

A Study of Multiserver Retrial Queues with Different Stages of Homogeneous Service

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Abstract- We discuss a queuing system with retrial of customers. Two models are discussed. First, we investigate single server queues in parallel, when the customer going to search and join the shorter of the two queues and in the second model we introduce the multiserver queue to multiserver retrial queue system. Multiserver provides different stages of homogeneous service in succession.

Keywords- Retrial Queue, Shortest queue, Multiserverqueue.

I. INTRODUCTION

1. Single Server Queues in Parallel:

In inter-arrival time, the arrival of the customers can be modeled according to different disciplines depending on each particular application.

Arriving customers join the shorter of the two queues with ties broken in any plausible manner. No jockeying between the queues is allowed. This situation is like in the ticket counter for theater, bank and vehicles going through toll booths etc., here we use inter-arrival time to obtain "bounds" for the probability distribution for the number of customer in the system, its expected value in equilibrium by linear programming techniques and obtained the value of lower and upper bounds, inform of equilibrium

2. The Mathematical Description:

A Poisson stream of customer with rate λ arrives at a service which consists of two single server queues in parallel. The service time of the customers is independent and exponentially distributed with rate. Both servers serve at an equal rate. Each server has associated queues if their sizes are unequal, otherwise he joins any queue. Jockeying between the queues is not allowed. This system which is known as the 'shortest queue', has received a lot of attention in the literature because it (with its generalizations to many servers and general service times) modes many real-life situations such as vehicles going through toll booths, jobs scheduled on a multiprocessor system, etc.,

On the other hand, no simple method for analyzing this system is known. The problem was originally introduced by Heights. Kingman proved that an equilibrium distribution exists whenever $\frac{\lambda}{\mu} \leq 2$.

Both Flatto and McKean treated μ the problem by applying techniques of complex function theory to obtain

representation for the general functions of the state probability. The above mentioned papers derive some asymptotic approximations for the state probability. The above mentioned papers derive sum asymptotic approximation for the state probabilities for large number of customers in the system, and for any traffic. Connolly discussed the finite -waiting-room version of the problem.

In current paper, we derive upper and lower bounds for the state probabilities, tail probabilities and mean number of customers in the is equilibrium.

The bounds are derived by considering a subset of the balance equation, which bounds for the tail programs which in turn produce bounds. We also derive lower bounds for the tail distribution by comparison with an M/M/2 system. These rather elementary techniques produce bound which are within ten of the true values for

$$1 \leq \frac{\lambda}{\mu} \leq 2,$$

and which are asymptotically tight in heavy traffic

3. Preliminary Results:

The state space consists of pairs (i, j) where $i, j = 0, 1, 2, \dots$ and $i \geq j$. We say that the system is in (i, j) if the number of customers in the longer queue is "i" and the number of customer in the shorter queue is "j" (number in queue includes the customers in service, if any) note that under this description servers are not associated with a particular component of the state vector and there is no need to specify what happens when a customer finds that both queues are of equal length. Clearly the system behaves as Markov chains on the state space with transition intensities as described in figure 1.1

Let

$$a = \frac{\lambda}{\mu}$$

We assume that $a \leq 2$, which implies that the system is stable.

Let P_{ij} be the equilibrium probability of the state (i, j) .
Let $\pi_n = \sum_{i+j=n} P_{ij}$, $n = 0, 1, 2, \dots$

Thus π_n is the probability that there are exactly n customers in the system in equilibrium. Denote by q_k the sum of probabilities in the k^{th} time is diagonal.

$$q_k = \sum_{i=0}^{\infty} P_{i+k, i} \quad k = 0, 1, 2$$

And let $q^* = \sum_{k=1}^{\infty} q_k$ be the probability that the queues are unequal.

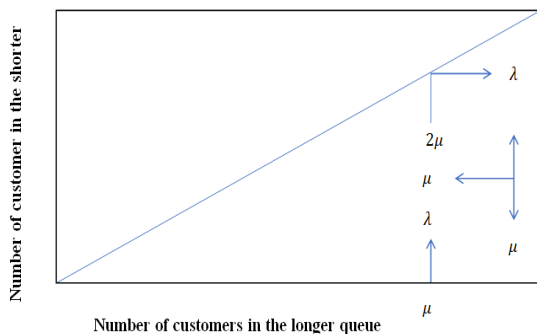


Fig 1. Transition Intensities for the “Shortest Queue”.

It is well known that for a Markov chain in equilibrium if the state space is divided into two disjoint subsets, then the ‘intensity flows’ from one subset into the other, are equal for both subsets.

First, for a fixed i , we divide the states into those with first component less than i , and those with first component greater than or equal to i . We get

$$\lambda P_{i-1, i-1} = \mu \sum_{j=0}^{i-1} P_{ij}, \quad i = 1, 2, 3,$$

Summing (1.1) over ‘ i ’ we get

$$\lambda q_0 = \mu q^* \quad (1.2)$$

Since, we have

$$q_0 + q^* = 1 \quad (1.3)$$

Next, we define the ‘diagonals’

$$D_k = \bigcup_{i=0}^{\infty} (i+k, i), \quad k = 0, 1, 2, \quad (1.4)$$

And let us separate the diagonals $D_0, D_1, D_2, \dots, D_k$ from

$$D_{k+1}, D_{k+2}, \dots$$

The resulting cut yield

$$(1+a)q_0 = (2+a)q_0 - 2p_{00}, \quad \text{for } k = 0,$$

$$(1+a)q_{k+1} = q_k - P_{k0}, \quad \text{for } k = 1, 2, 3,$$

Summing (1.5) and (1.6) over k we get

$$(1+a)q^* = (2+a)q_0 + q^* - (2P_{00} + \sum_{k=1}^{\infty} P_{k0}),$$

and subtracting the values of q_0 and q^* , we get the following result.

$$2P_{00} + \sum_{k=1}^{\infty} P_{k0} = 2 - a, \quad \text{for } 0 \leq a \leq 2, \quad (1.7)$$

It is introduced to note that q_0, q_1, \dots Exist even when $a \geq 2$, if we interpret q_k as the limit of the probability of being in D_k at time t , when $t \rightarrow \infty$ independent of the initial conditional. To see that, when $a \geq 2$ the probability that any queue is empty at time t converges to 0 for $t \rightarrow \infty$. thus the process behave asymptotically as birth-and-death process on the diagonals D_k , with the birth rate being μ and the death rate being $\lambda + \mu$ for $k \geq 0$, and for $k = 0$, the birth rate being $\lambda + 2\mu$ and the death rate 0. Thus a limiting distribution exists, independent of the initial conditions, and it can be calculated explicitly as follows.

II. CONCLUSION

The approach of the probability to its heavy traffic limit is very steep, so that this limit, which is easily computable, will not be a good approximation for most values of the load. Asymmetry in the service rates tends to extend to increase this probability in light traffic, but to decrease it in moderate to heavy traffic.

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