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A Study On Anti Ramsey Coloring Problems

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Abstract- Let ar(G, H) be the maximum number of colors such that there exists an edgecoloring of G with ar(G, H) colors such that each subgraph isomorphic to H has atleast two edges in the same color. We call ar(G, H) the Anti-Ramsey number for a pair of graphs ar(G, H). In this paper, we determine the Anti-Ramsey number for special graphs.

Keywords- Lollipop, Barbell, Triangular Snake Graphs.
Mathematical Classification : 05C15

I. INTRODUCTION

A Graph coloring is an assignment of labels, called colors, to the vertices of a graph such that no two adjacent vertices share the same color. Other types of colorings on graphs also exists, most notably edge colorings that may be subject to various constraints. The study of Graph colorings has historically been linked closely to that of planar graphs and the four colored theorem, which is also the most famous graph coloring problem. Those problems provided the original motivation for the development of Algebraic Graph Theory and the study of graph invariants such as those discussed. Applications for solved problems have been found in areas such as computer science, information theory, and complexity theory. Many day-to-day problems, like minimizing conflicts in scheduling are also equivalent to Graph coloring.

An edge-colored graph is called rainbow if all the colors on its edges are distinct. Let ar (G, H) be the largest number of colors such that there exists an edge-coloring of G with ar(G, H) colors such that each subgraph isomorphic to H has atleast two edge in the same color. For a pair of graphs(G, H), ar(G, H) is called the Anti-Ramsey number. Erdos and Simonovits introduced this concept in 1973 and it has been the subject of numerous studies. This paper deals the AntiRamsey number for some special graphs and also shows the exact value for particular graphs. For graph terminology, see[4].

A Lollipop Graph (Lm,n) is a graph consisting of a complete graph (clique) on m vertices and a path graph on n vertices, connected with a bridge. A Barbell Graph (Bn) is a special type of undirected graph consisting of two non-overlapping n − vertex cliques together with a single edge that has an endpoint in each clique. A Triangular Snake Graph (T Sn) is obtained from a pathu1, u2, . . . , un by joining ui and ui+1 to a new vertex vi for $1 \leq i \leq n$, that is every edge of a path is replaced by a triangle.

II. MAIN RESULT

Theorem 2.1. The Anti Ramsey number of a Lollipop Graph (Lm,n), ar(Lm,n, H) = m 2 + n − 1 where H = Sm; m ≥ 3 and n = 1, 2, 3.... m + n − 1 where H = C3 or Tm,r; n > m; n = 3, 4, 5, . . . ; m, r = 1, 2, 3, ... Proof. Csae 1: Let H = Sm the subgraph in Lm,n. In Lm,n, color exactly two edges with same color(blue) and the residue m 2 + n − 2 edges of Lm,n can be colored using different color. Entirely, we use m 2 + n − 1 colors. Suppose ar(Lm,n) = m 2 + n. Thus our assumption shows that there forms a rainbow coloring in Lm,n. This is a contradiction. Hence ar(Lm,n, Sm) = m 2 + n − 1.

Case 2: Let H = C3 or T3,r be the subgraph in Lm,n. In Lm,n, we color m 2 + n edges of Lm,n with same color(blue) and the residue m + n − 2 edges of Lm,n using different colors.Entirely, we use m + n − 1 colors. Suppose ar(Lm,n, H) = n + m. By hypothesis, we could able to color m 2 − m + 1 edges of Lm,n with same color(blue) and remaining m + n − 1 edges of Lm,n using different colors. Thus we obtain the subgraph that involve one edge from blue colored edge and rest of the edges from m + n − 1 edges. This shows that there forms a rainbow in a subgraphs. This is a contradiction. Hence ar(Lm,n, H) = m + n − 1.

Theorem 2.2. The Anti Ramsey number of a Barbell Graph (Bn) ar(Bn, H) = 2(n 2) − 1 where H = Lm,r or Tm,r and m = n; r = 1, 2, 3, . . . ; n = 3, 4, 5, . . . Proof. Let H = Lm,r or Tm,r be the subgraph in Bn. In Bn, color exactly three edges with same color(blue) and the residue 2n 2 − 2 edges of Bn can be colored using different colors. Entirely, we use 2n 2 − 1 colors. Suppose ar(Bn, H) = 2n 2 . By hypothesis, we could able to color exactly two with same color (blue) and residue 2n 2 − 1 edges with different color. Thus we obtain a subgraph that include one edge from blue colored edge and rest of edges from 2n 2 − 1 edges. This shows that there forms a rainbow in a subgraphs. This is a contradiction. Hence ar(Bn, Lm,r or Tm,r) = 2 n 2 − 1.

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Theorem 2.3. The Anti-Ramsey number of Triangular Snake Graph (T Sn), ar(T Sn, H) = (n−1) 2 + 1 where H = C3, orT3,r; n ≥ 3; r = 1, 2, 3, . . . n where H = S4 ∀ n ≥ 5 and n isodd (n−1) 2 + 2 where H = S3 ; n ≥ 5. Proof. Case 1: Let H = C3, orTm,r be the subgraph in T Sn. In T Sn, we color (n−1) edges of T Sn with same color(blue) and the residue (n−1) 2 edges of T Sn using different colors. Entirely, we use (n−1) 2 +1 colors. Suppose ar(T Sn, H) = (n−1) 2 + 2. By hypothesis, we colud able to color n − 2 same color(blue) and residue (n−1) 2 + 1 edges of T Sn using different colors. We obtain a subgraph that include one edge from those blue colored edge and remaining edges from (n−1) 2 + 1 edges.

This shows that there forms a rainbow coloring in a subgraphs. Hence ar(T Sn, C3, Tm,r) = (n−1) 2 + 1. Case 2: Let H = S4 be the subgraph in T Sn. In T Sn, we color (n−1) 2 edges of T Sn with same color(blue) and the residue (n−1) edges of T Sn by using different colors. Entirely, we use n colors. Suppose ar(T Sn, S4) = n + 1. By hypothesis we could able to color (n−3) 2 with same color(blue) and the residue n edges of T Sn using different colors. Thus we obtain a subgraph that involve one edge from blue colored edge and rest of edges from n edges. This shows that there forms a rainbow coloring of subgraph S4, which is a contradiction. Hence ar(T Sn, S4) = n. Case 3: Let H = S3 the subgraph in T Sn.

In T Sn, color (n − 2) edges with same color(blue) and the residue (n−1) 2 + 1 edges of T Sn using different colors. Entirely, we use (n−1) 2 + 2 colors. Suppose ar(T Sn, S3) = (n−1) 2 + 3. By hypothesis, we could able to color (n− 3) edges of T Sn with same color(blue) and the residue (n−1) 2 + 2 edges of T Sn using different colors. Thus we obtain a subgraph that involve one edge from those blue colored edge and rest of edges from (n−1) 2 + 2 edges. This shows that there forms a rainbow in a subgraph S3. This is a contradiction. Hence ar(T Sn, S3) = (n−1) 2 + 2.

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