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Vertex Edge and Equitable Vertex Edge Domination of Topologically Indexable Graphs

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Abstract -Let G = (V, E) be a simple graph. Let τ be a topology on V(G). We define a topology on V(G) and given a topology on a non-empty finite set V, and graph G(T) with vertex set V is called topological graph. In this paper we have identified the Perfect domination number, Vertex – Edge domination number, Equitable vertex-edge domination number Maximal vertex-edge domination number of graph G(T), also an attempt is made to identify the topologically indexable total Hamiltonian line graph.

Keywords-Topological graph, Global Perfect Domination, Global Vertex – Edge Domination, Maximal vertex-edge domination, equitable vertex – edge domination, Total Hamiltonian Line Graph.

I. INTRODUCTION

Let G = (V, E) be a graph with a vertex set V (G) and edge set E (G). B.D. Acharya has introduced set-indexer [1] of a graph G as an injective set assignment f: v →2x such that the induced set assignment f: E→2x defined by is also injective. Instead of allowing X to be arbitrary, we consider the topologies on V (G).Suppose is a topology on V(G) induced by {N(u) , u V(G)} as subbasis.
A– indexer of G = (V, E) is an injective set assignment f: V →τ such that the induced set assignment f *: E →τ(X) defined by f *(uv) = f (u). f (v) is 1- 1, Where. denotes symmetric difference.

A graph G is said to be – indexible if G admits an injective vertex and an injective induced function such that f (G) = {N[u] : u V(G)}. We call such f as indexer of G. For topology on V (G) we can take a topology generated by N[a] as subbasis if G is not complete. If G is complete then the topology is generated by N (a) (or) topology generated by all doubletons which are adjacent (or) we can take the topology T (G) generated by the sets S = {ф} {V} { {N(v)} : v G, deg v2 } { {N(v)} : v G, deg v =1 and adjacent points of v has degree ≥2} or │V(G)│= 2. [7]

Also if u v is an induced sub graph of G then {u,v} S and N(u) , N(v) S, such that is defined as f(u) f(v).Throughout the paper the topology on V(G) is generated by {N[a] , a V(G)} as subbasis if G is not complete, if G is complete topology is generated by {N(a) , a V(G)} .

1. Definition
Let X be a non empty set and T a topology on X. The graph G for which V(G) = X and in which v1,v2 V(G) are adjacent if { v1,v2 } T is called a topological graph and is denoted by G(T).[8]
Definition
Let G be a graph. The topology on V(G) generated by {(vi, vj) / vi, vj (E)} V(G) as subbasis is called the topology of the graph and is denoted by T(G).[8]
Note:
Clearly G (T (G)) contains G as a sub graph but not as an induced sub graph [8] .
Some domination parameter used in G (T (G)):
The following are some of the domination parameters used in this paper, and following domination numbers are identified for G (T (G)).

2. Definition
The set D is a dominating set if every vertex v V is either an element of D or is adjacent to an element of D. The minimum cardinality dominating set of G is said to be domination number and is denoted by γ (G). [6] The maximum cardinality of a minimal dominating set of a graph G is called the upper domination numbr and is denoted by (G).

3. Definition
A Subset S V (G) is a perfect domination set if for every vertex v V – S, where N (v) is the open neighborhood of v such that it is the set { u / u V – {v} and (u,v) E} .
The minimum cardinality perfect dominating set is called perfect domination number and is denoted by .The maximum cardinality perfect domination number is denoted by
A dominating set S V (G) of a graph is said to be Global perfect dominating set [5] if S is a Perfect dominating set of both G and its compliment .

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The Minimum cardinality Global perfect dominating set is called Global perfect domination number and is denoted by .The Maximal cardinality global perfect domination number is denoted by

4. Definition

A set S V(G) is a vertex-edge dominating set if for all edges e E(G), there exist a vertex v S such that v dominates e. Otherwise for a graph G = (V,E) a vertex u V(G) ve-dominates an edge vw E(G) if (i) u = v or u = w (u is incident to vw), or (ii) uv or uw is an edge in G (u is incident to an edge that is adjacent to vw).

The minimum cardinality of a vertex-edge dominating set is called vertex-edge domination number of a graph G and is denoted by ve (G).

The maximum cardinality of a vertex-edge dominating set is called as upper vertex-edge domination number and is denoted by ve (G).

A subset S of V is global vertex-edge dominating set if S is vertex-edge dominating set in both G and.

The minimum cardinality of a global vertex-edge dominating set is called global vertex-edge domination number [2] of a graph G and is denoted by γgve(G).

The maximum cardinality of a global vertex-edge dominating set is called as upper global vertex-edge domination number and is denoted by Γgve (G).

Note: For any connected graph G the ve - domination number is γve(G) and for

ve – domination number γve then γgve(G) = max { γve(G) , γve}.

5. Definition

A set S V(G) is a Maximal vertex-edge dominating set if (i) S is a vertex – edge dominating set, (ii) V(G) - S is not a vertex – edge dominating set.

The minimum cardinality of a Maximal vertex-edge dominating set is called Maximal vertex-edge domination number of a graph G and is denoted by γ mve (G).

The maximum cardinality of a Maximal vertex-edge dominating set is called as upper Maximal vertex-edge domination number and is denoted by Γ mve (G).

6. Definition

A set S V (G) is an equitable vertex-edge dominating set if

S is a vertex – edge dominating set,

For every v V - S there exist a vertex u S such that uv E(G) and

│deg(u) – deg(v) │≤ 1.

The minimum cardinality of a Equitable vertex-edge dominating set is called Equitable vertex- edge domination number of a graph G and is denoted by γ e ve (G).

The maximum cardinality of a Equitable vertex-edge dominating set is called as upper Equitable vertex-edge domination number and is denoted by Γ e ve (G).

Total Hamiltonian Line graph, Topology and Topological graph: 1.8

Let G = (V,E) be a simple connected graph , construct line graph L(G) of G, such that whose vertices correspond to the edges of G, and two vertices of L(G) are joined by an edge if and only if the corresponding edges in G are adjacent. Identify Hamiltonian path of line graph, which is HL(G), it covers all the vertices of line graph and it need not be unique. Construct the total graph T(HL(G)) of Hamiltonian path, the vertex set of total graph is the set of all elements of HL(G), and two vertices are adjacent if and only if the corresponding elements are associated in HL(G).The graph THL(G) is called Total Hamiltonian Line graph.

Let G be a Total Hamiltonian Line graph (THLG). The topology on V(G) generated by {(vi,vj) / vi,vj (E)} V(G) as sub basis is called the topology of Total Hamiltonian Line graph and is denoted by T(THLG).

Let X be a non empty set and T a topology on X. The graph THLG for which

V(THLG) = X and in which v1,v2 V(THLG) are adjacent if { v1,v2 } T is called a topological graph and is denoted by G[T(THLG)].

II. MAIN RESULTS

A graph G is said to be self morphic if G is isomorphic to topological graph of the topology of G.

1. Result

Total Hamiltonian line graph of path Pn is isomorphic to topological graph of the topology of THLPn.

2. Result

Total Hamiltonian line graph of Cycle Cn is isomorphic to topological graph of the topology of THLCn.

3. Result

Total Hamiltonian line graph of complete graph Kn is isomorphic to topological graph of the topology of THLKn.

4. Result

Total Hamiltonian line graph of Star graph K 1,n-1 is isomorphic to topological graph of the topology of THLK 1,n-1 .

5. Result

Total Hamiltonian line graph of Complete bi partite graph K n,m is isomorphic to topological graph of the topology of THLK n,m .

6. Result

For any graph G, The Total Hamiltonian line graph is isomorphic to topological graph of the topology of THLG. Therefore Total Hamiltonian line graph is Self morphic.

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III. SOME DOMINATION PARAMETERS ON G (T (G))

1. Theorem

Global Perfect domination number of G (T (Cn)) is given by = n.

Proof:

From the above result: 1 it is clear that G (T (Cn)) = Kn. We will prove the result for Kn. Thus in a complete graph all the vertices are adjacent to each other. Only one vertex is needed to dominate all the other vertices.

But compliment of complete graph consists of only the isolated vertices, Therefore all the n vertices are dominating vertices, by the note we have = n.

2. Theorem

Global vertex-edge domination number of, G (T (Cn)) is given by = 1.

Proof:

From the above result: 1 it is clear that G (T (Cn)) = Kn. Thus we will prove the result for Kn. In a complete graph all the edges are adjacent to each other , such that only one vertex is needed to dominate all the edges.

Hence vertex-edge domination number of Kn is 1.

But in all the vertices are isolated such that vertex-edge domination number of is 0.By the note we have = 1.

3. Theorem

Equitable Vertex-edge domination number of G (T (Cn)) is 1.

Proof:

From the above result: 1 it is clear that G (T (Cn)) = Kn. Thus we will prove the result for Kn. In a complete graph all the edges are adjacent to each other, such that only one vertex is needed to dominate all the edges. Hence vertex-edge domination number of K n is 1.Therefore {S} is a vertex –edge dominating set

Since in a complete graph all vertices are of same degree, │deg (u) – deg (v) │= 0 which is ≤ 1, where u S and v V - S.

4. Theorem

Perfect domination number of G (T (Pn)) is given byGlobal perfect domination number of path Pn is n

Proof:

From the above result: 2 it is clear that G (T (Pn)) = Pn. Thus we will prove the result for Pn.

Also non adjacent vertices are adjacent in the compliment of Pn, all n vertices will be considered so that it will satisfy the perfect domination condition. Hence we have

Theorem: 3.5

Global vertex-edge domination number of G (T (Pn)) is given by.

Proof:

From the above result: 2 it is clear that G (T (Pn)) = Pn. Thus we will prove the result for Pn.

Let us prove the result by method of mathematical induction. When n = 6, the result is true and is trivial from the following graph,

γve(P6) = 2

γve = 2 There fore

Also

By induction hypothesis, let us assume that it is true for all n = k.

i.e,

By conditional hypothesis if global ve-domination number of P6 and Pk aretrue, then it is true for Pk+1.

Here the result is true since n = k is a path with k vertices and k-1 edges,

n = k+1 is a path with k+1 vertices and k edges, also for every u,v S , d(u,v) ≤ 4.

Thus by mathematical induction it is true for all positive n, such that

5. Theorem

Maximal vertex – edge domination of G(T(Pn)) is given by

for all n = 12, 13, 14, ….

Proof:

From the above result: 2 it is clear that G (T (Pn)) = Pn. Thus we will prove the result for Pn.

Let Pn be a path with vertex set V (G), where n = 12, 13, 14, …. We can prove the result by method of mathematical induction.

Case (i): (for n is multiple of 4)

Consider P12, path with 12 vertices, and then we have mve (P12) = .

Assume that the result is true for n = 12, and is trivial from the following graph.

mve (P12) = 8

From the definition V-S is not a ve-dominating set, therefore there exist at least one vertex c V-S such that d (a,c) 5,where a V-S.

By induction hypothesis assume that the result is true for n = 4k, where k = 3,4,5…,

Let us assume if k = 4 then P4k implies P 4(4) = P16.

mve (P16).

The result is trivial from the following graph.

mve (P16) = 9

Thus we have γ mve (P4k)

Consider n = 4(k +1), where k = 3,4,5…,

As we assumed k = 4, P4 (k+1) implies P 4(4 + 1) = P20.

mve (P4(4+!)) = γ mve (P20) =

The result is true from the following graph.

mve (P20) = 10

Thus we have mve (P4 (k+!))

The result is true for all n = k+1.

Thus by mathematical induction the result is true for all n, where

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Case (ii): (for n-1 is multiple of 4)
Consider P13, path with 13 vertices, and then we have mve (P13)
Assume that the result is true for n = 13, and is trivial from the following graph.
mve (P13) = 8

From the definition V-S is not a ve-dominating set, therefore there exist at least one vertex c V-S such that d (a,c) 5,where a V-S.
By induction hypothesis assume that the result is true for n = 4k+1,where k = 3,4,5…,
then we have mve (P4k+ 1)
Consider n = 4(k +1) +1, where k = 3,4,5…,
mve (P4(k+!)+1) =
Thus by mathematical induction the result is true for all n, where
Case (iii): (for n - 2 is multiple of 4)
Consider P14, path with 14 vertices, and then we have mve (P14)
Assume that the result is true for n = 14, and is trivial from the following graph.
mve (P14) = 8
From the definition V-S is not a ve-dominating set, therefore there exist at least one vertex c V-S such that d (a,c) 5,where a V-S.
By induction hypothesis assume that the result is true for n = 4k+2, where k = 3,4,5…,
then we have mve (P4k+2)
Consider n = 4(k +1) +2, where k = 3,4,5…,
mve (P4(k+!) +2)
Thus by mathematical induction the result is true for all n, where
Case (iv): (for n-3 is multiple of 4)
Consider P15, path with 15 vertices, and then we have mve (P15)
Assume that the result is true for n = 15, and is trivial from the following graph.
mve (P15) = 8
From the definition V-S is not a ve-dominating set, therefore there exist at least one vertex c V-S such that d (a,c) 5,where a V-S.
By induction hypothesis assume that the result is true for n = 4k+ 3, where k = 3,4,5…,
then we have mve (P4k+3)
Consider n = 4(k +1) +3, where k = 3,4,5…,
mve (P4(k+!)+3) =
Thus by mathematical induction the result is true for all n, where
Theorem: 3.7
Equitable Vertex-edge domination number of G(T(Pn)) is
for all n = 2, 3, 4, ……

Proof:

Let Pn be a Path with vertex set V (G), where n = 2,3,4, …. We can prove the result by method of mathematical induction.

Case (i): (for n is multiple of 4)
Consider P4, a path with4 vertices, and then we have γ e ve (P4)
From the definition S is vertex – edge dominating set and │deg (u) – deg (v) │≤ 1, where u S and v V – S.
Assume that the result is true for n = 4, is trivial from the above definition and following graph.

By induction hypothesis the result is true for n = 4k where k = 1, 2, 3,…..
ve (P4k)
Consider n = 4(k +1), where k = 1, 2, 3…, Where γ e ve (P4 (k+1)) is union of γ e ve (P4k) and γ e ve (P4)
ve (P4 (k+1)) = γ e ve (P4k) + γ e ve (P4)

Thus by mathematical induction the result is true for all n, where
Case (ii): (for n-1 is multiple of 4)
Consider P5, a path with5 vertex, and then we have γ e ve (P5)
From the definition S is vertex – edge dominating set and │deg (u) – deg (v) │≤ 1, where u S and v V – S.
Assume that the result is true for n = 5, is trivial from the above definition and following graph.

By induction hypothesis the result is true for n = 4k + 1 where k = 1, 2, 3,…..
ve (P4k +1)
Consider n = 4(k +1) + 1, where k = 1, 2, 3…,
Where ve (P4 (k+1) +1) is union of γ e ve (P4k +1) and ve (P4)
ve (P4 (k+1) + 1) = ve (P4k + 1) + γ e ve (P4)

Thus by mathematical induction the result is true for all n, where
Case (iii): (for n +1 is multiple of 4)
Consider P3, a path with3 vertex, and then we have γ e ve (P3)
From the definition S is vertex – edge dominating set and │deg (u) – deg (v) │≤ 1, where u S and v V – S.
Assume that the result is true for n = 3, is trivial from the above definition and following graph.

By induction hypothesis the result is true for n = k-1, γ e ve (P4k - 1)
Consider n = 4(k +1) - 1, where k = 1, 2, 3…, Where γ e ve (P4 (k+1) - 1) is union of
ve (P4k - 1) and ve (P4)
ve (P4 (k+1)) = ve (P4k - 1) + ve (P4)

Thus by mathematical induction the result is true for all n, where
Case (iv): (for n+2 is multiple of 4)

Consider P2, a path with2 vertex, and then we have ve (P2)
From the definition S is vertex – edge dominating set and │deg (u) – deg (v) │≤ 1, where u S and v V – S.
Assume that the result is true for n = 2, is trivial from the above definition and following graph.

By induction hypothesis the result is true for n = k – 2, e ve (P4k -2)
Consider n = 4(k +1) -2 , where k = 1, 2, 3…, Where ve (P4 (k+1) – 2) is union of
γ e ve (P4k - 2) and ve (P ve (P4 (k+1) - 2) = ve (P4k - 2) + ve (P4)

Thus by mathematical induction the result is true for all n, where

IV. CONCLUSION

In this paper G (T (G)) is constructed for Cn, Pn, and total Hamiltonian line graph, identified the gp(G(T)), gve (G(T)), mve(G(T)), vee (G(T)). Cartesian product of other family of graphs and characterization of the parameters will be studied in our next paper.

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