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Nano Totally Semi Open Maps in Nano Topological Space

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***Abstract* - The theory of Nano topology [3] proposed by Lellis Thivagar and Richard is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. Aim of this paper is we introduced in Nano totally open maps in Nano topological space and also studied about Nano semi totally open Maps and Nano totally semi open maps in Nano topological space.**

***Keywords*- Nano totally open map, Nano semi totally open map, Nano totally semi open map, Nano clopen set.**

I. INTRODUCTION

The theory of Nano topology [3] proposed by Lellis Thivagar and Richard is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. The elements of a Nano topological space are called the Nano open set. It originates from the Greek word „Nanos" which means „dwarf" in its modern scientific sense, an order to magnititude-one billionth. The Topology is named as Nano topology so because of its size, since it has at most five elements. The author has defined Nano topological space in terms of Lower and upper approximations. He also introduced certain weak form of Nano open set [3] such as Nano open set, Nano semi-open sets and Nano pre open sets. Further he introduced continuity [4] which is the core concept of topology in Nano topological space.

II. PRELIMINARY

Throughout this paper $(M, \tau_R(X))$ (or X) represent Nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(M, \tau_R(X))$, Ncl (G) and Nint (G) denote the Nano closure of *H* and the Nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 1[2] Let M be a non-empty finite set of objects called universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernibility with one another. The pair (M, R) is said to be the Approximation Space.

Let $X \subseteq M$.

1. The lower approximation of X with respect to R is the set of all objects which can be certain classified as X with respect to R and is denoted by $L_R(X$

$$L_R(X) = \cup_{x\in M} \{R(X) / R(X) \leq X\}$$

2. The upper approximation of X with respect to R is the set of objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$

$$U_R(X) = \cup_{x\in M} \{R(X) / R(X) \cap X \neq \emptyset\}$$

3. The boundary region of x with respect to R is the se of all objects which can be classified neither as X nor not X with respect to R and it is denoted by $B_R(X)$ $B_R(X) = U_R(X) - L_R(X)$

Definition 2 Let M be the universe, R be an equivalence relation on M and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq M$. Then the property $\tau_R(X)$ satisfies the following axioms

1. U and $\emptyset$ belongs to $\tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U is called the NANO TOPOLORY on U with respect to X. $(M, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are celled as Nano-open sets and complements of Nano-open sets are called Nano-closed sets.

Definition 3 If $(M, \tau_R(X))$ is a Nano topological space and if $G \subseteq U$, then

1. The NANO-INTERIOR of H is defined as the union of all Nano-open subsets of G and it is dented by Nint (G). i.e.) Nint (G) is the largest Nano-open set contained in G.
2. The NANO-CLOSURE of G is defined as the intersection of all Nano-closed sets containing G and it is denoted by Ncl (G). i.e.) Ncl (G) is the smallest Nano-closed set containing G.
3. The set H is called NANO CLOPEN if it is both Nano-open and Nano-closed and is denoted by Nco (G).

Definition 4 If $(M, \tau_R(X))$ is a Nano topological space with respect to X and $G \subseteq M$ is called Nano Semi-open if $G \subseteq Ncl(Nint(G))$ and it is denoted by Nso $(M, \tau_R(X))$.

The complement of Nano Semi-open is called Nano Semi-closed and it is denoted by Nscl (M, $\tau_R(X)$).

III.NANO SEMI TOTLLY OPEN MAPS AND THEIR BASIC PROPERTIES

In this section, Nano semi totally open maps are defined and some of the properties are analyzed.

Definition 1

A function f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) is said to be NANO SEMI TOTLLY OPEN MAP if the image of every Nano semi open set in U is Nano clopen in V.

Example 1

Consider the Nano topological space (M, $\tau_R(X)$) and (N, $\tau_{R'}(Y)$) where M = N = {a, b, c, d} X = {a, c}
Y = {b, c}. M/R = {{a}, {b}, {c, d}} and N/R' = {{a}, {b}, {b, d}}
Then $\tau_R(X)$ = {U, ∅, {a}} and $\tau_{R'}(Y)$ = {U, ∅, {a}, {b, c, d}}
NSO (M, $\tau_R(X)$) = {U, ∅, {a}} and NCO (N, $\tau_{R'}(Y)$) = {U, ∅, {a}, {b, c, d}}
Define f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) is given by
f (a) = a, f (b) = c, f(c) = b, f (d) = d
f^{1}(a) = a, f^{-1}(b) = c, f^{1}(c) = b, f^{-1}(d) = d
Here, image of Nano semi open set in M is Nano clopen in N.
Hence, f is Nano semi totally open map.

Theorem 1

If a bijection f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) is Nano semi totally open map then the image of the Nano semi closed set in M is Nano clopen in N.
Proof:
Let D be a Nano semi closed in M.
Then D^C is Nano semi open in M.
Since f is Nano semi totally open, f (D^C) is Nano clopen in N.
⇒ f (D) is Nano clopen in N.

Theorem 2

A surjective f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) is Nano semi totally open if and only if for each subset A of (M, $\tau_{R'}(Y)$) and for each Nano semi closed set E containing f^{-1}(A), there is a Nano clopen set C of (M, $\tau_{R'}(Y)$) such that B ⊂ G and f^{-1} (C) ⊂ E.
Proof:
Assume that is a surjective Nano semi totally open map.
Le A⊂ (N, $\tau_{R'}(Y)$).
Let E be a Nano semi closed set in M such that f^{-1} (A) ⊂ E.Since E is Nano semi closed in M E^C is Nano semi open in M.Since f is Nano semi totally open, f (E^C) is clopen in N.⇒ (N, $\tau_{R'}(Y)$) – f (E^C) is Nano clopen in N.
Let C = (N, $\tau_{R'}(Y)$) – f (E^C).
Then C is Nano clopen set in N containing A such that f^{-1} (C) ⊂ E.
Conversely,
Suppose E is Nano semi open in M.
Then f^{-1} ((N, $\tau_{R'}(Y)$) – f (E)) ⊂ E^C and E^C is Nano semi closed M.
By hypothesis, there exist s Nano clopen set C of N such that (N, $\tau_{R'}(Y)$) – f (E)) ⊂ C
⇒ f^{-1} (C) ⊂ E^C
⇒ E ⊂ (f^{-1} (C)$)^C$
(N, $\tau_{R'}(Y)$) – C ⊂ f(E) f(f^{-1} (C)) ⊂ (N, $\tau_{R'}(Y)$) – C.
⇒ f(E) = (N, $\tau_{R'}(Y)$) – C, which is Nano clopen in N.
Hence, f is Nano semi totally open map.

Theorem 3

For any bijective function f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) the following are equivalent

- Inverse of f is Nano semi totally continuous function.
- f is Nano semi totally open map.

Proof:
(a) ⇒ (b)
Assume that the inverse function f^{-1}: (N, $\tau_{R'}(Y)$) → (M, $\tau_R(X)$) is Nano semi totally continuous function.
Let O be a Nano semi open in M.
Since f^{-1} is Nano semi totally continuous, $(f^{-1})^{-1}$(O) = f(O) is Nano clopen in N.
Hence f is Nano semi totally open map.
(b) ⇒ (a)
Assume that f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) is Nano semi totally open map.
Let O be a Nano semi open set in M.
Since f is Nano semi totally open map, f(O) is Nano clopen in N.
i.e.) $(f^{-1})^{-1}$(O) is Nano clopen in N.
Therefore, f^{-1} is Nano semi totally continuous function.

IV.NANO TOTALLY OPEN MAPS AND THEIR BASIC PROPERTIES

In this section, Nano totally open maps are defined and some of the properties are analyzed.

Definition1

A function f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) is said to be NANO TOTLLY OPEN MAP if the image of every Nano open set in M is Nano clopen in N.

Example 1

Consider the Nano topological space (M, $\tau_R(X)$) and (N, $\tau_{R'}(Y)$) where M = N = {a, b, c, d} X = {a, c}
Y = {b, c}. M/R = {{a}, {b}, {b, d}} and N/R' = {{a}, {b}, {c, d}}
Then $\tau_R(X)$ = {U, ∅, {a}} and $\tau_{R'}(Y)$ = {U, ∅, {a}, {b, c, d}}
NCO (N, $\tau_{R'}(Y)$) = {U, ∅, {a}, {b, c, d}}
Define f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) is given by
f (a) = a, f (b) = c, f(c) = b, f (d) = d
f^{1}(a) = a, f^{-1}(b) = c, f^{1}(c) = b, f^{-1}(d) = d
Here, image of Nano open set in M is Nano clopen in N.
Hence, f is Nano totally open map.

Theorem1

A surjective f: (M, $\tau_R(X)$) → (N, $\tau_{R'}(Y)$) is Nano totally open if and only if for each subset B of (N, $\tau_{R'}(Y)$) and for each Nano closed set E containing f^{-1} (B), there is a Nano clopen set S of (N, $\tau_{R'}$(Y)) such that B $\subset$ S and f^{-1} (S) $\subset$ F..

Theorem 2

The composition of two Nano totally open maps is again a Nano totally open map.

Proof:

Suppose f: (M, τ_R(X)) $\rightarrow$ (N, $\tau_{R'}$(Y)) and g: (N, $\tau_{R'}$ (Y)) $\rightarrow$ (P, $\tau_{R''}$ (z)) are two Nano totally open maps.

Let K be a Nano open set in M.

Since f is Nano totally open map, f (K) is Nano clopen in N. Hence it is Nano open in N.

$\Rightarrow$f (K) is Nano open in N.

Since g is Nano semi totally open map, g (f (K)) is Nano clopen in P.

Hence, g∘f is Nano semi totally open map.

Theorem 3

For any bijective function

f: (M, τ_R(X)) $\rightarrow$ (N, $\tau_{R'}$(Y)) the following are equivalent

- Inverse of f is Nano totally continuous function.
- f is Nano totally open map.

Definition 2

A function f: (M, τ_R(X)) $\rightarrow$ (N, $\tau_{R'}$ (Y)) is said to be NANO PRE OPN MAP if image of ever Nano open set in M is Nano open in N.

Example 2

Consider the Nano topological space (M, τ_R(X)) and (N, $\tau_{R'}$(Y)) where M = N = {a, b, c, d} X = {a, b}

Y = {a, c}. M/R = {{a}, {b}, {b, d}} and N/R' = {{a}, {b}, {C, d}}

Then τ_R(X) = {U, ∅, {a}, {b, d}, {a, b, d}} and $\tau_{R'}$ (Y) = {U, ∅, {a}, {c, d}, {a, c, d}}

Define f: (M, τ_R(X)) $\rightarrow$ (N, $\tau_{R'}$(Y)) is given by

f (a) = a, f (b) = c, f(c) = b, f (d) = d

f^{-1}(a) = a, f^{-1}(b) = c, f^{-1}(c) = b, f^{-1}(d) = d

Here, image of Nano open set in M is Nano open in N.

Hence, f is Nano pre open map

Theorem 4

If f: (M, τ_R(X)) $\rightarrow$ (N, $\tau_{R'}$(Y)) is Nano pre open map and g: (N, $\tau_{R'}$ (Y)) $\rightarrow$ (P, $\tau_{R''}$ (z)) is Nano totally open map then g∘f: (M, τ_R(X)) $\rightarrow$ (P, $\tau_{R''}$ (z)) is Nano totally open map.

Proof:

Let S be a Nano open set in M.

Since f is Nano pre open map, f(S) is Nano open in N.

Since g is Nano totally open map, g(f(S)) is Nano clopen in P.

i.e.) (g∘f) (S) is Nano clopen in P.

Hence, g∘f is Nano totally open map.

V. NANO TOTALLY SEMI OPEN MAPS AND THEIR BASIC PROPERTIES

In this section, Nano totally semi open maps are defined and some of the properties are analyzed.

Definition 1

A Function F: (M, T_r(X)) $\rightarrow$ (N, $T_{r'}$(Y)) Is Said To Be Nano Totlly Semi Open Map If The Image Of Every Nano Open Set In M Is Nano Semi Clopen In N.

Theorem 1

If A Bijection F: (M, T_r(X)) $\rightarrow$ (N, $T_{r'}$ (Y)) Is Nano Totally Semi Open Map Then The Image Of The Nano Closed Set In M Is Nano Semi Clopen In N.

Proof:

Let Q Be A Nano Closed In M.

Then Q^c Is Nano Open In M.

Since F Is Nano Totally Semi Open, F (Q^c) Is Nano Semi Clopen In N.

$\Rightarrow$ F (Q) Is Nano Semi Clopen In N.

Theorem 2

For Any Bijective Function F: (M, T_r(X)) $\rightarrow$ (N, $T_{r'}$ (Y)) The Following Are Equivalent

- Inverse Of F Is Nano Totally Semi Continuous Function.
- F Is Nano Totally Semi Open Map.

Theorem 3

If F: (M, T_r(X)) $\rightarrow$ (N, $T_{r'}$(Y)) In Nano Totally Semi Open Map And G: (N, $T_{r'}$ (Y)) $\rightarrow$ (P, $T_{r''}$ (Z)) Is Nano Totally Open Map Then G∘F: (M, T_r(X)) $\rightarrow$ (P, $T_{r''}$ (Z)) Is Nano Semi Totally Open Map.

Theorem 4

Let (M, T_r(X)) $\rightarrow$ (N, $T_{r'}$(Y)) And

- G: (N, $T_{r'}$ (Y)) $\rightarrow$ (P, $T_{r''}$ (Z)) Be Two Functions Such That G∘F Is Nano Semi Totally Open Map Then If f is irresolute and surjective, then g is Nano totally semi open map
- If g is Nano totally continuous and injective, then f is Nano totally semi open map.

Proof:

Let (M, τ_R(X)) $\rightarrow$ (N, $\tau_{R'}$(Y)) and g: (N, $\tau_{R'}$ (Y)) $\rightarrow$ (P, $\tau_{R''}$ (z) be two functions such that g∘f is Nano semi totally open map.

- Suppose f is irresolute and surjective.
  Claim: g is Nano totally semi open map.
  Let R be Nano open in N.
  Then R is Nano semi open in N.
  Since f is irresolute, f^{-1} (R) is Nano semi open in M.
  Since g∘f is Nano semi totally open map,
  (g∘f) (f^{-1} (R)) Nano Semi clopen in P.
  $\Rightarrow$ g (R) is Nano semi clopen in P.
  Then g is Nano totally semi open map.
- Suppose g is Nano totally continuous and injective.
  Claim: f is Nano totally semi open map.
  We have, f (J) = g^{-1}((g∘f) (J)) is true for every subset J of M.
  Let S be Nano open in M.
  $\Rightarrow$ S is Nano semi open in M.
  Therefore, (g∘f) (S) is Nano clopen and Nano open in P.
  Since g is Nano totally continuous,
  g^{-1} ((g∘f) (S)) = f (S) is Nano clopen in N.
  $\Rightarrow$ f (S) is Nano semi clopen in N.

Hence, f is Nano totally semi open map.

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