International Journal of Scientific Research & Engineering Trends
Volume 6, Issue 4, July-Aug-2020, ISSN (Online): 2395-566X

Exact Series Solutionfor Vibrationof Restrained Composite Plate

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***Abstract*-The main objective of the present paper is to derive an exact series solution to derive the egin values and eigen functions of the fourth order homogeneous partial differential equations PDES of composite plate. The plates are studied under various types of classical and restrained boundary conditions BCS. Due to the additional terms in the governing PDES and the complex substations of BCS, the deduced mathematical complexity makes the publications of this type in literature very few. Although a huge amount of publications are concerned with the vibration of composite plate, the analytic and closed form solutions have a little attention from the publishers. The previous review of publications shows that the method of separation of variables is one of the most universal powerful techniques for solving the linear PDE. The present paper offers an accurate analytical solution which reduces the labor may be needed for carrying out the alternative numerical solutions. The effectiveness of the derived method shows the capability to provide an explicit closed form solution for higher order linear PDE incorporating mixed derivatives under complex boundary conditions. The present approach is utilized to create the closed form solution of composite plates with many types of boundary conditions. Many numerical cases are demonstrated by calculating the eigen values and mode shapes for a different modes of anisotropic plates with different edges of supports.**

***Keywords*-Eigen Functions, Equation of Motion, Power Series, Composite, Restraint Plate.**

I.INTRODUCTION

Closed form solution of linear PDES of many engineering problems is always a priority of attention for mathematicians and specialized publishers. Back in 1823, Navier offered the exact solution of bi-harmonic PDE of rectangular plate bending with simply supported edges [1]. In 1899, a single Fourier series is utilized by Levy [2] to derive a mathematical technique to study the bending problems of rectangular plate with two opposite simply supported edges. Exact closed form solution of PDE still for long time devoted to the simplest cases in many physical phenomenon and engineering applications.

Although a huge amount of publications are devoted to the analytic solution of the bi-harmonic PDE under classical BCS, the closed form solutions of higher order PDE incorporating mixed derivatives under complex boundary conditions are very few in literature [3]. The aim of the present study is to overcome the complexity due to variation of plate material and boundary conditions BCS.

The simplest plate theory, based on various simplified assumptions, has been used extensively for bending analysis of plates [4].However, simple plate theory is accompanied with simple type of governing PDE such as the bi-harmonic PDE [5], [6]. Adding some terms and coefficients due to considering the effect of composite materials, makes the solution of plate is difficult [7]-[9]. On the other hand, non-classical boundary conditions such as partially restrained boundary conditions increase the mathematical complexity of the solution [10]-[13]. In various techniques, solution of the equation of motion with higher order normally involves complicated numerical procedures [14], [15]. Only few cases, such as simply supported edges of special isotropic plates, closed form solutions exist [16]. Vibrations of various types of laminated plates subjected to classical boundary conditions are extensively studied in literature [18], [19]. Restrained boundary conditions against rotation or translation for edges of isotropic pate attracted many researchers in the recent literature [5], [11]. However, anisotropic and orthotropic plates with edges elastically restrained have very few publications available in the literature [3].

Recently, many studies on the exact solutions of linear PDES of plates have been published [20]. Publishers' attempts on the exact series solutions of plates with general boundary conditions had been previously done by using various methods such as Fourier series, superposition method and finite integral transform. Exact closed form for deflection of restrained orthotropic plated has been studied by Farag et al [21] using the computer aided Algebraic solution. Ashour [3] uses the finite strip

International Journal of Scientific Research & Engineering Trends
Volume 6, Issue 4, July-Aug-2020, ISSN (Online): 2395-566X

transition matrix technique to study buckling and vibration of cross-ply laminated restrained plate. Three-dimensional elasticity approach is applied by Setoodeh and Karami [22] to analyze composite plates with edges of elastic stiffness.

Liew et al. [23] studied the free vibration of symmetric cross-ply laminated restrained plates by Ritz method. Analytical method of solution for plates with additional parameters such as variable thickness is studied by Farag [24]. Anisotropic elastic plates with holes and cracks are studied by Hsieh et al [25] under various types of boundary conditions. Kang et al [26], introduce exact solutions for the vibrations plates having two opposite simply supported edges subjected to in-plane moments.

Also, Kuang et al [27] offered an exact solution for anisotropic plate with an elliptic hole subjected to remote loading. A new exact series solution for vibration of restrained orthotropic plates is offered by Zhang et al [28] using the method of finite integral transforms. Ragb et al [29], analyzed the composite plates using the method of moving least squares differential quadrature. Yongbin Ma el al [30], studied, analytically, the wave propagation model for forced vibration of orthotropic composite plate and introduced an agnatical method for the solution. Based on Fourier series, Xiang Liua et al [31] established an accurate analytical method for buckling of orthotropic plates. A. H. A. Hassan and Naci Kurgan [32] used developed Kantorovich method to analyze buckling of skew plate with elastic base. An analytical spectral stiffness method is offered by Xiang Liu et al [33] to investigate buckling of plate resting on Winkler subgrade base. In the present paper, an analytical method based on series solution of the fourth order partial differential equation is developed.

The solution method depends on expressing the displacement in a trigonometric series containing two separate functions. These two functions are determined according to the studied PDE and the proposed BC's. One of the two functions is the shape function of vibrating beam while the other is a series solution of reduced ordinary differential equation. The introduced approach relies on the device of separation of variables and series expansion of the unknown function. Special attention in this paper is focused on applying the accomplished technique to solve the DE's of vibration of orthotropic and anisotropic plate subjected to restrained conditions.

The obtained solution for partial differential equation is utilized to create the closed form solution of composite plate with various types of classical and restrained boundary conditions BC's. Natural frequency parameters and mode shapes of composite plates under the effect of restrained boundary conditions and composite materials are studied by the created approach through many cases of study.

II. EQUATIN OF MOTION FOR VIBRATION OF COMPOSITE PLATE

The dimension-less equation of motion of composite plate is:

$$D_{11}\frac{\partial^4 w}{\partial\zeta^4}+4\tau\frac{\partial^2}{\partial\zeta\,\partial\eta}[D_{16}\frac{\partial^2 w}{\partial\zeta^2}+\tau^2 D_{26}\frac{\partial^2 w}{\partial\eta^2}]+2\tau^2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial\zeta^2\partial\eta^2}+\tau^4 D_{22}\frac{\partial^4 w}{\partial\eta^4}+\rho h a^4\frac{\partial^2 w}{\partial t^2}=0 \quad (1)$$

where
w is the dimensionless displacement of plate, t denotes time, ρ is plate mass per unit volume and h is the plate thickness, $\tau=\frac{a}{b}$ is the aspect ratio and a,b are the dimensions of plate in ζ,η directions respectively.

The composite plate constants are:

$$D_{ij}=\int_{-h/2}^{h/2}\overline{Q_{ij}}^{\,k} z_k^2\,dz \qquad ;i,j=1,2,6 \quad (2)$$

where z_k is the distance from bottom of k^{th} layer to the middle plane of plate while is defined by $\overline{Q_{ij}}^{\,k}$:

$$\overline{Q_{ij}}^{\,k}=T(Q)_k T^T \quad (3)$$

where

$$T=\begin{bmatrix}\cos\theta_k{}^2 & \sin\theta_k{}^2 & \sin\theta_k\cos\theta_k\\ \sin\theta_k{}^2 & \cos\theta_k{}^2 & -\sin\theta_k\cos\theta_k\\ -2m\cos\theta_k & 2\sin\theta_k\cos\theta_k & \cos\theta_k{}^2-\sin\theta_k{}^2\end{bmatrix} \quad (4)$$

and θ_k is the principle axis orientation angle of the laminating layer, and T^T means transpose of matrix T. Matrix $(Q)_k$ is the material matrix constant for k^{th} layer in the material principle coordinates where:

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}},\ Q_{22}=\frac{E_{22}}{1-\nu_{21}\nu_{12}}$$
$$Q_{12}=Q_{21}=\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}}=\frac{\nu_{21}E_{11}}{1-\nu_{21}\nu_{12}}$$
$$Q_{66}=G_{12},\ Q_{16}=Q_{26}=0 \quad (5)$$

where E_{11},E_{22} are the Young's modulus in the directions of layer principle axes 1,2 respectively. is the plane shear

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modulus and, are Poisson's ratiosThe plate dimensionless displacement $w = w(\zeta,\eta,t)$ is assumed to be in the form:

$$w(\zeta,\eta) = \sum_{m=1}^{M} g_m(\zeta) f_m(\eta) \sin \omega t \qquad (6)$$

where $g_m(\zeta)$ is a known basic function satisfying the boundary conditions of plate at the two edges $(\zeta = 0,1)$. Equation (6) is applied [3], [5] to reduce Eq. (1) to:

$$\sum_{m=1}^{M} \{ \tau^4 \alpha_m D_{22} f_m''''(\eta) + 4\tau^3 \beta_m (D_{16}+D_{26}) f_m'''(\eta) + 2\tau^2 \gamma_m (D_{12}+2D_{66}) f_m''(\eta) + 4\tau p_m (D_{16}+D_{26}) f_m'(\eta) + (q_m C_o - \omega^2 \rho h a^4 \alpha_m) f_m(\eta) \} = 0 \qquad (7)$$

where

$$\alpha_{mn} = \int_0^1 g_m g_m d\zeta \,,\; \beta_{mn} = \int_0^1 g_m g'_m d\zeta \,,\; \gamma_{mn} = \int_0^1 g_m g''_m d\zeta \,,\; p_{mn} = \int_0^1 g_m g'''_m d\zeta \,,\; q_{mn} = \int_0^1 g_m g''''_m d\zeta \qquad (8)$$

The series solution of the reduced the ordinary differential equation (7) yields:

$$w(\zeta,\eta) = \sum_{m=1}^{M} g_m(\zeta) \left[f_m(0) + \sum_{k=1}^{K} f_m^{(k)}(0) \frac{\eta^k}{k!} \right] \qquad (9)$$

where

$$f_m^{(k)}(0) = \frac{d^k f}{d\eta^k} \;; k = 1,2,3,\ldots\ldots\ldots K \qquad (10)$$

Consequently:

$$w(\zeta,\eta) = \sum_{m=1}^{M} g_m(\zeta) \{ f_m(0) + f_m'(0)\eta + f_m''(0)\frac{\eta^2}{2!} + f_m'''(0)\frac{\eta^3}{3!} + \Re_0 \} \qquad (11)$$

where

$$\Re_0 = \sum_{k=4}^{K} f_m^{(k)}(0) \frac{\eta^k}{k!} \qquad (12)$$

The truncated power series function $\Re_0$,with variable η, has constantcoefficients $f_m^{(k)}(0);\; k = 4,5,6,\ldots\ldots\ldots K$ depending on the four initial values $f_m(0),\, f_m'(0)$ $f_m''(0),\, f_m'''(0)$ so that:

$$f_m^{(k)}(\eta) = \frac{d}{d\eta} f_m^{(k-1)}(\eta);\quad k = 5,6,7,\ldots\ldots K \qquad (13)$$

and

$$f_m^{(4)}(\eta) = -\frac{4\beta_m (D_{16}+D_{26})}{\alpha_m \tau D_{22}} f_m'''(\eta) - \frac{2\gamma_m (D_{12}+2D_{66})}{\alpha_m \tau^2 D_{22}} f_m''(\eta) - \frac{4 p_m (D_{16}+D_{26})}{\alpha_m \tau^3 D_{22}} f_m'(\eta) - \frac{(q_m \frac{D_{11}}{D_{22}} - \lambda^2 \alpha_m)}{\alpha_m \tau^4} f_m(\eta) \qquad (14)$$

where

$$\lambda_m^2 = \omega^2 a^4 \frac{\overline{m}}{D_{22}},\; \overline{m} = \rho h$$

Initial values $f_m(0),\, f_m'(0),\, f_m''(0),\, f_m'''(0)$ are accomplished from the known boundary condition at $(\eta = 0,1)$.

Reformatting $\Re_0$ in term of these initial values, one can establish the final solution incorporating the kth power degree of η such that:

$$\begin{aligned} w(\zeta,\eta) = \sum_{m=1}^{M} g_m(\zeta) \{ & f_m(0) + f_m'(0)\eta + f_m''(0)\frac{\eta^2}{2!} \\ & + f_m'''(0)\frac{\eta^3}{3!} - \frac{1}{A_4}[A_3 f_m'''(0) + A_2 f_m''(0) + A_1 f_m'(0) \\ & + A_0 f_m(0)]\frac{\eta^4}{4!} - \frac{1}{A_4}[-\frac{A_3}{A_4}[A_3 f_m'''(0) + A_2 f_m''(0) \\ & + A_1 f_m'(0) + A_0 f_m(0)] + A_2 f_m'''(0) + A_1 f_m''(0) \\ & + A_0 f_m'(0)]\frac{\eta^5}{5!} - \frac{1}{A_4}[-\frac{A_3}{A_4}[-\frac{1}{A_4}[A_3 f_m'''(0) + A_2 f_m''(0) \\ & + A_1 f_m'(0) + A_0 f_m(0)] - \frac{A_2}{A_4}[A_3 f_m'''(0) + A_2 f_m''(0) \\ & + A_1 f_m'(0) + A_0 f_m(0)] + A_1 f_m'''(0) + A_0 f_m''(0)]\frac{\eta^6}{6!} + \ldots\ldots \\ & \ldots\ldots + f_m^{(k)}(0).\frac{\eta^k}{k!} \} \end{aligned} \qquad (15)$$

where

$$A_0 = (q_m \frac{D_{11}}{D_{22}} - \lambda_m^2 \alpha_m),\; A_1 = 4\tau\, p_m (D_{16}+D_{26}),$$
$$A_2 = 2\tau^2 \gamma_m (D_{12}+2D_{66}),\; A_3 = 4\tau^3 \beta_m (D_{16}+D_{26}),$$
$$A_4 = \tau^4 \alpha_m D_{22}$$

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III. SHAPE FUNCTION AND BOUNDARY CONDITIONS

The shape function $g_m(\zeta)$, most commonly used [5], [11], is:

$$g_m(\zeta) = B_1 \sin(\mu_m \zeta) + B_2 \cos(\mu_m \zeta) + B_3 \sinh(\mu_m \zeta) + B_4 \cosh(\mu_m \zeta) \qquad (16)$$

where μ_m is a parameter and B_1, B_2, B_3, B_4 are constants to be determined [5] when the known boundary conditions at the edges of support at $(\zeta = 0,1)$ are satisfied. The General boundaries implemented in this paper are elastically restrained against rotation Er or translation Et. The conditions of simply supported S, clamped C, free F Edges are deduced as particular cases of the general conditions of the boundaries Erand Et. More convenience formulae of boundary conditions [24] are used here. Equations (15) represents the general solutions of the governing partial differential equations of vibration of plates under many types of boundary conditions at $(\zeta = 0,1)$ and $(\eta = 0,1)$.

Applying the formulated boundary conditions [24] at $(\zeta = 0,1)$, one can obtain the basic function coefficients of equation (16) and consequently the integral values of equations (8). Also, if the proposed boundary conditions are satisfied at $(\eta = 0)$, two initial values from $f_m(0)$, $f'_m(0)$, $f''_m(0)$ or $f'''_m(0)$ will be known. The two remaining unknowns are determined by satisfying the boundary conditions at $(\eta = 1)$.

Satisfying boundary conditions at $(\eta = 1)$, one can achieve two homogenous algebraic equations containing the two remaining unknowns. The eigen values and eigen functions of the characteristic equation of these homogenous equations are the natural frequency parameters λ_{mn} and mode shape $w_{mn}(\zeta, \eta)$ of composite plate.

IV. CASE STUDY

Here, the study will focus to the cases of composite plate with edges simply supported, clamped and/or elastically restrained against rotation. The restrained boundary conditions against rotation of orthotropic plate are:

$$g(\zeta) = 0, \quad g'(\zeta) + \delta_T\, g''(\zeta) = 0 \ \text{ at } \zeta = 0 \qquad (17)$$
$$g(\zeta) = 0, \quad g'(\zeta) - \delta_T\, g''(\zeta) = 0 \ \text{ at } \zeta = 1 \qquad (18)$$

and

$$f(\eta) = 0, \quad f'(\eta) + \delta_L\, f''(\eta) = 0 \ \text{ at } \eta = 0 \qquad (19)$$
$$f(\eta) = 0, \quad f'(\eta) - \delta_L\, f''(\eta) = 0 \ \text{ at } \eta = 1 \qquad (20)$$

where, δ_T , δ_L are the moduli of restraint respectively at $\zeta = (0,1)$, $\eta = (0,1)$. Boundary conditions of simply supported edge and clamped edge can be achieved when the modulus of restraint against rotation at this edge tends to ∞ and 0 respectively. Applying, boundary conditions of Eqs (17), (18) at $\zeta = (0,1)$ in Eq. (16), one can determine the constants B_1, B_2, B_3, B_4 and μ_m. On the other side, the reduced ODE (7) is solved as an initial value problem under the boundary conditions of Eqs (19), (20) to achieve the longitudinal function $f(\eta)$. To carry out the solution, the four initial values $f(0) = 0$, $f'(\eta) = \phi_{RL} \nabla_1$, $f''(0) = \nabla_1$, and $f'''(0) = \nabla_2$ of $f(\eta)$ at $\eta = 0$ are applied. The two constants ∇_1, ∇_2 are determined from satisfying the known boundary conditions at $\eta = 1$.

To touch the explicit and implicit solutions that can obtained from equation (15) under specified boundary conditions, several cases are studied in this section. Some cases such as carbon-epoxy plate (T300/N5208) [25] are considered. In the present cases study, the plate thickness is 0 .75 mm, and the material properties of an orthotropic carbon-epoxy plate are:

$E_{11} = 181 GPa$, $E_{22} = 10.3\, GPa$, $G_{12} = 7.17 GPa$, $\nu_{12} = \nu_{21} = 0.28$

The bending stiffness's calculated from this materials properties and plate thickness are:
$D11 = 409.08$, $D22 = 23.28$, $D12 = 6.52$, $D16 = D26 = 0$ and $D66 = 16.13$

1. Case of Square Orthotropic Plate CCCC
The explicit eigen values and mode shapes are obtained for truncation number $k = 20$. The results are achieved for the first three modes as follows:

1.1- First Mode $m = 1$
The explicit eigen values $\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}, \ldots\ldots\ldots \lambda_{1n}$ are obtained from:

$$0.0057069951\lambda_1^{2} + 23.548308 + 4.5710619(10)^{-7}\lambda_1^{4} - 1.6847265(10)^{-11}\lambda_1^{6} + 1.1511158(10)^{-15}\lambda_1^{8} - 8.8050265(10)^{-15}\lambda_1^{-20} + 5.938257(10)^{-15}\lambda_1^{12} + 1.0895634(10)^{-28}\lambda_1^{14} + 1.2839980(10)^{-33}\lambda_1^{16} = 0 \qquad (21)$$

In this case, the first three values of λ_{1n} are:
$\lambda_{11} = 97.2518290$, $\lambda_{12} = 109.9865128$,
$\lambda_{13} = 110.3751452$

The following equation describes the mode shape of plate corresponding to $\lambda_{11} = 97.2518299$ which is:

$$w_{11}(\zeta,\eta) = [0.5\zeta^2 - 1.08249920\zeta^3 + .990239226\zeta^4 - 1.286319831\zeta^5 + 1.885145978\zeta^6 - 1.749144857\zeta^7 + 1.267152400\zeta^8 - .914460981\zeta^9 + 0.631031390\zeta^{10} - 0.372595033\zeta^{11} + 0.198142671\zeta^{12} - 0.098995053\zeta^{13} + 0.046689888\zeta^{14} - 0.020216711\zeta^{15} + 0.008218366\zeta^{16} - 0.003139893335\zeta^{17} + 0.001142188306\zeta^{18} - 0.0003904328387\zeta^{19}][\sin(1.5\pi\eta) - \sinh(1.5\pi\eta) - 1.017803746\cos(1.5\pi\eta) + 1.017803746\cosh(1.5\pi\eta)] \quad (22)$$

Then, the mode shape, Fig.1, is:

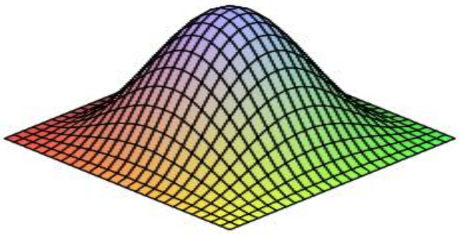

Fig. 1 CCCC Plate Mode Shape, $\lambda_{11} = 97.2518299$, $m = 1$, $n = 1$.

Similarly, the mode shape of plate when $\lambda_{12} = 109.9865182$ is shown in Fig. 2 and described by the function:

$$w_{12}(\zeta,\eta) = [5000000000\zeta^2 - 1.41017376\zeta^3 + .9902393432\zeta^4 - 1.675691454\zeta^5 + 5.550588132\zeta^6 - 6.709108934\zeta^7 + 4.378293991\zeta^8 - 4.116103490\zeta^9 + 4.935408880\zeta^{10} - 3.796245897\zeta^{11} + 2.15328615\zeta^{12} - 1.401465090\zeta^{13} + 0.9861571932\zeta^{14} - .5562611833\zeta^{15} + .2668208493\zeta^{16} - .1327989734\zeta^{17} + 0.06680276608\zeta^{18} - 0.02974848329\zeta^{19}][\sin(1.5\pi\eta) - \sinh(1.5\pi\eta) - 1.017803746\cos(1.5\pi\eta) + 1.017803746\cosh(1.5\pi\eta)] \quad (23)$$

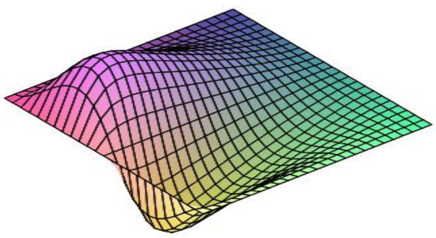

Fig. 2 CCCC Plate Mode Shape, $\lambda_{12} = 109.9865182$, $m = 1$, $n = 2$.

1.2- Second Mode $m = 2$

The explicit eigen values $\lambda_{21}, \lambda_{22}, \lambda_{23}, \lambda_{24}, \ldots\ldots\ldots \lambda_{2n}$ are obtained from:

$$7.644634941(10)^5 - 75.34205782\lambda_2^2 + 0.292062574(10)^{-2}\lambda_2^4 - 5.309246972(10)^{-8}\lambda_2^6 + 3.638407441(10)^{-13}\lambda_2^8 + 1.388083961(10)^{-18}\lambda_2^{10} - 2.174119277(10)^{-23}\lambda_2^{12} - 1.022276480(10)^{-28}\lambda_2^{14} + 1.283998061(10)^{-33}\lambda_2^{16} = 0 \quad (24)$$

The first two values of λ_{2n} are:

$\lambda_{21} = 249.8456446$, $\lambda_{22} = 258.1382675$

And the mode shape for $\lambda_{21} = 249.8456446$ is:

$$w_{21}(\zeta,\eta) = [.5000000000\zeta^2 + .8713804435\zeta^3 + 3.734445199\zeta^4 + 3.904947015\zeta^5 + 4.990212575\zeta^6 + 3.727177445\zeta^7 - 1.882863895\zeta^8 - 1.093793825\zeta^9 - 6.271183160\zeta^{10} - 2.980683475\zeta^{11} - 3.554375945\zeta^{12} - 1.429483245\zeta^{13} - .5913606095\zeta^{14} - .2061200315\zeta^{15} + .1404552295\zeta^{16} + 0.04319648040\zeta^{17} + 0.07689118181\zeta^{18} + 0.02115834818\zeta^{19}][\sin(2.5\pi\eta) - \sinh(2.5\pi\eta) - .9992232922\cos(2.5\pi\eta) + .9992232922\cosh(2.5\pi\eta)] \quad (25)$$

This function yields the mode shape illustrated in Fig. 3.

2. Case of Square Elastically Restrained- Simply Supported Orthotropic Plate ErSErS

The studied plate in this case is square and simply supported S at two opposite edges while the other edges are restrained against rotation Er with restraint coefficientcoefficient δ. Considering $m = 1$ and a truncation number $k = 12$, one can achieve implicit equation of the plate parameter λ_{1n} as a function of restraint coefficient δ such as:

$$F(\lambda_1,\delta) = 1.770039494 - 035.3923400\delta + 180.9214150\delta^2 - [0.0012986062 + 0.03064708838\delta + 0.20159645\delta^2]\lambda_1^2 + [1.932304010(10)^{-7} + 0.768361790(10)^{-5}\delta + 0.00008920383847\delta^2]\lambda_1^4 + [2.037868010(10)^{-11} - 8.851891310(10)^{-10}\delta - 2.206726297(10)^{-8}\delta^2]\lambda_1^6 + [6.903689410(10)^{-15} + 2.761475791(10)^{-13}\delta + 2.623401991(10)^{-12}\delta^2]\lambda_1^8 = 0 \quad (26)$$

3. Case of Square Elastically Restrained- Clamped Orthotropic Plate ErCErC

Similarly, The plate under study is square and clamped supported C at two opposite edges while the other edges are restrained against rotation Er with restraint coefficient δ. Considering $m=1$ and a truncation number $k=12$, one can achieve implicit equation of the plate parameter λ_{1n} as a function of restraint coefficient δ such as:

$$\begin{aligned} F(\lambda_1,\delta) &= 29.5489009 + 1627.971007\,\delta \\ &+ 20937.88851\,\delta^2 + (-0.009013528 \\ &- .6332350116\,\delta - 8.619248553\,\delta^2)\lambda_1^2 \\ &+ (0.0000014631853 20 + 0.000102727196\,\delta \\ &+ 0.00135591526\,\delta^2)\lambda_1^4 + (-1.53453878(10)^{-10} \\ &- 8.31318323(10)^{-9}\,\delta - 9.65785705(10)^{-8}\,\delta^2)\lambda_1^6 \\ &+ (6.90368946(10)^{-15} + 2.76147580(10)^{-13}\,\delta \\ &+ 2.62340200(10)^{-12}\,\delta^2)\lambda_1^8 = 0 \end{aligned} \quad (27)$$

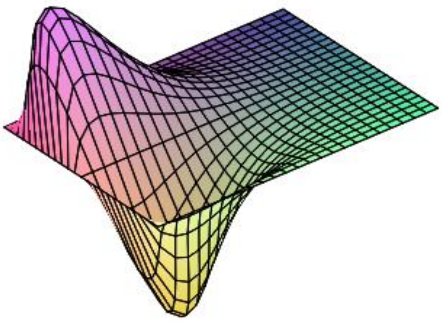

Fig. 3. CCCC Plate Mode Shape, $\lambda_{21}=249.845646$, $m=2$, $n=1$.

V. CONCLUSION

A novel series technique is presented to obtain the egin values and eigen functions of the fourth order homogeneous partial differential equations PDES of composite plate. The direct exact outcomes derived for the basic element employing the proposed technique may help to provide an insight into the investigation of composite structures. Due to mixed derivative involved in the governing PDE of composite and direct substitutions of complex boundary conditions, the analytical implantation becomes more complex and publications of this type in literature become rare. The present exact technique reduces the excessive effort needed for carrying out the alternative numerical solutions. The effectiveness of the achieved technique shows the capability to provide an explicit closed form solution for composite plate governed by a higher order linear PDE incorporating mixed derivatives under complex boundary conditions. Different cases study are discussed by calculating the eigen values and mode shapes of the vibration modes for a composite plates with different types of boundary conditions.

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