

# Vibration and Buckling Analysis of Cracked Composite Beam

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**Abstract**-Cracks in structural members lead to local changes in their stiffness and consequently their static and dynamic behaviour is altered. The influence of cracks on dynamic characteristics like natural frequencies, modes of vibration of structures has been the subject of many investigations. However studies related to behavior of composite cracked structures subject to in-plane loads are scarce in literature. Present work deals with the vibration and buckling analysis of a cantilever beam made from graphite fiber reinforced polyimide with a transverse one-edge non-propagating open crack using the finite element method. The undamaged parts of the beam are modeled by beam finite elements with three nodes and three degrees of freedom at the node. An overall additional flexibility matrix is added to the flexibility matrix of the corresponding non-cracked composite beam element to obtain the total flexibility matrix, and therefore the stiffness matrix in line with previous studies. The vibration of cracked composite beam is computed using the present formulation and is compared with the previous results. The effects of various parameters like crack location, crack depth, volume fraction of fibers and fibers orientations upon the changes of the natural frequencies of the beam are studied. It is found that, presence of crack in a beam decreases the natural frequency which is more pronounced when the crack is near the fixed support and the crack depth is more. The natural frequency of the cracked beam is found to be maximum at about 45% of volume fraction of fibres and the frequency for any depth of crack increases with the increase of angle of fibres. The static buckling load of a cracked composite beam is found to be decreasing with the presence of a crack and the decrease is more severe with increase in crack depth for any location of the crack. Furthermore, the buckling load of the beam decreased with increase in angle of the fibres and is maximum at 0 degree orientation.

**Keywords**-Cracks, Fibres, Angle, Beam

## I. INTRODUCTION

Composites as structural material are being used in aerospace, military and civilian applications because of their tailor made properties. The ability of these materials to be designed to suit the specific needs for different structures makes them highly desirable.

Improvement in design, materials and manufacturing technology enhance the application of composite structures. The suitability of a particular composite material depends on the nature of applications and needs. The technology has been explored extensively for aerospace and civil engineering applications, which require high strength and stiffness to weight ratio materials.

### 1. Scope of the present Investigation:

The main aim of this thesis is to work out a composite beam finite element with a non-propagating one-edge open crack. It has been assumed that the crack changes only the stiffness of the element whereas the mass of the element is

unchanged. For theoretical modeling of cracked composite beam dimensions, crack locations, crack depth and material properties is specified.

In this work an “overall additional flexibility matrix”, instead of the “local additional flexibility matrix” is added to the flexibility matrix of the corresponding non-cracked composite beam element to obtain the total flexibility matrix, and therefore the stiffness matrix in the line with the other researchers.

By using the present model the following effects due to the crack of the cantilever composite beam have been analyzed. The influence of the volume fraction of fibers, magnitude, location of the crack, angle of fibers upon the bending natural frequencies of the cantilever cracked composite beam.

The effects of above parameters on buckling analysis of cracked composite beam. The present results are compared with previous studies and the new results are obtained in the MATLAB environment.

## II. LITERATURE REVIEWS

### 1. Introduction:

The widespread use of composite structures in aerospace applications has stimulated many researchers to study various aspects of their structural behaviour. These materials are particularly widely used in situations where a large strength-to-weight ratio is required.

Similarly to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. These result in local changes of the stiffness of elements for such materials and consequently their dynamic characteristics are altered.

This problem is well understood in case of constructing elements made of isotropic materials, while data concerning the influence of fatigue cracks on the dynamics of composite elements are scarce in the available literature.

### 2. Review on vibration of cracked composite beam:

A local flexibility will reduce the stiffness of a structural member, thus reducing its natural frequency. For small crack depths the change (decrease) in natural frequency is proportional to the square of the crack depth ratio.

## III. THEORY AND FORMULATIONS

### 1. Introduction:

Structures are weakened by cracks. When the crack size increases in course of time, the structure becomes weaker than its previous condition. Finally, the structure may breakdown due to a minute crack.

The basic configuration of the problem investigated here is a composite beam of any boundary condition with a transverse one-edge non-propagating open crack. However, a typical cracked cantilever composite beam, which has tremendous applications in aerospace structures and high-speed turbine machinery, is considered.

- The following aspects of the crack greatly influence the dynamic response of the structure.
- The position of a crack in a cracked composite beam
- The depth of crack in a cracked composite beam
- The angle of fibers in a cracked composite beam
- The volume fraction of fibers in a cracked composite beam

### 2. The Methodology:

The governing equations for the vibration analysis of the composite beam with an open one-edge transverse crack are developed. An additional flexibility matrix is added to the flexibility matrix of the corresponding composite beam element to obtain the total flexibility matrix and therefore the stiffness matrix is obtained by Krawczuk&Ostachowicz (1995).

The assumptions made in the analysis are:

- The analysis is linear. This implies constitutive relations in generalized Hooke's law for the materials are linear.
- The Euler–Bernoulli beam model is assumed.
- The damping has not been considered in this study.
- The crack is assumed to be an open crack and have uniform depth „a“.

### 3. Computational Procedure for a Cracked Composite Beam:

A computer program is developed to perform all the necessary computations in MATLAB environment. In the initialization phase, geometry and material parameters are specified.

For example for a Euler–Bernoulli composite beam model with localized crack, material parameters like modulus of elasticity, the modulus of rigidity, the Poisson ratio and the mass density of the composite beam material and geometric parameters like dimensions of the composite beam, also the specifications of the damage like size of the crack, location of the crack and extent of crack are supplied as input data to the computer program.

## IV. RESULTS AND DISCUSSIONS

Effect of an open edge transverse crack on various parameters of a composite beam like vibration and buckling are studied and compared with previously studied results. The formulation is then validated and extended for other problems.

### 1. Introduction:

In order to check the accuracy of the present analysis, the case considered in Krawczuk&Ostachowicz (1995) is adopted here. The beam assumed to be made of unidirectional graphite fiber-reinforced polyamide.

The geometrical characteristics and material properties of the beam are chosen as the same of those used in Krawczuk&Ostachowicz (1995). The material properties of the graphite fiber-reinforced polyamide composite, in terms of fibers and matrix, is identified by the indices f and m, respectively, are in Table-4.1

Table 1. Properties of the graphite fibre- reinforced polyamide composite.

Modulus of Elasticity	$E_m$	2.756 GPa
	$E_f$	275.6 GPa
Modulus of Rigidity	$G_m$	1.036 GPa
	$G_f$	114.8 GPa
Poisson's Ratio		0.33
		0.2
Mass density	$\rho_m$	1600 kg/m <sup>3</sup>
	$\rho_f$	1900 kg/m <sup>3</sup>

The geometrical characteristics, the length (L), height(H) and width (B) of the composite beam, are taken as 1.0 m, 0.025 m and 0.05m respectively.

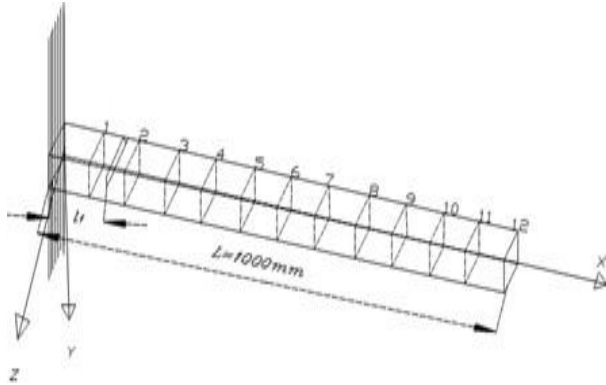


Fig1.Geometry of cantilever cracked composite beam with 12 elements.

In this chapter, the results of vibration and buckling analysis of composite beam structure with or without crack are presented using the formulation given in Chapter-3.

Each of the cracked composite beam problems is presented separately for the following studies:

- Convergence Studies
- Comparison with Previous Studies
- Numerical Result
- Vibration and Buckling Analysis of results of composite beam with single crack
- Vibration Analysis of results of composite beam with multiple cracks

## 2. Convergence Study:

The convergence study is carried out for the free vibration of cracked composite beam and omitted here for sake of brevity. Based on this study, a mesh of 12 elements shows good convergence of numerical solutions for free vibration of cracked composite beam, which is shown in Fig.4.2.

Table 2.Convergence of non-dimensional free vibration frequencies of cracked composite beam for different angle of fibers.

Mesh Division	Non dimensional frequencies for different angle of fibers "α"(degrees)		
	α = 0	α = 15	α = 30
2 elements	1.5982	1.6703	1.7125
4 elements	1.6815	1.7255	1.7732
8 elements	1.6995	1.7257	1.7748
12 elements	1.7055	1.7245	1.7743
Krawczuk & Ostachowicz (1995)	1.7055	1.7245	1.7743

## 3. Comparison with Previous Studies:

Quantitative results on the effects of various parameters on the vibration and buckling analysis of cracked composite are presented.

### 3.1Vibration analysis studies:

Table 3. Comparison of First three Non- dimensional natural frequencies of the non cracked composite beam as a function of the angle of fibers α, where Value of V=0.10 and 0.30.

Angle of Fibers (degree)	Volume Fraction V	Present Analysis			Krawczuk & Ostachowicz (1995)		
		1 <sup>st</sup> Non-dimensional Nat. freq	2 <sup>nd</sup> Non-dimensional Nat. freq	3 <sup>rd</sup> Non-dimensional Nat. freq	1 <sup>st</sup> Non-dimensional Nat. freq	2 <sup>nd</sup> Non-dimensional Nat. freq	3 <sup>rd</sup> Non-dimensional Nat. freq
0	0.10	1.8798	4.8566	7.6681	1.85145	4.52827	7.71888
15		1.8243	4.5300	7.4841	1.81768	4.51477	7.51418
30		1.6655	4.1530	6.9033	1.65453	4.12945	6.89687
45		1.4342	3.5854	5.9833	1.38995	3.53323	5.97735
60		1.2083	3.0230	5.0513	1.15370	3.01580	5.01780
75		1.0998	2.7514	4.5973	1.08133	2.74320	4.57040
90		1.0881	2.7205	4.5410	1.08007	2.71020	4.51710
0	0.30	1.8771	4.6113	7.5073	1.85145	4.52827	7.64894
15		1.8188	4.4873	7.3447	1.81768	4.44477	7.37372
30		1.6484	4.0982	6.7804	1.65453	4.02945	6.92680
45		1.3886	3.4482	5.7818	1.38995	3.43323	5.85710
60		1.1068	2.7684	4.6260	1.15370	2.71580	4.76640
75		0.948	2.3713	3.9632	1.08133	2.27052	4.04030
90		0.9307	2.3263	3.8831	1.08007	2.21720	3.97620

The presented method has been applied for the free vibration analysis of a non-cracked and cracked composite cantilever beam. Free vibration analysis of a cantilever cracked composite beam has been examined by Krawczuk&Ostachowicz (1995) using finite element method (FEM).

In this study the results obtained with present element are compared with the results of Krawczuk&Ostachowicz. Throughout this investigation, 12 elements are used in modeling the cracked composite beam. In addition, the three lowest eigen-frequencies for various values of the angle of the fiber (α) and the volume fraction of fibers (V) are determined and given in Table-4.3

### 3.2 Buckling analysis studies:

In this buckling analysis study, the results of non- cracked composite beam obtained with the present element are compared with the analytical results of Reddy (1997) and Ozturk&Sabuncu (2005).

Table-4.5 shows the comparison of present results of buckling load with the results of Reddy (1997) and Ozturk&Sabuncu (2005) for various values of the angle of the fiber (α). As seen from the tables agreements are good.

The buckling loads are normalized according to the following relation;

Table 4. Buckling loads of a non-cracked composite beam for angle of fiber = 0, 30, 60 and 90 degree.

Angle of fibers (degree)	Present FEM	Ozturk & Sabuncu (2005)	Reddy (1997)
0	4.9984	5.1404	5.14
30	1.6632	-	-
60	0.3891	-	-
90	0.2006	0.2056	0.205

#### 4. Numerical Results:

After obtaining the comparison with previous study and validating the formulation with the existing literatures, the results for non-dimensional natural frequencies of the non-cracked composite beam as a function of the angle of fibers ( $\alpha$ ) are presented.

The changes of the two first natural frequencies of the beam due to the crack as functions of the angle of fibers ( $\alpha$ ) and volume fraction of fiber are analyzed and buckling analysis is carried out for free vibration of a composite beam with single crack for various crack positions and crack depths.

Similarly, the three first natural frequencies of the composite beam due to the crack as functions of the angle of fibers ( $\alpha$ ) and volume fraction of fiber are analyzed for free vibration of a composite beam with multiple cracks for various crack positions. The beam assumed to be made of unidirectional graphite fiber-reinforced polyamide.

The geometrical characteristics and the material properties of the graphite fiber-reinforced polyamide composite beam are chosen as the same of those used in Ozturk&Sabuncu (2005). The material properties of the graphite fiber-reinforced polyamide composite are

$E_{11} = 129.207\text{GPa}$ ,  $E_{22} = 9.42512\text{GPa}$ ,  
 $G_{12} = 5.15658\text{GPa}$ ,  $G_{13} = 4.3053\text{GPa}$ ,  $G_{23} = 2.5414\text{GPa}$ ,

$$\nu_{12} = \nu_{13} = \nu_{23} = 0.3$$

The geometrical characteristics, the length (L), height(H) and width (B) of the composite beam were chosen as 1.0m, .009525m and 0.0127m, respectively.

The crack is located at  $x/L = 0.1, 0.2, 0.4, 0.6$  and  $0.8$ .

Relative crack depth ( $a/H$ ) = 0.2, 0.4 and 0.6

#### 4.1 Vibration analysis of results of composite beam with single crack:

Firstly, the present method has been applied for the free vibration analysis of a non-cracked composite cantilever

beam by using twelve elements FE model of the same length.

The three lowest non-dimensional natural frequencies for various values of the angle of fibers ( $\alpha$ ) and the volume fraction of fibers (V) are determined and tabulated in Table 4.6, 4.7, 4.8 and 4.9. The results are also plotted in Figures 4.7 to 4.10.

As the angle of the fiber increases from  $0^\circ$  to  $90^\circ$ , the non-dimensional natural frequency decreases. It is also found that the rate of decrease in non dimensional natural frequency with increase in volume fraction of fibers is more as the volume approaches approximately 45%.

Table 5. Numerical Result of First three non- dimensional natural frequencies of the non-cracked composite beam as a function of the angle of fibers  $\alpha$ , where Value of  $V=0.02$

Angle of fibers (degree)	Present analysis		
	1 <sup>st</sup> Non-Dimensional Frequency	2 <sup>nd</sup> Non-dimensional Frequency	3 <sup>rd</sup> Non-dimensional Frequency
0	2.6780	6.2190	10.3191
15	2.6605	6.3681	10.2488
30	2.6286	6.1887	10.0580
45	2.5081	5.8617	9.7541
60	2.2001	5.0901	9.0700
75	1.8163	4.4184	8.1543
90	1.6045	4.0736	7.4392

Table 6. Numerical Result of First three Non- dimensional natural frequencies of the non-cracked composite beam as a function of the angle of fibers  $\alpha$ , where Value of  $V=0.1$

Angle of fibers (degree)	Present analysis		
	1 <sup>st</sup> Non-Dimensional Frequency	2 <sup>nd</sup> Non-dimensional Frequency	3 <sup>rd</sup> Non-dimensional Frequency
0	2.4957	6.1828	10.1850
15	2.4674	6.1118	10.0653
30	2.3785	6.1887	9.6933
45	2.2201	5.8898	9.0445
60	1.9891	5.4987	8.1248
75	1.7186	4.9374	7.1049
90	1.5685	4.2808	6.5070

Table 7. Numerical Result of First three Non- dimensional natural frequencies of the non-cracked composite beam as a function of the angle of fibers  $\alpha$ , where Value of  $V=0.30$

Angle of fibers (degree)	Present analysis		
	1 <sup>st</sup> Non-Dimensional Frequency	2 <sup>nd</sup> Non-dimensional Frequency	3 <sup>rd</sup> Non-dimensional Frequency
0	2.4929	6.1383	10.0266
15	2.4401	6.0128	9.8280
30	2.2873	5.6482	9.2407
45	2.0519	5.0821	8.2490
60	1.7649	4.3856	7.1315
75	1.4848	3.6997	6.1433
90	1.3486	3.3634	5.5950



## V. CONCLUSION

The following conclusions can be made from the present investigations of the composite beam finite element having transverse non-propagating one-edge open crack. This element is versatile and can be used for static and dynamic analysis of a composite or isotropic beam.

From the present investigations it can be concluded that the natural frequencies of vibration of a cracked composite beam is not only the functions of the crack locations and crack depths but also the functions of the angle of fibers and the volume fraction of the fibers.

The presence of a transverse crack reduces the natural frequencies of the composite beam. The rate of decrease in the natural frequency of the cracked composite beam increases as the crack position approaches the fixed end. The intensity of the reduction in the frequency increases with the increase in the crack depth ratio.

This reduction in natural frequency along with the mode shapes of vibrations can be used to detect the crack location and its depth. When, the angle of fibers ( $\alpha$ ) increase the values of the natural frequencies also increase. The most difference in frequency occurs when the angle of fiber ( $\alpha$ ) is 0 degree. This is due to the fact that the flexibility of the composite beam due to crack is a function of the angle between the crack and the reinforcing fibers.

The effect of cracks is more pronounced near the fixed end than at far free end. It is concluded that the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end, respectively.

The decrease of the non-dimensional natural frequencies depends on the volume fraction of the fibers. The non-dimensional natural frequency is maximum when the volume fraction of fiber is approximately 45%. This is due to the fact that the flexibility of a composite beam due to crack is a function of the volume fraction of the fibers. Buckling load of a cracked composite beam decrease with increase of crack depth for crack at any particular location due to reduction of stiffness.

When, angle of fibers increase the values of the buckling loads decrease. This is due to the fact that for 0 degree orientation of fibers, the buckling plane normal to the fibers is of maximum stiffness and for other orientations stiffness is less hence buckling load is less.

## VI. SCOPE OF FUTURE WORK

The vibration and stability results obtained using this formulation can be verified by conducting experiments. The dynamic stability of the composite beam with cracks Static and dynamic stability of reinforced

concrete beam with cracks. The dynamic stability of beam by introducing slant cracks (inclined cracks) in place of transverse crack.

## REFERENCES

- [1] Adams R. D., Cawley, P. C., Pye J. and Stone J. (1978). A vibration testing for non-destructively assessing the integrity of the structures. *Journal of Mechanical Engineering Sciences* 20, 93-100.
- [2] Banerjee J. R. (2001). Frequency equation and mode shape formulae for composite Timoshenko beams. *Composite Structures* 51 (2001) 381-388.
- [3] Binici B. (2005) Vibration of beams with multiple open cracks subjected to axial force. *J Sound Vib*; 287(1-2):277-95.
- [4] Broek D. *Elementary Engineering Fracture Mechanics*. Martinus Nijhoff; 1986.
- [5] Chandrupatla T. R., Belegundu A. D. *Introduction to finite elements in engineering*. New Jersey: Prentice-Hall; 1991.
- [6] Dimarogonas A.D. (1996), *Vibration of Cracked Structures: A State of the Art Review*. *Engineering Fracture Mechanics*, 55(5), 831- 857.
- [7] Ghoneam S. M. (1995). Dynamic analysis of open cracked laminated composite beams. *Composite Structures* 32 (1995) 3-11.
- [8] Gounaris G.D., Papadopoulos CA, Dimarogonas AD. (1996). Crack identification in beams by coupled response measurement. *Comput Struct*; 58(2):299-305.
- [9] Goyal Vijay K., Kapania Rakesh K. (2008). Dynamic stability of laminated beams subjected to non-conservative loading. *Thin-Walled Structures* 46 (2008) 1359-1369.
- [10] Hamada A. Abd El-Hamid (1998). An investigation into the eigen-nature of cracked composite beams. *Composite Structure* Vol. 38, No. 1 - 4, pp. 45-55.
- [11] Jones R. M. *Mechanics of composite materials*. Taylor & Francis Press; 1999.
- [12] Kaihong Wang, Daniel J. Inmana & Charles R. Farrar (2005). Modeling and analysis of a cracked composite cantilever beam vibrating in coupled bending and torsion. *Journal of Sound and Vibration* 284 (2005) 23-49.