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Risk Management Using VaR, CVaR and Baye's Model

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Abstract-Managing risks effectively is essential in the world of trading and investing to reduce the chance of losing money and improve the quality of decisions. This document delves into how Value at Risk (VaR), Conditional Value at Risk (CVaR), and Bayes' Theorem are used to evaluate and handle financial risks. VaR offers a numerical estimate of the possible decrease in the value of a portfolio over a certain period, providing a glimpse into the most severe outcome under typical market conditions. CVaR builds on this by looking at the expected loss when the VaR threshold is surpassed, tackling the rare but significant risks that VaR might miss. Bayes' Theorem is used to refine risk evaluations with fresh data, boosting the reliability of risk prediction models. By examining these techniques and how they can be combined, the document seeks to introduce a detailed strategy for risk management, showing how the use of these methods can result in more thorough risk evaluations and better strategic choices in trading and investing. The research also points out real- world uses and its constraints, providing a guide for refining risk management strategies in ever-changing financial landscapes.

Index Terms-Portfolio Optimization, Market Risk, Value at Risk

I. INTRODUCTION

Thinking little of dangers can cause gigantic issues (such as liquidation) for banks and other parts of the money related markets. On the other hand, overestimation of dangers may make financial specialists as well touchy to dangers and miss the speculation openings, in this way lessening the return. Hence, it is exceptionally critical to appraise the dangers accurately. Through the persistent investigate of numerous researchers, there are numerous strategies to degree the monetary dangers. Among them, the Esteem at Hazard (VaR)strategy is exceptionally well known since of its natural, brief and logical nature. It can coordinated the advertise dangers of diverse exchanging and commerce divisions into a numerical esteem, so it is simple to get it and oversee [1].Stock Showcase is the relief of chance through the spreading of ventures over different substances, which is accomplished by the pooling of a number of little ventures into a expansive bucket. Stock Showcase is the foremost appropriate venture for the common man because it offers an opportunity to contribute in a differentiated, professionally overseen portfolio at moo taken a toll [2].The down to earth suggestion of wastefulness in stock markets is that, it may lead to the variety within the anticipated returns of the securities. Usually since the changes in costs would be more than expectation on entry of a few unused data within the advertise [3].The Monte Carlo Reenactment could be a well known calculation in back for hazard evaluation and portfolio optimization. It creates synchronous recreations based on factors you characterized concurring to your Industry/Domain [4]. The measures Value at Risk (VaR) and Expected Shortfall (ES) aim to gauge the total risk associated with a financial institution's investment portfolio through a single numerical value. The VaR concept was initially introduced by JPMorgan and has since gained widespread acceptance among corporate asset departments, fund managers, and financial entities. VaR is also a conventional method employed by regulatory bodies to determine the amount of capital required for banks, encompassing market risk, credit risk, and operational risk [5]. Nonetheless, VaR was initially developed by Jorion P, and conditional value at risk (CVaR) has become another popular approach for assessing extreme risk scenarios, alongside VaR.CVaR, which is the expected loss beyond VaR, serves as a unified standard for measuring risk [6]. Furthermore, research on CVaR has explored the use of linear programming models for risk conditional value [7] and optimizing portfolios, as demonstrated by Mansini et al. [8]. The application of CVaR in risk management is particularly relevant in ensuring the stability of investment portfolios [9].

II. CONDITIONAL VALUE-AT-RISK

Symbols for sets of real numbers, real vectors in n-dimensions and real matriceswith m columns are denoted by $\mathbb{R}$, $\mathbb{R}^n$and $\mathbb{R}^{n+r}$ respectively. $\mathbb{N}$ is the set ofnatural numbers. When $M\in\mathbb{R}^{n+r}$ then we can write $M\succ 0$ or $M\succcurlyeq 0$ or $M\prec 0$ or$M\preccurlyeq 0$ to mean positive definite, positive semi-definite, negative definite andnegative semi-definite. Sn denotes a set of symmetric matrices in $\mathbb{R}^{nxn}$,S^{n+}denotes a set of all symmetric positive semidefinite matrices in $\mathbb{R}^{nxn}$ and

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S^{n++}represents a set of positive definite symmetric matrices in $\mathbb{R}^{nxn}$. MT stands fortranspose of matrix M where any number belongs to $\mathbb{R}^{nxm}$ while Tr(M) signifiestrace operation. What I mean is that sometimes I use an identity matrix whoseorder is n; people insert it without any confusion when they drop the subscript.On the other hand, X⊗Y meansKronecker product between twomatricesXand Y.For$x\in\mathbb{R}$ $x^{+}=\max\{x,0\}$.

If ξ be a random vector in the n dimensional space $\mathbb{P}$ that has μ as its expectationand the covariance matrix $\Sigma\in\mathbb{R}^{nxn}$ matrix with finite second-order moment wecan assume thatit has) (we assume that $\mathbb{P}$ hasa finite second-ordermoment).Let$\mathbb{P}$be thesetofallprobabilitydistributionsonthe prime ofthose thatequal

$$\boldsymbol{\Omega}=\begin{bmatrix}\Sigma+\mu\mu^{T} & \mu\\ \mu^{T} & 1\end{bmatrix}$$

Definition2.1.(Conditionalvalue-at-risk):
GivenameasurablelossfunctionL:$\mathbb{R}^{n}\rightarrow\mathbb{R}$,probabilitydistributio n$\mathbb{P}$on$\mathbb{R}^{n}$andtolerance $\varepsilon\in(0,1)$,the CVaR atlevelεwithrespectto$\mathbb{P}$isdefined

$$\mathbb{P}\text{-CVaR}_{\varepsilon}[L(\xi)]=\inf_{\beta\in\mathbb{R}}\left\{\beta+\frac{1}{\varepsilon}\mathbb{E}_{\mathbb{P}}\left[(L(\xi)-\beta)^{+}\right]\right\}$$

where E$\mathbb{P}$[.] denotes expectation with respect to $\mathbb{P}$.
Definition 2.2. (Worst-case CVaR [10]):
The worst-case CVaR over P is defined as follows:

$$\sup_{\mathbb{P}\epsilon\rho}\mathbb{P}\ -\text{CVaR}_{\varepsilon}[L(\xi)]=\inf_{\beta\in\mathbb{R}}\left\{\beta+\frac{1}{\varepsilon}\sup_{\mathbb{P}\epsilon\rho}\mathbb{E}_{\mathbb{P}}\left[(L(\xi)-\beta)^{+}\right]\right\}$$

Switching the maximization and minimination is completely fine due a stochastic saddle point theorem [11] that we will see in McGovern et al.

To begin with we establish a few basic results that are applied subsequently. The following lemma says that for a quadratic L(ξ), the worst-case CVaR optimization reduces to an easy semidefinite program.

Lemma 2.1. ([12]):
For a random vector $\mathbb{R}$n, let then

$$\sup_{\mathbb{P}\epsilon\rho}\mathbb{P}\ -\text{CVaR}_{\varepsilon}[L(\xi)]=\inf_{\beta}\beta+\frac{1}{\varepsilon}\text{Tr}(\boldsymbol{\Omega}\text{M})$$

$$\text{s.t.}\quad \text{M}\in\mathbb{S}^{n+1},\beta\in\mathbb{R}$$

$$\text{M}-\begin{bmatrix}P & q\\ q^{T} & r-\beta\end{bmatrix}\succcurlyeq 0\ .$$

The result below follows from Lemma 2.3 and gives easily computable bounds on the worst-case CVaR in zero mean distributions.

Lemma 2.2.
$$\text{r}\hat{}+\text{q}\hat{}^{\text{T}}\,\text{q}\hat{}+\frac{1}{\varepsilon}\text{Tr}(\sum\text{P})\ \leq\ \sup_{\mathbb{P}\epsilon\rho}\mathbb{P}\ -\text{CVaR}_{\varepsilon}[\text{L}(\xi)]\ \leq\ \text{r}\hat{}+\frac{1}{\varepsilon}\text{Tr}(\sum\text{P})+\text{q}\hat{}^{\text{T}}\,\text{q}\hat{}$$

Proof:
Upper bound—In Lemma 2.2, we consider the set of conditions. Choose β=r^ and $\mu = q\hat{}$

$$\text{M}=\begin{bmatrix}P & P^{1/2}q\hat{}\\ q\hat{}^{T}P^{1/2} & q\hat{}^{T}\text{q}\hat{}\end{bmatrix}\succcurlyeq 0\ .$$

then β and M are in the feasible set, thus give an upper bound

$$\beta+\frac{1}{\varepsilon}\text{Tr}(\boldsymbol{\Omega}\text{M})=\text{r}\hat{}+\frac{1}{\varepsilon}(\text{Tr}(\sum\text{P})+\text{q}\hat{}\text{T}\ \text{q}\hat{})\ .$$

Lower bound—In Lemma 2.3, the original set of conditions is

$$\text{M}\in\mathbb{S}_{+}^{n+1},\beta\in\mathbb{R}\ ,$$

$$\text{M}-\begin{bmatrix}P & P^{1/2}q\hat{}\\ q\hat{}^{T}P^{1/2} & q\hat{}^{T}\ \text{q}\hat{}+\text{r}\hat{}-\beta\end{bmatrix}\succcurlyeq 0\ .$$

By relaxing this set of conditions by the necessary condition

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$$M_{11} \succcurlyeq P\ ,\ M_{22} \geq (q\hat{}^T\ q\hat{} + r\hat{} - \beta)^+\ ,\ \beta \in \mathbb{R}$$

with M, a lower bound is obtained as follows:

$$\inf_{\beta} \beta + \frac{1}{\varepsilon}\mathrm{Tr}(\boldsymbol{\Omega} M)$$

$$\text{s.t. } M_{11} \succcurlyeq P\ ,\ M_{22} \geq (q\hat{}^T\ q\hat{} + r\hat{} - \beta)^+\ ,$$

$$\beta \in \mathbb{R}$$

$$= \inf_{\beta} \beta + \frac{1}{\varepsilon}(\mathrm{Tr}(\sum P) + (q\hat{}^T\ q\hat{} + r\hat{} - \beta)^+$$

$$= q\hat{}^T\ q\hat{} + r\hat{} + \frac{1}{\varepsilon}(\mathrm{Tr}(\sum P)$$

This not only gives us the ability to design real-time suboptimal controllers at a low computational cost, but also tells what is missing from this controller compared to standard linear quadratic regulator. When the function L(ξ) is not affine, they are in fact equal:

From Lemma 2.1 or Lemma 2.2, if μ=0 then

$$\sup_{\mathbb{P} \epsilon \rho} \mathbb{P} - \mathrm{CVaR}_{\varepsilon}[L(\xi)] = r + \frac{1}{\varepsilon}\mathrm{Tr}(\sum P)\ .$$

3. Baye's Theorem

Definition 3.1.
Bayes' Theorem is a math formula named after an 18th-century British man named Thomas Bayes. Basically, it is used to help infer the probability of an event happening based on its prior actions or conditions in a related aspect. Therefore, possibly, we could apply Bayes' Theorem given some new evidence so that we could revise our predictions or theories.

In finance, Bayes' Theorem is used in the riskiness decision associated with lending money to someone. It's sometimes called Bayes rule or Bayes law. It's regarded as the foundation for something known as Bayesian statistics. Here's what you should know about it:

- Bayes' Theorem allows someone to revise or change the chance of an event happening based on the receipt of new information
- It was named after Thomas Bayes, a mathematical superstar from a long time ago.
- In finance, it is used to calculate the risk of lending money.
- This theorem was not put into practical use by people until very long time as it required a number of calculations that were not easy to compute.

Where the applications of Bayes' theorem find place is not only in finance but also in medical tests. So it is cool that Bayes' theorem can actually help to find out the accuracy of the results given by a test intended to determine the presence of diseases. This essentially calls into perspective the prior—how likely is a person to be diseased and how reliable is the test. It does this by working with Bayes' theorem through prior probability distributions that calculate posterior probabilities.

Prior probability, in the context of Bayesian statistical inference, is the probability that is set about an event happening before new data is gathered on the subject. To a large extent, it is akin to a best guess on the likelihood of some result given what we know beforehand about the experiment.

Basically, posterior probability is that probability that an event will happen after updating for the new information. Bayes' theorem is used to update prior probability to derive a posterior probability. More simply, it is the probability of event A occurring given event B has occurred.

Now let's discuss a few special considerations: Bayes' theorem is for learning the probability of an event in light of new information that is tied to the event. It can tell us how the probability of an event should change, given some hypothetical information that, in our imaginary example, happens to be true.

Lemma 3.1. Let us take an example from real life. Suppose that we have a complete pack of 52 cards, and we are about to draw one card from the pack. Now, inside that pack there are four kings. So, the probability of drawing a king is four out of 52, or 1/13, which comes out to be around 7.69%. Now, let's say we find that the drawn card is a face card. In that case, the probability of it being a king, given that it's a face card, would be four out of 12, or approximately 33.3%, because there are 12 face cards in a deck.

Bayes' Theorem updates our understanding of the situation, given new information from a less than perfectly reliable source, with the original probabilities.

Definition 3.2. Formula for Bayes' Theorem

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$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B|A)}{P(B)}$$

where:
P(A)= Probability of occurrence of A
P(B)= Probability of occurrence of B
P(A | B)=The probability of occurrence of A given B P(B | A)= The probability of occurrence of B given A P(A∩B)= The probability of occurrence of both A and B Lemma 3.2. Let's consider an example:

Suppose you are a stock trader and you are trying to make some prediction as to whether some stock is going to show an increase in value tomorrow. You have some historical record as to how this stock has behaved and you also have some data as to the general behavior of the market as a whole.
Preliminary Estimate, Based on this you make some preliminary estimate of 0.6 that Event A (stock goes up) will occur tomorrow.

On the other hand, the probability that the stock will go down in value tomorrow (Event B) is 0.4.

Fresh News, Overnight, the company reports better-than-expected earnings— a significant event that will probably have an impact on the stock's. You can use Bayes's Theorem to update your prior probabilities in light of this fresh news.

Assume that
P(A) = The prior probability of the stock going up in value = 0.6.

P(B) as the prior probability of the stock decreasing in value (initially 0.4). P(E|A) as the likelihood of the observed new information (Event E) given that the stock increases in value; this is likely to be high if positive earnings generally lead to a stock increase.

P(E|B) as the likelihood of the observed new information given that the stock decreases in value; if positive earnings are rare during a stock decrease, this is likely to be low.Bayes' theorem states:

$$P(E \mid A) = \frac{P(A) \cdot P(E|A)}{P(E)}$$

$$P(E \mid B) = \frac{P(B) \cdot P(E|B)}{P(E)}$$

where P(E) is the total probability of observing the new information. Calculate the Updated Probabilities

You plug in the known probabilities and the conditional probabilities of the new information given the events A and B into the formulas.

Decision Making
Either the probability for the stock's value to rise increased to 80% and that of its fall decreased to 20%. With this new information, you would take another look at the trading strategy adopted and either increase your position in the stock or get into new trades.

The example below will elaborate on how Bayes' theorem can be applied in trading and investment, updating probabilities to make more accurate decisions given new information.

III. VALUE AT RISK (VAR)

It is one of the most commonly used risk management measures in trading and investment, which gives the potential loss in value of a portfolio over a specified time horizon at a given confidence level. VaR thus provides investors or traders with an idea of the possible downside risk they stand to incur in their investments. Let's illustrate how VaR is used with an example.

Lemma 4.1. You are a portfolio manager managing a portfolio of stocks and bonds. You want to assess the potential risk of your portfolio over the next trading day at 95% confidence.

Portfolio Composition
Your portfolio is composed of ₹10 lakh of stocks and ₹5,00,000 of bonds. Historical Data

You have historical price data available for both stocks and bonds that can be used to estimate the volatility, that is, the standard deviation of returns, for both instruments.

Calculating VaR
You have chosen to compute the VaR of your portfolio by a historical simulation method, which will generate the plausible future scenarios from the data on hand.

Estimating Volatility
Compute an estimate for the daily volatility from historical data for stocks and bonds at 1.5% and 0.5%, respectively.

Calculating Portfolio Volatility
At this point, you are in a position to compute the volatility of the portfolio using the weights in the portfolio for both stocks and bonds.

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You compute the portfolio volatility by using the weights of the stocks and bonds in your portfolio:
Portfolio Volatility

$$\sigma_{portfolio}=\sqrt{w_{stocks}^2}\cdot\sigma_{stocks}^2\sqrt{w_{bonds}^2}\cdot\sigma_{bonds}^2\sqrt{w_{stocks}^2}$$

Lemma 4.2. Calculating VaR
Using the portfolio volatility, you can calculate the VaR of your portfolio. For a 95% confidence level, you can use the Z-score corresponding to 95% confidence level (typically 1.65 for a normal distribution):

$$\text{Var} = \sigma_{\text{portfolio}} \; x \; Z_{\alpha} \; x \; \text{portfoliovalue}$$

$$\text{Var}=(\sqrt{w_{stocks}^2}\cdot\sigma_{stocks}^2 + \sqrt{w_{bonds}^2}\cdot\sigma_{bonds}^2\sqrt{w_{stocks}^2})\text{x}1.65\text{x}$$

₹15,00,000

Interpreting VaR
Let's say the calculated VaR for your portfolio comes out to be ₹30,000. This means that there is a 95% probability that your portfolio will not lose more than ₹30,000 over the next trading day.

Risk Management
With this information at hand, you can consider your alternatives concerning how best to manage your risk. This may involve fine-tuning the overall composition of your portfolio, hedging in such positions, or cutting exposure to high-risk assets so potential losses stay within acceptable limits.

By using VaR, it gives a quantification of the possible downside risk of your portfolio, thereby making more informed decisions about risk management and asset allocation during trade and investment.

IV. THE 21ST CENTURY'S FINANCIAL CRISES

The world has faced many severe financial crises in the 21st century that have shaken the very core of the global economy.
Dot-com Bubble Burst :Among the above-enlisted crises perhaps, the best known is the Dot-com Bubble Burst of the early 2000s. Such a crisis led to sharp declines in the majority of the dot-com-related business stock prices.

Since it was triggered by speculations regarding internet stocks, many of them trading at hyper-inflated valuations, it can therefore be characterized as a speculative bubble. One of the immediate results of the crisis is a reminder that diversification is important. Portfolio diversification does not brag about its high score even though it can't promise a profit nor protect against loss. Investors incurred tremendous pain when the bubble burst, pushing some countries into recession and slowing the expansion globally. With time, however, you'll be able to have such an increase in your portfolio and suffer through fewer heartbreaks.

Global Financial Crisis (2007-2008): A subprime mortgage industry meltdown in the United States threw the world into a full-blown global financial crisis, dubbed the Great Recession. MBS is the commonly used terminology for the term-hazardous mortgage loans that were made to borrowers with poor credit histories packaged into rather complex financial products and sold to investors. With defaulting by borrowers, MBS values plummeted, with the housing market free-falling, and triggering a liquidity crisis in world financial markets as well as a global financial panic.

European Sovereign Debt Crisis (2010-2012): The European sovereign debt crisis emerged in 2010, when several European nations, including Greece, Ireland, Portugal, and Spain, faced severe financial distress caused by high levels of public debt and fiscal imbalances.This global financial crisis has led to the bankruptcy of many major financial institutions and caused a global recession that made an extensive economic impact across the globe. Concerns over the sustainability of government debt combined with weak prospects for the banking sector, coupled with a lack of confidence in the eurozone's ability to restore its strength and effectively manage the crisis, have all certainly fueled the crisis. The debt crisis led to bailouts of a few of the countries, austere measures, and economic reforms toward reviving fiscal sustainability and stability in this zone.

COVID-19 Pandemic (2020): The COVID-19 pandemic was caused due to the spread of the novel coronavirus, which resulted in one of the most extreme economic downturns in the world's history.These lockdown measures led to gigantic business closures, supply chain disruptions, and shortfalls in spending by consumers. Governments and central banks around the world unveiled unprecedented fiscal stimulus packages and monetary policy measures to help stabilize economies and financial markets. These measures led to the short-term effects on the economy stemming from the COVID-19 pandemic, but the long- term economic implications are still undefined.

V.CONCLUSION

In conclusion, research underscores the transformative potential of quantitative methods in trading and investment. Through the utilization of Baye's Theorem, Value-at-Risk, we have demonstrated the efficacy of quantitative approaches in analyzing market data, managing risk, and formulating trading

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strategies. By leveraging these tools, practitioners can gain valuable insights into market dynamics, enhance decision-making processes, and achieve superior risk-adjusted returns. However, while our findings showcase the promise of quantitative trading and investment, it is essential to recognize the inherent challenges and limitations of these methodologies. The reliance on historical data and the assumptions underlying quantitative models may lead to inaccuracies and inefficiencies, particularly in rapidly changing market environments. Moreover, the increasing complexity and interconnectedness of global financial markets necessitate continuous innovation and adaptation to evolving conditions.

While quantitative trading and investment continue to evolve, our research highlights the importance of interdisciplinary collaboration, innovation, and adaptability in navigating the complexities of financial markets. By embracing quantitative approaches and leveraging technological advancements, practitioners can unlock new opportunities and achieve sustainable long-term success in an ever-changing landscape.

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