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Dynamic Analysis of Thermal Stresses in a Semi-Infinite Solid Circular Cylinder

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Abstract- This paper presents an analysis of the thermoelastic response of a semi-infinite solid circular cylinder subjected to an arbitrary initial heat input on its lower surface, while the curved surface is thermally insulated. The study employs a dynamic approach based on potential functions to model the system. The resulting expressions for temperature distribution and thermal stresses are derived using Bessel's functions. To demonstrate the applicability of the model, copper (pure) is selected as the material, and the outcomes are visualized graphically, highlighting the thermal and mechanical behavior under dynamic conditions.

Index Terms- heat input, dynamic, thermoelastic, cylinder, Bessel's functions.

I. INTRODUCTION

In recent years, the study of dynamic thermal stresses has gained significant importance due to its relevance in engineering applications operating under extreme thermal conditions. These studies have played a pivotal role in the design of advanced technologies, including nuclear reactors and aerodynamic structures for aircraft. Numerous researchers have addressed the issue of dynamic thermal stresses in infinite cylinders. Dhaliwal and Chaudhary [1] employed integral transforms and parameter variation methods to solve the dynamic thermoelastic problem in cylindrical regions for a one-dimensional case. Wadhawan [3] applied the Laplace Transform technique to address dynamic thermoelasticity in isotropic media. Boiko [5] examined the axisymmetric dynamic problem for a half-space with boundary conditions of the second kind, considering finite heat propagation velocity. Noda et al. [6] and Chandrasekaraiah and Keshavan [7] explored thermoelastic problems in transversely isotropic bodies. Germanovich et al. [8] analyzed a dynamic thermoelastic problem for a half-space with distributed heat sources under axial symmetry, providing solutions in asymptotic form. Sharma and Grover [9] investigated body wave propagation in rotating thermoelastic media, while Venkatesan and Ponnusamy [10] focused on wave propagation in a generalized thermoelastic solid cylinder immersed in a fluid. Deshmukh et al. [11] studied displacement and thermal stresses in a thin circular plate under steady-state temperature distribution due to constant heat generation, utilizing a quasi-static approach. Kumar and Mukhopadhyay [12] examined the impact of thermal relaxation time on plane wave propagation in two-temperature thermoelasticity. Abd-Alla et al. [13] addressed the propagation of Rayleigh waves in generalized magneto-thermoelastic orthotropic materials under initial stress and gravity fields, and in a separate study [14], they analyzed dynamic thermal stresses in an infinite circular cylinder. Kedar et al. [15] investigated quasi-static thermal stresses in a semi-infinite circular cylinder.Warbhe et al. [16] conducted a detailed study on fractional heat conduction in a thin hollow circular disk, analyzing the resultant thermal deflection using fractional calculus to model heat transfer. In another work, Warbhe et al. [17] investigated fractional heat conduction in a thin circular plate with constant temperature distribution, focusing on the thermal stresses induced by this scenario. Additionally, Tripathi et al. [18] extended the application of fractional order thermoelasticity to thin circular plates, exploring the deflection behaviors in such systems.

In this paper, we analyze dynamic thermal stresses in a semi-infinite solid circular cylinder subjected to an arbitrary initial heat input on the lower surface, while the curved surface remains thermally insulated. To solve the elasto-dynamic problem, we employ the method of potential functions and derive exact solutions without imposing any simplifying assumptions on the physical quantities in the governing equations. The mathematical model is developed for pure copper material, and the resulting thermoelastic behavior is analyzed in detail.

II. FORMULATION OF THE PROBLEM

Consider a semi-infinite solid circular cylinder defined as $0<r<a, 0<z<\infty$. Let the lower surface be subjected to arbitrary initial temperature and a curved boundary surface $r=a$ is at zero heat flux. Under these more realistic prescribed conditions the dynamic thermal stresses in the solid circular cylinder are required to be determined.

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We take the axis of symmetry as the z axis and the origin of the system of co-ordinates at the middle plane. The problem is studied using the cylindrical polar co-ordinates (r,ϕ,z). Due to the rotational symmetry about the z axis of the problem, all quantities are independent of the co-ordinate ϕ.

The displacement vector, thus, has the form $\bar{u}=(u,0,w)$.
The classical equations of dynamic thermoelasticity of a homogeneous isotropic body in the absence of body forces, in terms of the vector displacement $\vec{u}$ and the temperature $T(r,z,t)$ in non-dimensional form are [14]

$$(\lambda+\mu)\nabla(\nabla.\bar{u})+\mu\nabla^2\bar{u}-\rho v^2\frac{\partial^2\bar{u}}{\partial t^2}=(3\lambda+2\mu)\alpha a\nabla T \tag{1}$$

$$\nabla^2 T=\frac{1}{K}\frac{\partial T}{\partial t} \tag{2}$$

where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$, $K=\frac{k}{\rho c_E}$ and k is the coefficient of thermal conductivity, ρ is the density, c_E is the specific heat, v and α are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the solid circular cylinder respectively, λ and μ are Lame's constants.

The boundary conditions imposed on the problem are as follows

$T(r,z,t)=0$, for $z=\infty$

$$T(r,z,0)=F(r),\quad \text{for } z=0 \tag{4}$$

$\frac{\partial T(r,z,t)}{\partial r}=0$, for $r=a$

The curved surface of the solid circular cylinder is assumed to be fixed

$$u(r,z,t)=w(r,z,t)=0\ ,\quad r=a\ ,t>0 \tag{6}$$

Solution of the Heat Conduction Equation:

To obtain the expression for temperature $T(r,z,t)$, we assume

$$T(r,z,t)=e^{-z}\sum_{n=1}^{\infty}f_n(t)J_0(\lambda_n r)$$

Using equation (7) in equation (3), we get,

$$f_n(t)=A_n\,e^{-\alpha(\lambda_n^2-1)t}$$

Thus, the expression of the temperature in equation (7) becomes,

$$T(r,z,t)=e^{-z}\sum_{n=1}^{\infty}A_n\,e^{-\alpha(\lambda_n^2-1)t}J_0(\lambda_n r)$$

Using condition (5) in equation (9), one obtains

$$F(r)=\sum_{n=1}^{\infty}A_n\,J_0(\lambda_n r)$$

Using the orthogonality property of Bessel functions, one obtains

$$A_n=\frac{2}{a^2J_0^2(\lambda_n a)}\int_0^a r'F(r')J_0(\lambda_n r')dr'$$

Hence, the temperature distribution in the solid circular cylinder is obtained as

$$T(r,z,t)=e^{-z}\sum_{n=1}^{\infty}\left[\frac{2}{a^2J_0^2(\lambda_n a)}\int_0^a r'F(r')J_0(\lambda_n r')dr'\right]e^{-\alpha(\lambda_n^2-1)t}J_0(\lambda_n r)$$

Solution of Thermoelastic problem:

By Helmholtz theorem, the displacement vector $\vec{u}$ can be expressed as,

$$\bar{u}=\nabla\varphi+\nabla\times\bar{\psi} \tag{13}$$

where the two potentials φ and $\bar{\psi}$ are the Lame's potentials representing irrotational and rotational parts of the displacement vector $\vec{u}$ respectively. (3)

Using equation (13) in equation (2), we get,

$$\nabla^2\varphi-\frac{1}{c_L^2}\frac{\partial^2\varphi}{\partial t^2}=mT \quad (5) \tag{14}$$

$$\nabla^2\bar{\psi}-\frac{1}{c_T^2}\frac{\partial^2\bar{\psi}}{\partial t^2}=0 \tag{15}$$

where $m=\frac{(3\lambda+2\mu)}{(\lambda+2\mu)}\alpha aT_0$, $c_L^2=\frac{(\lambda+2\mu)}{\rho v^2}$ and $c_T^2=\frac{\mu}{\rho v^2}$ are the velocities of the longitudinal and (7) where $\lambda_1,\lambda_2,\ldots$ are the pos transverse waves respectively.

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It is possible to take only one component of the vector $\vec{\psi}$ to be non-zero, i.e., $\vec{\psi}$ can be written as $\vec{\psi} = (0, \psi,_r, 0)$

Where $\psi,_r = \frac{\partial \psi}{\partial r}$ and ψ satisfies the wave equation (15). Therefore, ψ has the general form

$$\psi = \sum_{n=1}^{\infty} D_n J_0\left(r(p_n^2 - q_n^2 c_T^{-2})^{1/2}\right) e^{-p_n z - q_n t} \qquad (16)$$

where D_n, p_n, q_n are constants to be determined from the boundary conditions.

To obtain the particular integral of equation (14), T is eliminated from equations (3) and equation (14), we get,

$$\left(\nabla^2 - \frac{1}{K}\frac{\partial}{\partial t}\right)\left(\nabla^2 - \frac{1}{c_L^2}\frac{\partial^2}{\partial t^2}\right)\varphi = 0 \qquad (17)$$

We take $\varphi = \varphi_1 + \varphi_2$, where φ_1 and φ_2 satisfy the following equations:

$$\left(\nabla^2 - \frac{1}{c_L^2}\frac{\partial^2}{\partial t^2}\right)\varphi_1 = 0 \qquad (18)$$

$$\left(\nabla^2 - \frac{1}{K}\frac{\partial}{\partial t}\right)\varphi_2 = 0 \qquad (19)$$

Also φ is the solution to equation (14), therefore,

$$\left(\nabla^2 - \frac{1}{c_L^2}\frac{\partial^2}{\partial t^2}\right)\varphi_2 = mT \qquad (20)$$

φ_2 can be determined by subtracting equation (20) from equation (19) as follows:

$$\left(\frac{\partial^2}{\partial t^2} - b^2\frac{\partial^2}{\partial t^2}\right)\varphi_2 = -mc_L^2 T \qquad (21)$$

Where $b^2 = c_L^2 / K$

On integrating the differential equation (21), we get,

$$\varphi_2 = F(r,z) + G(r,z)e^{b^2 t} - \frac{mc_L^2}{b^2}\left[e^{b^2 t}\int e^{-b^2 t} T dt - \int T dt\right] \qquad (22)$$

To satisfy the physical conditions in the above equation, where $b^2 > 0$ must satisfy the following condition:
$G(r,z) = 0$

Now, φ_2 being a solution to equation (19), substituting equation (22) into equation (19), one can get,

$$\nabla^2 F(r,z) = 0$$

which has a solution in the form:

$$F(r,z) = \sum_{n=1}^{\infty} C_n J_0(\beta_n z) e^{-\beta_n z} \qquad (23)$$

Where C_n and β_n are arbitrary constants.

For φ_1, the solution to equation (18) can be written in the form:

$$\varphi_1 = \sum_{n=1}^{\infty} B_n J_0\left(r(e_n^2 - d_n^2 c_L^{-2})^{1/2}\right) e^{-e_n z - d_n t} + t\sum_{n=1}^{\infty} b_n J_0(\delta_n r) e^{-\delta_n z} \qquad (24)$$

where B_n, e_n, d_n, b_n and δ_n are arbitrary constants.

Combining equations (22), (23), and (24), one can get the complete solution to equation
(14) as follows:

$$\varphi = \sum_{n=1}^{\infty} B_n J_0\left(r(e_n^2 - d_n^2 c_L^{-2})^{1/2}\right) e^{-e_n z - d_n t} + t\sum_{n=1}^{\infty} b_n J_0(\delta_n r) e^{-\delta_n z} + \sum_{n=1}^{\infty} C_n J_0(\beta_n z) e^{-\beta_n z} - \frac{mc_L^2}{b^2}\left[e^{b^2 t}\int e^{-b^2 t} T dt - \int T dt\right] \qquad (25)$$

The displacement components in terms of φ and ψ are

$$u = \frac{\partial \varphi}{\partial r} - \frac{\partial^2 \psi}{\partial r \partial z} \qquad (26)$$

$$w = \frac{\partial \varphi}{\partial z} - \frac{\partial^2 \psi}{\partial z^2} + \frac{1}{c_T^2}\frac{\partial^2 \psi}{\partial t^2} \qquad (27)$$

$$u_\varphi = 0$$

By using the stress-strain relations

$$\sigma_{ij} = 2\mu e_{ij} + \lambda \delta_{ij} e_{kk}$$

where $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \alpha \delta_{ij} T$, one obtains

$$\frac{\sigma_{rr}}{2\mu}=\frac{\partial^2\varphi}{\partial r^2}-\frac{\partial^3\psi}{\partial r^2\partial z}-\nabla^2\varphi+\frac{1}{2c_T^{\,2}}\frac{\partial^2\varphi}{\partial t^2} \qquad (27)$$

$$\frac{\sigma_{rz}}{2\mu}=\frac{\partial^2\varphi}{\partial r\partial z}-\frac{\partial^3\psi}{\partial r\partial z^2}+\frac{1}{2c_T^{\,2}}\frac{\partial^3\psi}{\partial r\partial t^2} \qquad (28)$$

$$\frac{\sigma_{\theta\theta}}{2\mu}=\frac{1}{r}\frac{\partial\varphi}{\partial r}-\nabla^2\varphi+\frac{1}{2c_T^{\,2}}\frac{\partial^2\varphi}{\partial t^2}-\frac{1}{r}\frac{\partial^2\psi}{\partial r\partial z} \qquad (29)$$

$$\frac{\sigma_{zz}}{2\mu}=\frac{\partial^2\varphi}{\partial z^2}-\nabla^2\varphi+\frac{1}{2c_T^{\,2}}\frac{\partial^2\varphi}{\partial t^2}-\frac{\partial^3\psi}{\partial z^3}+\frac{1}{2c_T^{\,2}}\frac{\partial^3\psi}{\partial z\partial t^2} \qquad (30)$$

$$\sigma_{r\theta}=\sigma_{\theta z}=0 \qquad (31)$$

In order to determine the arbitrary constants in equation (16) and equation (25), the boundary conditions must be studied in detail.

III. BOUNDARY CONDITIONS

It can be obtained easily from equations (12) and (25):

$$\begin{aligned}\varphi(r,z,t)=&\sum_{n=1}^{\infty}B_n J_0\left(r(e_n^{\,2}-d_n^{\,2}c_L^{\,-2})^{1/2}\right)e^{-e_nz-d_nt}+t\sum_{n=1}^{\infty}b_nJ_0(\delta_nr)e^{-\delta_nz}\\&+\sum_{n=1}^{\infty}C_nJ_0(\beta_nz)e^{-\beta_nz}-mc_L^{\,2}\sum_{n=1}^{\infty}A_n\,e^{-k\left(\lambda_n^{\,2}-1\right)t-z}\frac{J_0\left(\lambda_nr\right)}{\left[k(\lambda_n^{\,2}-1)\right]\left[k(\lambda_n^{\,2}-1)+b^2\right]}\end{aligned} \qquad (32)$$

By using φ from equation (32) and ψ from equation (16) and applying the boundary condi-tions (1), we get,

$$\begin{cases}\sum_{n=1}^{\infty}B_n(e_n^{\,2}-d_n^{\,2}c_T^{\,-2})^{1/2}J_1\left(a(e_n^{\,2}-d_n^{\,2}c_T^{\,-2})^{1/2}\right)e^{-e_nz-d_nt}+t\sum_{n=1}^{\infty}b_n\delta_nJ_1(\delta_na)e^{-\delta_nz}\\+\sum_{n=1}^{\infty}C_n\beta_nJ_1(\beta_na)e^{-\beta_nz}-mc_T^{\,2}\sum_{n=1}^{\infty}\frac{A_n\lambda_nJ_1\left(\lambda_na\right)}{\left[k(\lambda_n^{\,2}-1)\right]\left[k(\lambda_n^{\,2}-1)+b^2\right]}e^{-k\left(\lambda_n^{\,2}-1\right)t-z}\\+\sum_{n=1}^{\infty}D_np_n(p_n^{\,2}-q_n^{\,2}c_L^{\,-2})^{1/2}J_1\left(a(p_n^{\,2}-q_n^{\,2}c_L^{\,-2})^{1/2}\right)e^{-p_nz-q_nt}=0\end{cases} \qquad (33)$$

$$\begin{cases}\sum_{n=1}^{\infty}-e_nB_nJ_0\left(a(e_n^{\,2}-d_n^{\,2}c_L^{\,-2})^{1/2}\right)e^{-e_nz-d_nt}-t\sum_{n=1}^{\infty}b_n\delta_nJ_0(\delta_na)e^{-\delta_nz}\\-\sum_{n=1}^{\infty}C_n\beta_nJ_0(\beta_na)e^{-\beta_nz}-mc_L^{\,2}\sum_{n=1}^{\infty}\frac{A_nJ_0\left(\lambda_na\right)}{\left[k(\lambda_n^{\,2}-1)\right]\left[k(\lambda_n^{\,2}-1)+b^2\right]}e^{-k\left(\lambda_n^{\,2}-1\right)t-z}\\-\sum_{n=1}^{\infty}D_np_n^{\,2}J_0\left(a(p_n^{\,2}-q_n^{\,2}c_T^{\,-2})^{1/2}\right)e^{-p_nz-q_nt}\\+\frac{1}{c_T^{\,2}}\sum_{n=1}^{\infty}D_nq_n^{\,2}J_0\left(a(p_n^{\,2}-q_n^{\,2}c_T^{\,-2})^{1/2}\right)e^{-p_nz-q_nt}=0\end{cases} \qquad (34)$$

By taking $e_n=p_n=1$, $d_n=q_n=k\left(\lambda_n^{\,2}-1\right)$, $\beta_n=\delta_n$, and equating the coefficient of $\exp(-\delta_nz)$, $t\exp(-\delta_nz)$ and $\exp\left\{-k\left(\lambda_n^{\,2}-1\right)t-z\right\}$ to zero, one gets from the above equations,

$$B_n=\frac{mc_L^2A\left(R_2J_0(\lambda_na)J_1(aR_2)-\lambda_nR_2J_1(\lambda_na)J_0(aR_2)\left[1-k^2\left(\lambda_n^{\,2}-1\right)^2c_T^{-2}\right]\right)}{\Delta}$$

$$D_n=\frac{mc_L^2A\left(R_2J_0(\lambda_na)J_1(aR_2)-\lambda_nJ_1(\lambda_na)J_0(aR_2)\right)}{\Delta}$$

$$\Delta=R_2J_0(aR_1)J_1(aR_2)\left[1-k^2(\lambda_n^{\,2}-1)^2c_T^{-2}\right]-R_1J_1(aR_1)J_0(aR_2)$$

$$R_1=\left[1-k^2(\lambda_n^{\,2}-1)^2c_T^{-2}\right]^{1/2}$$

$$R_2=\left[1-k^2(\lambda_n^{\,2}-1)^2c_L^{-2}\right]^{1/2}$$

$$A=\frac{A_n}{\left[k(\lambda_n^{\,2}-1)\left\{k(\lambda_n^{\,2}-1)+b^2\right\}\right]}$$

Then the displacement and stress components are given as follows:

$$u=\sum_{n=1}^{\infty}\left\{-B_nR_2J_1\left(rR_2\right)+mc_L^{\,2}A\lambda_nJ_1\left(\lambda_nr\right)-D_nR_1J_1\left(rR_1\right)\right\}e^{-k(\lambda_n^{\,2}-1)t-z} \qquad (35)$$

$$w=\sum_{n=1}^{\infty}\left\{-B_nJ_0\left(rR_2\right)-mc_L^{\,2}AJ_0\left(\lambda_nr\right)-D_nJ_0\left(rR_1\right)\right\}e^{-k(\lambda_n^{\,2}-1)t-z} \qquad (36)$$

$$\frac{\sigma_{rr}}{2\mu}=\sum_{n=1}^{\infty}\left\{\begin{aligned}&B_n\left\{\frac{R_2}{r}J_1\left(rR_2\right)+J_0\left(rR_2\right)\left(\frac{\left(k^2(\lambda_n^{\,2}-1)^2\right)-2c_T^{\,2}}{2c_T^{\,2}}\right)\right\}\\&+mc_L^{\,2}A\left\{J_0(\lambda_nr)\left(\frac{2c_T^{\,2}-\left(k^2(\lambda_n^{\,2}-1)^2\right)}{2c_T^{\,2}}\right)+\frac{\lambda_nJ_1(\lambda_nr)}{r}\right\}\\&+D_nR_1\left\{R_1J_0\left(r\,R_1\right)-\frac{J_1\left(rR_1\right)}{r}\right\}\end{aligned}\right\}e^{-z-k(\lambda_n^{\,2}-1)t} \qquad (37)$$

$$\frac{\sigma_{rz}}{2\mu}=\sum_{n=1}^{\infty}\left\{B_nR_2J_1\left(rR_2\right)-mc_L^{\,2}A\lambda_nJ_1(\lambda_nr)+D_nR_1J_1\left(rR_1\right)\left\{\frac{c_T^{\,2}-k^2(\lambda_n^{\,2}-1)^2}{c_T^{\,2}}\right\}\right\}e^{-z-k(\lambda_n^{\,2}-1)t} \qquad (38)$$

$$\frac{\sigma_{zz}}{2\mu}=\sum_{n=1}^{\infty}\left\{\begin{aligned}&B_nJ_0\left(rR_2\right)\left(\frac{\left(k^2(\lambda_n^{\,2}-1)^2\right)}{2c_T^{\,2}}+R_2^{\,2}\right)+mc_L^{\,2}AJ_0(\lambda_nr)\left(\lambda_n^{\,2}-\frac{\left(k^2(\lambda_n^{\,2}-1)^2\right)}{2c_T^{\,2}}\right)\\&+D_n\left\{1-\frac{\left(k^2(\lambda_n^{\,2}-1)^2\right)}{c_T^{\,2}}\right\}J_0\left(rR_1\right)\end{aligned}\right\}e^{-z-k(\lambda_n^{\,2}-1)t} \qquad (39)$$

$$\frac{\sigma_{\theta\theta}}{2\mu}=\sum_{n=1}^{\infty}\left\{\begin{aligned}&B_n\left\{J_0\left(rR_2\right)\left(\frac{\left(k^2(\lambda_n^{\,2}-1)^2\right)}{2c_T^{\,2}}+R_2^{\,2}-1\right)-\frac{R_2}{r}J_1\left(rR_2\right)\right\}\\&-mc_L^{\,2}A\left\{J_0(\lambda_nr)\left(\lambda_n^{\,2}+\frac{\left(k^2(\lambda_n^{\,2}-1)^2\right)}{2c_T^{\,2}}-1\right)-\frac{\lambda_nJ_1(\lambda_nr)}{r}\right\}\\&-\frac{D_nR_1J_1\left(rR_1\right)}{r}\end{aligned}\right\}e^{-z-k(\lambda_n^{\,2}-1)t} \qquad (40)$$

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Numerical Calculations

Mathematical model is prepared with Copper material for purposes of numerical computations. The material constants of the problem are thus given in S.I. units [14]

$$\rho = 8954\ kg\,m^{-3}\ c_E = 383.1\,J.Kg^{-1}K^{-1},\ \mu = 3.86\times10^{10}\,Nm^{-2}, \alpha_t = 1.78\times10^{-5}\,K^{-1},$$

$$\lambda = 7.76\times10^{10}\,Nm^{-2}.$$

To study the mathematical thermoelastic behavior of a semi-infinite solid circular cylinder, we consider the following function

$$F(r) = \left(r^2 - a^2\right)^2$$

The computational mathematical software Matlab has been used to carry out the numerical calculations.

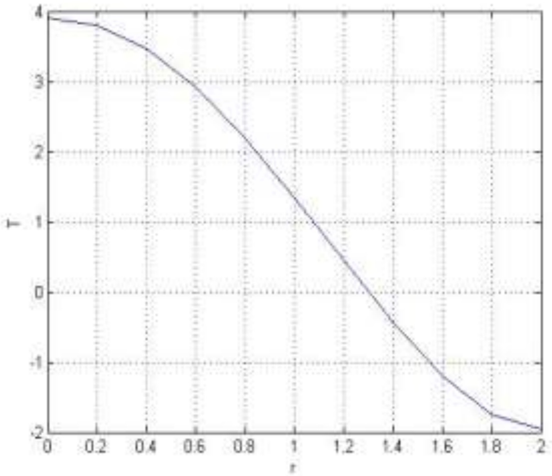

Fig.1 Temperature distribution at $z = 1$ and $t = 1$.

Fig. 1 shows the temperature distribution within a semi-infinite solid circular cylinder at z= 1 and t = 1, with the radial distance r varying from 0 to 2. The analysis demonstrates how the temperature decreases smoothly from the center of the cylinder r = 0 toward the boundary, reflecting the diffusion of heat under dynamic conditions.

Initially, the temperature is highest at the center and gradually declines as heat propagates outward. The insulated curved surface of the cylinder contributes to the decrease in temperature near the boundary, where the cooling effects become more pronounced.

This distribution highlights the dynamic nature of the thermal response in the system, influenced by the geometry and material properties.

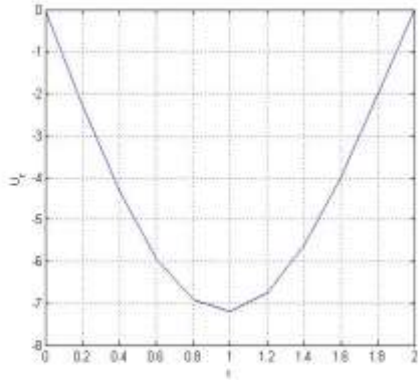

Fig 2. Radial component of displacement at $z = 1$

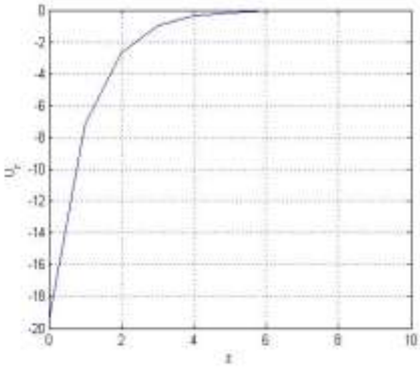

Fig 3. Radial component of displacement $r = 1$.

Fig. 2 shows the variation of with respect to the radial distance r at z = 1, where the displacement reaches its minimum near r = 1, indicating significant compressive stresses in this region. Fig. 3 illustrates the variation of along the axial direction z at r = 1, showing that displacement increases sharply near the surface and gradually levels off as z increases, highlighting the influence of the axial position on the deformation profile. Both figures reveal how thermal stresses affect displacement within the cylinder.

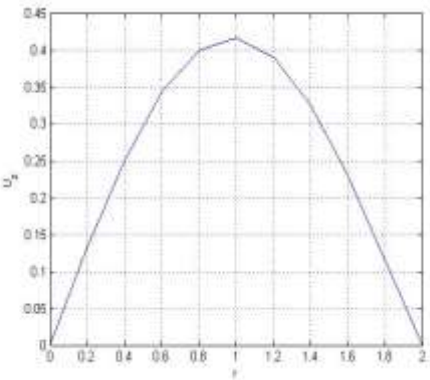

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Fig 4. Axial component of displacement $z = 1$.

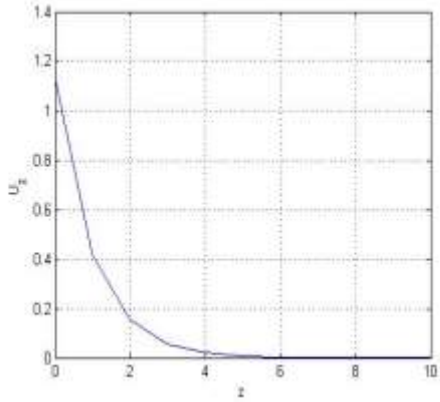

Fig 5. Axial component of displacement $r = 1$.

Fig. 4 and Fig. 5 illustrate the axial displacement component in a semi-infinite solid cylinder for a dynamic thermoelastic problem. In Fig. 4, exhibits a peak at r = 1 when z = 1, increasing from the origin and then gradually decaying towards the boundary. Fig. 5 shows that decreases sharply along the axial direction, with maximum displacement near z = 0, and rapidly diminishes as z increases. These results highlight the localized nature of the axial displacement near the surface and the center of the cylinder.

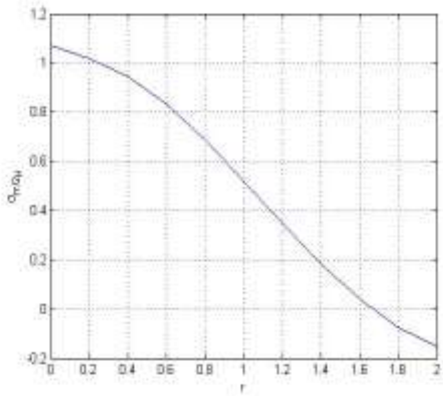

Fig 6. Radial Stress at $z = 1$.

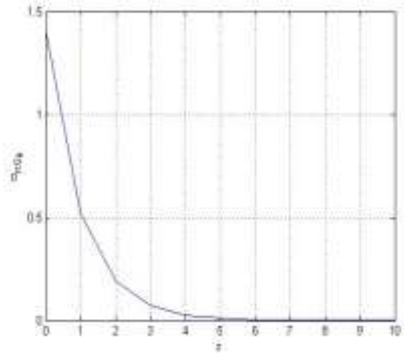

Fig. 7 Radial Stress at $r = 1$.

Fig. 6 and Fig. 7 present the radial stress component for a semi-infinite solid cylinder under dynamic thermoelastic conditions. In Fig. 6, the radial stress decreases steadily along the radial direction r, starting from a maximum at r = 0 and approaching zero near the boundary at r = 2 when z = 1. Fig. 7 shows that the radial stress decreases sharply in the axial direction, with a maximum at z = 0, and quickly approaches zero as z increases. This behavior highlights the significant stress concentration near the center and the surface.

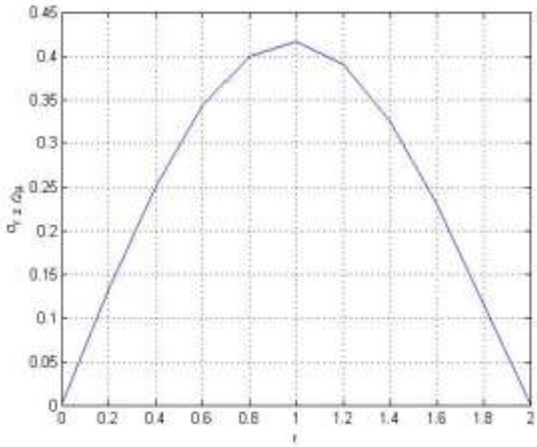

Fig. 8 Axial shear stress at $z = 1$.

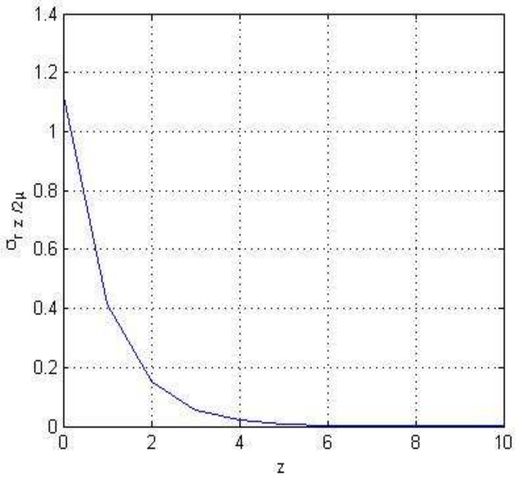

Fig 9 Axial shear stress at .

Fig. 8 and Fig. 9 depict the axial shear stress σ_{rz} in a semi-infinite solid cylinder under dynamic thermoelastic conditions. In Fig. 8, the shear stress increases along the radial direction r, reaching a peak near r = 1, and then decreases towards zero at the boundary when z = 1. Fig. 9 shows that the axial shear stress is highest at z = 0 and rapidly diminishes as z increases, becoming negligible beyond z = 3. These results indicate localized shear stress near the center of the cylinder, with both radial and axial variations.

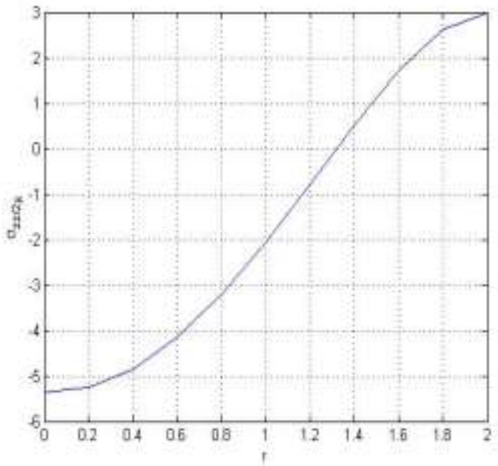

Fig 10 Axial stress at .

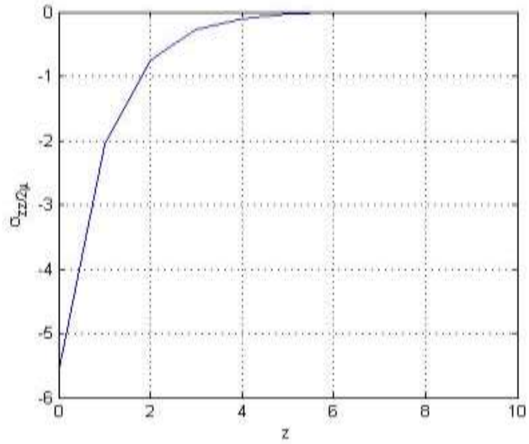

Fig 11 Axial stress at .

Fig. 10 illustrates that the axial stress increases with increasing r and Fig.11 shows that the axial stress increases gradually with increasing height and becomes steady after .

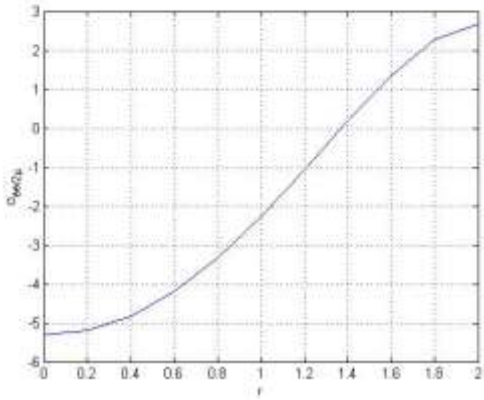

Fig.12 Angular stress at $z = 1$.

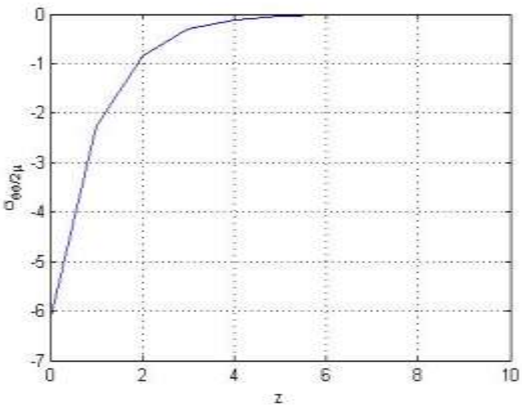

Fig. 13 Angular stress at $r = 1$.

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Fig. 12 shows that the angular stress increases gradually with increasing radius. The angular stresses become zero near and then increases with increasing .Fig. 13 depicts that the angular stresses are increasing with increasing .The stresses become negligible after and are steady afterwards.

The variations of the stresses are due to the effects of the inertia term. It can also be seen that the displacements and temperature satisfy the boundary conditions. The results seen are specific for this problem and may depict different trends because of the dependencies of the results on the thermal constants of the material taken up for observations.

IV. CONCLUSION

The study of the dynamic thermoelastic behavior of a semi-infinite solid circular cylinder reveals that temperature and displacement distributions are significantly influenced by the geometry and material properties. The temperature decreases smoothly from the center to the boundary, while displacement and stress profiles show localized variations, with significant compressive stresses near (r = 1) and sharp decreases in axial displacement and stress along the axial direction. The effects of inertia play a crucial role in these variations, and the results satisfy the boundary conditions, highlighting the specific thermal and mechanical responses of the material under dynamic conditions. These findings provide valuable insights for designing materials subjected to similar conditions, emphasizing the importance of considering both thermal and mechanical properties in engineering applications.

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