

Assessing Model Misspecification in Stochastic Linear Regression Analysis

Research Scholar Siddamsetty Upendra, Research Scholar R. Abbaiah

Department of Statistics,
Sri Venkateswara University, Tirupati, India

Abstract- This paper studies misspecification tests for stochastic linear regression models, including the Durbin-Watson test, Ramsey's regression specification error test, Lagrange's multiplier test, and UTTS' rainbow test. Specification errors arise when there are deviations from the underlying assumptions of a stochastic linear regression model, impacting associated inferences. Specifically, errors may occur in specifying the error vector (ϵ) and the data matrix (X). Common causes of specification errors involve including irrelevant independent variables or excluding relevant ones in the stochastic linear regression model. Previous research by Ivan Krivy et al. (2000) presented two stochastic algorithms for estimating parameters in nonlinear regression models. In a 1984 paper, Russell Davidson et al. developed a computational procedure for a variety of model specification tests. Ludger Ruschendorf et al. (1993) constructed nonlinear regression representations of general stochastic processes, focusing on specific representations for Markov chains and certain m -dependent sequences. This study contributes to the understanding of misspecification in stochastic linear regression models, utilizing a range of tests to identify errors in model assumptions and parameter estimation. The insights gained from these tests can enhance the accuracy and reliability of regression model inferences.

Index Terms- Misspecification, Durbin-Watson test, Ramsey's regression specification error test (RESET), Lagrange multiplier test (LM-test), and UTTS' Rainbow test.

I. INTRODUCTION

The relationship between data and the specification of a linear regression model is integral. The specification of a stochastic regression model holds significant implications for the necessary data and may be constrained by data limitations.

The specification of a stochastic regression model encompasses several key components:

- **Listing of Independent Variables:** This involves identifying independent variables, including lagged values, for each equation in the model.
- **Formulating the Functional Form:** Defining the functional form that relates independent variables to the dependent variables is a crucial step in the specification process.
- **Stochastic Properties of the Error Term:** This includes making assumptions about the unobservable error variables. Under the first two items, each specification yields estimates of the errors.

The first two aspects necessitate a substantial interaction between theory and data. The third aspect involves assumptions about the characteristics of unobservable error

variables, and each specification provides estimates of these errors.

In stochastic model building, the correct specification of the stochastic regression model is paramount. If the model is not accurately specified, the tests become invalid, and various stochastic statements derived from the model are no longer reliable. The precision of the estimates and the validity of the model's inferences depend on the accuracy of the model specification.

II. MISSPECIFICATION TEST FOR THE DURBIN-WATSON TEST STATISTIC IN STOCHASTIC LINEAR REGRESSION MODEL

The fundamental stochastic linear regression model is expressed as:

$$Y = X\beta + \epsilon \quad (1.1)$$

Here's a closer look at its components:

- Y : This signifies the dependent variable.
- X : Represents the matrix of independent variables.

- β : Denotes the vector of coefficients.
- ϵ : Characterizes the error term or residuals.
- $\epsilon \sim N(0, \sigma^2 I)$

In essence, the model encapsulates the interrelation between the dependent variable Y and the independent variable X , with the error term ϵ capturing unobserved factors influencing the dependent variable. Assumptions are made about the normality and homoscedasticity of the error distribution, with a consistent variance $\sigma^2 I$ maintained across all observations. The estimation of this model typically involves methodologies like Ordinary Least Squares (OLS).

One may obtain the OLS estimator $\hat{\beta}$ as the BLUE for β which is given by

$$\hat{\beta} = X^{-1}(X^T)^{-1}(X^T Y) \quad (1.2)$$

The OLS residual vector emends the difference of column vectors Y and $\hat{\beta}X$ i.e., Y and \hat{Y}

Suppose that X^* be an excluded relevant independent variable which is excluded from the stochastic linear regression model. Now arrange the OLS residual in an order according to the increasing values of excluded independent variables X^* .

Let the ordered OLS residual be denoted by $[e_{(1)} \ e_{(2)} \ \dots \ e_{(n)}]$

By using the Durbin –Watson statistic one may test the null hypothesis that the model is mis-specified.

The Durbin-Watson test statistic is given by

$$d^* = \frac{\sum_{i=2}^n [e_i - e_{i-1}]^2}{\sum_{i=1}^n e_i^2} \quad (1.3)$$

In the context of the Durbin–Watson test, if the calculated value of the Durbin–Watson statistic (d^*) is deemed significant based on the critical values provided in the Durbin–Watson table, it implies that there is evidence to accept the null hypothesis of misspecification in the stochastic linear regression model at the chosen level of significance. This significance indicates the presence of autocorrelation in the residuals, suggesting a departure from the expected independence of observations. Accepting the null hypothesis of misspecification implies that there may be issues with the assumed error structure in the model, prompting a closer examination and potential refinement of the regression model to enhance its validity.

III. RESET (RAMSEY’S REGRESSION SPECIFICATION ERROR TEST) FOR THE MISSPECIFICATION OF STOCHASTIC LINEAR REGRESSION MODEL

Consider a standard stochastic linear regression model, expressed as

$$Y_i = X_i' \beta + \epsilon_i; \ i=1, 2, \dots, n \quad (2.1)$$

where

Y_i is the dependent variable for observation i , X_i' represents the transpose of the i^{th} row of the matrix X denoting the original independent variables for observation i , β is a vector of unknown parameters, and ϵ_i is the random error term for observation i .

Now, let's augment this model by introducing an additional set of independent variables Z_i . The augmented stochastic linear regression model is then represented as

$$Y_i = X_i' \beta + Z_i' \Gamma + \epsilon_i; \ i=1, 2, \dots, n. \quad (2.2)$$

This augmented model, Z_i' signifies the transpose of the i^{th} row of the matrix Z , which contains the augmented independent variables for observation i . The vector Γ comprises unknown parameters associated with these augmented variables.

Matrix notation can be employed to denote the various components:

- Y is an $(n \times 1)$ vector encompassing all dependent variable observations.
- X is an $(n \times k)$ matrix containing the original independent variable observations.
- Z is an $(n \times m)$ matrix containing the augmented independent variable observations.
- β is a $(k \times 1)$ vector representing parameters related to the original variables.
- Γ is an $(m \times 1)$ vector representing parameters related to the augmented variables.
- ϵ is an $(n \times 1)$ vector representing random errors.

This augmented model provides the flexibility to incorporate additional independent variables Z_i to capture additional factors influencing the dependent variable Y_i beyond the original set of variables X_i . The parameters Γ account for the effect of these augmented variables on the dependent variable. Where Z_i may be chosen as independent variables which are responsible for the misspecification of the model. Under a mis-specification test, one may use F-test to

test $H_0 : \Gamma = 0$. Here the critical problem is the choice of Z_i 's. Often Z_i 's may be approximated by higher powers in X_i 's or higher powers of estimates $\hat{Y}_i = X_i' \hat{\beta}$, and the OLS estimator of β is obtained as $\hat{\beta}$. Therefore, the Ramsey RESET test involves the following steps:

- Perform a regression of Y_i on X_i' and calculate the estimated coefficients \hat{Y}_i' .
- Conduct a regression of Y_i on X_i' , \hat{Y}_i^2 , \hat{Y}_i^3 and \hat{Y}_i^4 , and obtain the coefficients \hat{Y}_i' .
- Utilize an F-test to examine if the coefficients of all the powers of \hat{Y}_i' are equal to zero.

Under the null hypothesis the F-test statistic defined as

$$F = \frac{[R^{*2} - R^2] / q}{(1 - R^{*2}) / (n - q)} \sim F_{[q, (n-q)]} \quad (2.3)$$

where R^2 = The coefficient of multiple determinative without Z_i 's

R^{*2} = The coefficient of multiple determinative including Z_i 's
 q = Number of Z_i 's in the model.

A Lagrange Multipliers Test is used to check the Miss-Specification of a Stochastic Linear Regression Model

The standard stochastic linear regression model is expressed as:

$$Y = X\beta + \epsilon \quad (3.1)$$

Here, Y is an $(n \times 1)$ vector, X is an $(n \times k)$ matrix, β is a $(k \times 1)$ vector, ϵ is an $(n \times 1)$ vector and $\epsilon \sim N(0, \sigma^2 I)$

The OLS estimator $\hat{\beta}$ as the BLUE for β which is given by

$$\hat{\beta} = (X'X)^{-1}(X'Y) \quad (3.2)$$

Assume that Z represents an $(n \times p)$ matrix of regressors that are incorporated in the model, and their coefficients are to be examined for potential mis-specification of the model.

Express the augmented stochastic model as:

$$Y = X\beta + z\delta + \nu \quad (3.3)$$

Here, δ is an $(p \times 1)$ vector of parameters associated with Z , and ν is a vector of errors.

By utilizing OLS estimation, one can estimate the restricted stochastic linear regression model (3.1) and obtain the OLS residual vector $[Y - \hat{Y}]$ where $\hat{Y}_i = X_i' \hat{\beta}$.

If we assume that the unrestricted regression model or augmented regression model (3.3) is the true regression model, then the OLS residual vector e should be related to Z .

This implies that we can regress e on all the regressors, including those in the restricted regression, resulting in:

$$Y = X\beta + Z\delta + \nu \quad (3.4)$$

Here, ν is an error term with the usual properties. For a large sample size, we have:

$$\chi^2 = nR^2 \stackrel{asy}{\sim} \chi_p^2 \quad (3.5)$$

Where p is the number of restrictions imposed by the stochastic restricted regression, and R^2 is the coefficient of multiple determination obtained from the augmented stochastic regression model.

If χ^2 is significant at the chosen level of significance, then we can reject the restricted stochastic regression model. Otherwise, we cannot reject it and should reject the augmented stochastic regression model. It is important to note that the Lagrange multiplier test serves as an alternative to Ramsey's RESET test.

IV. THE UTTS' RAINBOW TEST IS DESIGNED TO DETECT MISSPECIFICATION IN THE STOCHASTIC LINEAR REGRESSION MODEL

If the functional relation is not linear, the Rainbow test can be used to test for misspecification of the model. This test examines the null hypothesis of a linear relationship based on a diagonalization criterion.

In the standard stochastic linear regression model, denoted as

$$Y = X\beta + \epsilon \quad (4.1)$$

where Y is the dependent variable, X is the matrix of independent variables, β is the vector of coefficients, and ϵ is the error term, the ordinary least squares (OLS) residual sum of squares (RSS) is calculated as

$$e = Y - X \quad (4.2)$$

To perform the Rainbow test, one can obtain the OLS-RSS based on the center half of the sample, denoted as \tilde{e}^{\wedge} . The null and alternative hypotheses can be stated as Under And Under

Where e is the OLS-RSS and e' is its transpose. The Rainbow test statistic, denoted as F , is used to test H_0 . It is calculated as

$$F = \frac{((\tilde{e}^{\wedge})' \tilde{e}^{\wedge}) / (n/2)}{F[n/2, (n/2 - k)]} \sim F[n/2, (n/2 - k)]$$

where k is the number of independent variables, n is the sample size, and e is the OLS-RSS under H_0 .

To test the hypothesis that the regression model fits the data correctly, we use several test statistics such as F-statistic, UTTS, Chow test and Rainbow test. Among these, the F-statistic is most commonly used. It compares the calculated value of the F-statistic with its critical value to determine the fit of the middle half of the model. Although the UTTS and Chow test statistics are similar, they differ in sub-sample selection. On the other hand, the Rainbow test uses a data-driven approach to select the first half of observations by considering their distance from the mean. Overall, these test statistics help us confidently assess the fitness of the regression model to the data.

R-Code to perform the above Misspecification Tests for Stochastic Linear Regression Model

```
# Function to generate synthetic data and perform tests
generate_and_perform_tests <- function(num_observations = 500) {
  set.seed(123) # Set seed for reproducibility

  # Generate synthetic data for demonstration purposes
  advertising_expense <- rnorm(num_observations, mean = 100, sd = 20)
  customer_satisfaction <- rnorm(num_observations, mean = 80, sd = 10)
  error <- rnorm(num_observations, mean = 0, sd = 5)
  sales <- 97.83 + 0.114 * advertising_expense - 0.04386 *
customer_satisfaction + error

  # Create data frame
  supermarket_data <- data.frame(sales, advertising_expense,
customer_satisfaction)

  # Fit linear regression model
  model <- lm(sales ~ advertising_expense +
customer_satisfaction, data = supermarket_data)
  print(model)
```

```
print(summary(model))

# Perform Durbin-Watson Test
durbin_watson_statistic <- calculate_durbin_watson(model)
interpret_durbin_watson(durbin_watson_statistic)

# Perform RESET (Ramsey's Regression Specification Error Test)
reset_test_result <- perform_reset_test(model)
cat("\nRESET (Ramsey's Regression Specification Error Test) Result:\n")
cat(reset_test_result)

# Perform A Lagrange Multipliers Test
lagrange_test_result <- perform_lagrange_test(model)
cat("\nA Lagrange Multipliers Test Result:\n")
cat(lagrange_test_result)

# Perform UTTS' Rainbow Test
rainbow_test_result <- perform_rainbow_test(model)
cat("\nUTTS' Rainbow Test Result:\n")
cat(rainbow_test_result)
}

# Function to calculate Durbin-Watson test statistic
calculate_durbin_watson <- function(model) {
  # Extract residuals from the model
  residuals <- resid(model)

  # Calculate Durbin-Watson test statistic
  dw_statistic <- sum(diff(residuals)^2) / sum(residuals^2)

  return(dw_statistic)
}

# Function to interpret Durbin-Watson test result
interpret_durbin_watson <- function(dw_statistic) {
  cat("\nDurbin-Watson Test Result:\n")
  cat("Durbin-Watson Test Statistic:", dw_statistic, "\n")
  cat("Interpretation: ")
  if (dw_statistic < 1.5) {
    cat("There is evidence to suggest positive autocorrelation.\n")
  } else if (dw_statistic > 2.5) {
    cat("There is evidence to suggest negative autocorrelation.\n")
  } else {
    cat("There is no significant evidence of autocorrelation.\n")
  }
}

# Function to perform RESET (Ramsey's Regression Specification Error Test)
perform_reset_test <- function(model) {
  # Obtain fitted values from the model
```

```
fitted_values <- fitted(model)

# Fit a linear regression model with squared fitted values
lm_model <- lm(I(sales^2) ~ . + I(fitted_values^2), data =
model.frame(model))

# Perform RESET test
reset_test_result <- anova(lm_model)
p_value <- reset_test_result$`Pr(>F)`[2]

# Interpret RESET test result
if (p_value < 0.05) {
  return("There is evidence to reject the null hypothesis of
correct functional form specification. This suggests a potential
specification error.")
} else {
  return("There is no significant evidence to reject the null
hypothesis of correct functional form specification. The model
may be correctly specified.")
}
}

# Function to perform A Lagrange Multipliers Test
perform_lagrange_test <- function(model) {
  # Obtain residuals from the model
  residuals <- resid(model)

  # Fit a linear regression model with squared residuals
  lm_model <- lm(sales ~ . + I(residuals^2), data =
model.frame(model))

  # Perform Lagrange Multipliers Test
  lagrange_test_result <- anova(lm_model)
  p_value <- lagrange_test_result$`Pr(>F)`[2]

  # Interpret Lagrange Multipliers Test result
  if (p_value < 0.05) {
    return("There is evidence to reject the null hypothesis of
homoscedasticity. This suggests heteroscedasticity in the
model.")
  } else {
    return("There is no significant evidence to reject the null
hypothesis of homoscedasticity. The model may exhibit
homoscedasticity.")
  }
}

# Function to perform UTTS' Rainbow Test
perform_rainbow_test <- function(model) {
  # Obtain residuals and fitted values from the model
  residuals <- resid(model)
  fitted_values <- fitted(model)

  # Fit a linear regression model for squared residuals on fitted
values
```

```
lm_model <- lm(I(residuals^2) ~ fitted_values +
I(fitted_values^2), data = model.frame(model))

# Perform F-test for the added variables
f_test <- summary(lm_model)$fstatistic

# Extract p-value from F-test
p_value <- pf(f_test[1], f_test[2], f_test[3], lower.tail =
FALSE)

# Interpret UTTS' Rainbow Test result
if (p_value < 0.05) {
  return("There is evidence to reject the null hypothesis of
correct functional form specification. This suggests a potential
specification error.")
} else {
  return("There is no significant evidence to reject the null
hypothesis of correct functional form specification. The model
may be correctly specified.")
}
}

# Generate data and perform tests
generate_and_perform_tests(500)
```

V. RESULTS

```
Call:
lm(formula = sales ~ advertising_expense +
customer_satisfaction,
data = supermarket_data)

Coefficients:
(Intercept)                                advertising_expense
customer_satisfaction                        0.12969          -0.01729

Call:
lm(formula = sales ~ advertising_expense +
customer_satisfaction,
data = supermarket_data)

Residuals:
Min      1Q  Median      3Q      Max
-13.6005 -3.4430  0.0964  3.2865 16.4139

Coefficients:
(Intercept)      Estimate Std. Error t value Pr(>|t|)
(Intercept)      94.25643    2.16211  43.595 <2e-16 ***
advertising_expense 0.12969    0.01142  11.352 <2e-16 ***
customer_satisfaction -0.01729    0.02199  -0.786  0.432
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 4.958 on 497 degrees of freedom
Multiple R-squared: 0.2083, Adjusted R-squared: 0.2051

F-statistic: 65.38 on 2 and 497 DF, p-value: < 2.2e-16

Regression model

$$Y = 94.25643 + 0.12969 X_1 + (-0.01729) X_2$$

Here,

Y = Sales, X_1 = advertising_expense and X_2 = customer_satisfaction

Durbin-Watson Test Result

Durbin-Watson Test Statistic: 1.99841

Interpretation: There is no significant evidence of autocorrelation.

RESET (Ramsey's Regression Specification Error Test) Result

There is no significant evidence to reject the null hypothesis of correct functional form specification. The model may be correctly specified.

Lagrange Multipliers Test Result

There is no significant evidence to reject the null hypothesis of homoscedasticity. The model may exhibit homoscedasticity.

UTTS' Rainbow Test Result

There is no significant evidence to reject the null hypothesis of correct functional form specification. The model may be correctly specified

VI. CONCLUSION

The analysis employed various diagnostic tests to assess the accuracy of a stochastic linear regression model applied to supermarket sales data.

The regression model indicates that advertising expenses positively impact sales, with a coefficient estimate of 0.12969, while customer satisfaction has a negative coefficient estimate of -0.01729, though not statistically significant. The model's overall explanatory power, as measured by the adjusted R-squared value, is 20.51%, indicating that the included variables explain approximately 20.51% of the variation in sales.

Furthermore, diagnostic tests revealed important insights into the model's validity. The Durbin-Watson test statistic of 1.99841 suggests no significant evidence of autocorrelation in the residuals, indicating reasonable assumptions of independent errors. The RESET test, Lagrange Multipliers Test, and UTTS' Rainbow Test collectively indicate no significant evidence to reject the null hypothesis of correct functional form specification, suggesting that the model may

be correctly specified regarding its structure and functional form.

Overall, while the model demonstrates some explanatory power, further refinement may be necessary to enhance its predictive accuracy. Future research could explore alternative variables or modeling techniques to better capture the complex relationships influencing supermarket sales. Additionally, robustness checks and validation against out-of-sample data could provide further insights into the model's reliability and generalizability.

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