp.Chirp` system object, a linear chirp signal was created, designed to sweep linearly from 0 Hz to 10 Hz over a 100-second interval. With a sampling rate of 50 Hz, 5000 samples were produced to cover the entire duration. The SweepTime and TargetTime parameters were both set to 100 seconds, ensuring the chirp completes its frequency sweep precisely at the end of the signal duration. The resulting signal was then transposed into a row vector to align with the required input format for further analysis. The generated signal was transposed into a row vector to match the expected input format for subsequent analysis.

The MATLAB syntax used to generate the signal is:

```
hchirp = dsp.Chirp(...
'InitialFrequency', 0,...
'TargetFrequency', 10, ...
'TargetTime', 100, ...
'SweepTime', 100, ...
'SampleRate', 50, ...
'SamplesPerFrame', 5000);
```

To create the chirp signal, the below syntax for the step function was utilized on the dsp.Chirp System object, resulting in the complete 100-second signal. The obtained column vector was subsequently transposed.

```
chirpData = step(hchirp)';
```

Using the syntax = "plot(chirpData)"

The chirp signal was plotted as shown in the below picture:

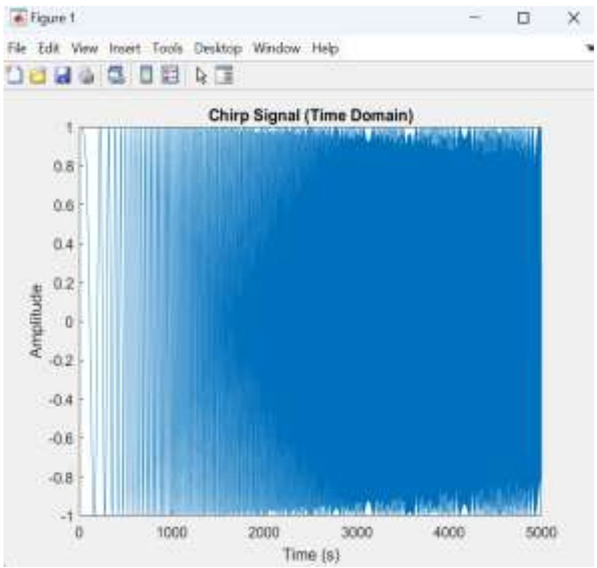

Fig.1: Chirp Signal Generation in MATLAB

2. Frequency Domain Analysis using FFT

To examine the chirp signal within the frequency domain, the first step involved identifying the number of samples present in the signal.

The chirp signal underwent a Fast Fourier Transform (FFT) to convert it into the frequency domain. The output was then centered using `fftshift` to enable a symmetrical display around 0 Hz. This shift realigns the zero-frequency

International Journal of Scientific Research & Engineering Trends
Volume 11, Issue 2, Mar-Apr-2025, ISSN (Online): 2395-566X

component to the center of the spectrum, making interpretation more intuitive.

The frequency axis was adjusted based on the sampling rate, resulting in a range from –25 Hz to +25 Hz, aligning with the Nyquist limits of the sampled signal. This step revealed the spread of frequency components across the spectrum, showing energy distribution from 0 to 10 Hz consistent with the chirp definition.

The above steps were executed using the below syntax:

```
N = length(chirpData);
fftchirp = fft(chirpData);
realfft = abs(fftshift(fftchirp)); % Center FFT
freq = (-N/2:N/2-1)*(50/N);
```

The magnitude of the centred FFT was taken using abs() Function in MATLAB to obtain the real-valued spectrum for plotting.

3. Time-Frequency Analysis using CWT

To examine the time-varying frequency traits of the chirp signal, the Continuous Wavelet Transform (CWT) was performed using the cwt function in MATLAB. The Morse wavelet, an analytic wavelet, was selected for this purpose, with the sampling rate configured at 50 Hz and the below syntax in MATLAB code was used:

```
[wt, f] = cwt (chirpData, 50, 'morse');
```

wt and f are the outputs of continuous wavelet transform, where wt consists of all the complex wavelet coefficients representing the signal's time-frequency content and f holds the corresponding frequency values in Hertz.

Hence, The Morse wavelet was selected for continuous wavelet transform (CWT) analysis because of its superior time-frequency resolution and its adaptability to the unique traits of linear chirp signals. Its analytic properties ensure the preservation of frequency content without introducing negative components, making it particularly well-suited for analyzing non-stationary signals.

III. RESULTS

The analysis of the synthetically generated linear chirp signal was performed in both the frequency and time-frequency domains to assess its spectral properties and how its frequency components change over time. This dual-domain evaluation provided a comprehensive understanding of the signal's behaviour, including its spectral characteristics and the gradual evolution of its frequency content.

1. Frequency Domain Analysis of the Chirp Signal

The Fast Fourier Transform (FFT) was employed on the chirp signal to derive its frequency-domain representation. This process provided valuable insights into the signal's spectral components, enabling a deeper understanding of its frequency characteristics.

As shown in Fig. 2 below, the resulting spectrum exhibits symmetry around 0 Hz due to the use of fftshift, which repositions the DC component at the centre. The energy is predominantly concentrated in the range of 0 Hz to 10 Hz, aligning perfectly with the frequency sweep range specified during the chirp signal generation.

This demonstrates that the spectral content of the signal is accurately confined within the anticipated Nyquist limits (±25 Hz), as determined by the 50 Hz sampling rate.

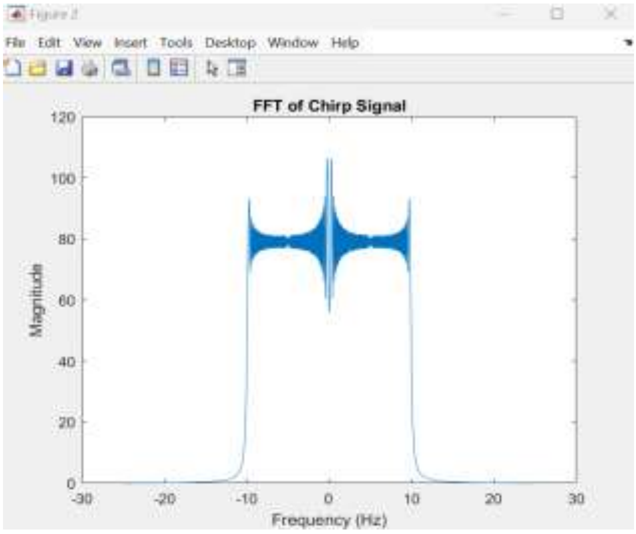

Fig. 2: Frequency-domain representation of the linear chirp signal using FFT.

The main lobe exhibits prominent peaks, reflecting strong frequency components in that range. Meanwhile, the side lobes represent spectral leakage, a common effect caused by the finite duration of signals. The symmetrical structure and concentrated energy validate the chirp's linearity and spectral consistency throughout the sweep.

The signal's peak magnitude, reaching around 110 units, represents the highest amplitude of its strongest frequency component. The primary frequency band spans symmetrically from approximately –10 Hz to +10 Hz, aligning with the linear chirp's frequency sweep from 0 Hz to 10 Hz.

Time-Frequency Domain Analysis of the Chirp Signal using CWT

To examine the time-varying frequency characteristics, the Continuous Wavelet Transform (CWT) was applied, utilizing the Morse wavelet for analysis. The findings are depicted in Fig. 3 below , plotting the evolution of the signal's frequency content over time.

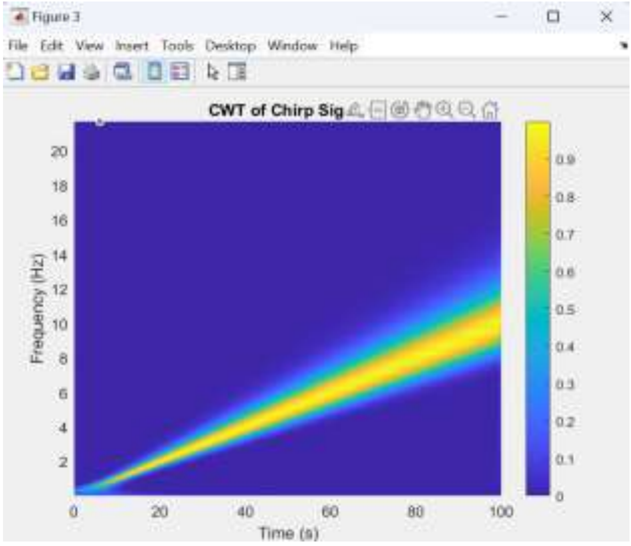

Fig.3 : Frequency time analysis of Chirp Signal using CWT

The time axis spans from 0 to 100 seconds, while the frequency resolution ranges from 0 Hz to approximately 22 Hz, with the majority of the signal's energy concentrated below 10 Hz. The CWT plot's ridge, represented by the brightest region in the above fig.3 , aligns diagonally from (0 s, ~0 Hz) to (100 s, ~10 Hz), clearly demonstrating the linear frequency modulation of the chirp over time.

The energy concentration around the ridge has a width of about ±1 Hz, signifying excellent time-frequency localization—ideal for accurately tracking frequency variations over time. Furthermore, the maximum wavelet coefficient magnitude, near 0.95–1.0 on the colour scale, highlights the significant energy concentration along the chirp's instantaneous frequency trajectory.

IV. CONCLUSION

The chirp signal was analyzed using both Fast Fourier Transform (FFT) and Continuous Wavelet Transform (CWT) methods, as illustrated in the provided figures. The FFT results (Fig.2) reveal the global frequency characteristics, showcasing pronounced spectral components symmetrically centered around 0 Hz, with sharp peaks exceeding magnitudes of 100. This confirms the dominance of frequencies within the range of approximately –10 Hz to +10 Hz. However, FFT lacks the ability to depict the evolution of frequency over time, making it less suitable for analyzing non-stationary signals like chirps. On the other hand, the CWT results (Fig.3) provide a detailed time–frequency perspective. The time axis highlights a linear increase in frequency from around 1 Hz to over 20 Hz throughout the 100-second signal duration. The scalogram's ridge, with its diagonal trajectory, demonstrates the chirp's dynamic frequency modulation, effectively capturing time-dependent changes in the frequency content. This temporal precision is crucial when analyzing signals with non-stationary components.

To sum up, the Continuous Wavelet Transform (CWT) outshines the Fast Fourier Transform (FFT) when analyzing non-stationary signals[8].

While FFT offers high resolution for stationary signals, the CWT excels in analyzing signals with time-dependent frequency variations. For this synthetically generated chirp signal, CWT proves to be the more appropriate tool, offering a comprehensive and interpretable representation of its dynamic behavior.

Its ability to provide precise time-frequency localization makes it a more effective tool for capturing the dynamic characteristics of non-stationary or non-periodic signals.

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