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Scrutiny on Fine Semi Totally Continuous Functions in Fine Topological Space

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***Abstract*- The properties of new class of functions, namely fine totally continuous function, fine semi totally continuous function and fine totally semi continuous functions in fine topological space are analyzed in this paper. The relation of these functions with already existing well known functions are studied.**

***Keywords*- Fine open set, fine semi open set, fine closed set, fine clopen set**

I. INTRODUCTION

Powar P. L. and Rajak K.11 have investigated a special case of generalized topological space called fine topological space .Functions and of course open functions stand among the most important notions in the whole of mathematical science. Many different forms of open function have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences.

II. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

Definition 1 [10, 11]

Let (X, τ) be a topological space we define $\tau(A\alpha) = \tau\alpha=\{G\alpha(\neq X):G\alpha\cap A\alpha \neq \varphi$,for $A\alpha\in\tau$ and $A\alpha \neq X$, φ for some $\alpha \in J$, where J is the index set}. Now, define $\tau f = \{\varphi, X, \cup\tau\alpha\}$. The above collection τf of subsets of X is called the fine collection of subsets of X and (X, τ, τf) is said to be the fine topological space X and generated by the topology τ on X. The element of τf are called fine open sets in (X, τ, τf) and the complement of fine open set is called fine closed sets and it is denoted by τfc.

Example1 [10, 11]

Consider a topological space X = {1, 2, 3} with the topology $\tau =\{X, \varphi, \{1\}\} =\{X, \phi, A\alpha\}$ where $A\alpha = \{1\}$. In view of Definition 2.1 we have, $\tau\alpha =\tau (A\alpha) = \tau \{1\} = \{\{1\}, \{1, 2\}, \{1, 3\}\}$. Then the fine collection is $\tau f = \{\phi, X, \cup\tau\alpha\} = \{\phi, X, \{1\}, \{1, 2\}, \{1, 3\}\}$.

We quote some important properties of fine topological spaces.

Lemma: 2.3. [10, 11]

Let (X, τ, τf) be a fine space then arbitrary union of fine open set in X is fine-open in X.

Lemma: 2.4. [10, 11]

The intersection of two fine-open sets need not be a fine-open set as the following example shows.

Example 2 [10,11]

Let X = {1, 2, 3} be a topological space with the topology $\tau = \{X, \phi, \{1\},\{2\}\}, \{1, 2\}\}$, $\tau f = \{X, \phi, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$. It is easy to see that, the above collection τf is not a topology. Since, $\{1, 3\} \cap \{2, 3\} = \{3\} = \tau f$. Hence, the collection of fine open sets in a fine space X does not form a topology on X, but it is a generalized topology on X.

Remark :2.6 [10,11]

In view of Definition 2.1 of generalized topological space and above Lemmas 2.3 and 2.4 it is apparent that (X, τ, τf) is a special case of generalized topological space. It may be noted specifically that the topological space plays a key role while defining the fine space as it is based on the topology of X but there is no topology in the back of generalized topological space.

Definition2 [10,11]

A subset A of a Fine space (X, τ, τf) is called Fine semi-open if $A \subseteq Fcl(Fint(A))$. The complement of Fine semi-open set is called Fine semi-closed. The Fine semi-closure of a subset A of Fine space X, denoted by Fscl(A), is defined to be the intersection of all Fine semi-closed sets containing A in Fine space X.

III. FINE TOTALLY CONTINUOUS FUNCTIONS AND THEIR SUBSTRATAL QUALITY

Definition 1

A function $f : (X, \tau, \tau f) \to (Y, \sigma, \sigma f)$ is said to be Fine Totally Continuous Function if inverse image of every fine open set in Y is fine clopen in X.

Example 1

Consider the fin space (X, τ, τf) & (Y, σ, σf) where X=Y=[1, 2, 3} with topology $\tau =\{X, \varphi, \{1\}\}$ and $\sigma = \{Y, \phi, \{1\},\{3\}\}$. Then $\tau f = \{ X, \varphi , \{1\}, \{1, 2\}, \{1, 3\}\}$ and $\sigma f = \{Y, \phi, \{1\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$.

Also $\sigma fc = \{ Y, \varphi, \{2, 3\}, \{1, 2\}, \{3\}, \{2, 3\},\{1, 3\},\{1, 2\}\}$. $FCO(Y) = \{ Y, \varphi, \{1\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$.

Define $f : (Y, \sigma, \sigma f) \to (X, \tau, \tau f)$ by f (1) = 1; f(2) = 2; f(3) = 3

Thus inverse image of every fine open set in x is fine clopen in Y.
Hence, f is fine totally continuous function.

Theorem1

A function f: (X, τ, τf) → (Y, σ, σf) is fine totally continuous function iff inverse image of every fine closed subset of Y is fine clopen in X.

Proof

Suppose f: (X, τ, τf) → (Y, σ, σf) is fine totally continuous function.
Let F be fine closed in Y.
Then FC is fine open in Y.
Since f is fine totally continuous, f-1 (FC) is fine clopen in X.
⇒ f-1 (F) is fine clopen in X.
Conversely,
Suppose inverse image of every fine closed subset of Y is fine clopen in X.
Let G be fine open in .Y
Then GC is fine closed in Y.
By hypothesis, f-1 (GC) = (f-1(G))C is fine clopen in X.
i.e.) f-1 (G) is fine clopen in X.
Hence, f is fine totally continuous function.

Theorem 2

The composition of two fine totally continuous functions is fine totally continuous:.

Proof

Let f: (X, τ, τf) → (Y, σ, σf) and g: (Y, σ, σf) → (Z, μ, μf) be any two fine totally continuous functions.
Let V be fine open in Z.
Since g is fine totally continuous, g-1(V) is fine clopen in Y.
⇒ g--1(V) is fine open in Y.
Since f is fine totally continuous, f-1g-1(V) is fine clopen in X.i.e.) (g∘f)-1 is fine clopen in X.
Hence g∘f is fine totally continuous.

IV. FINE SEMI TOTALLY CONTINUOUS FUNCTION AND THEIR SUBSTRATAL QUALITY

Definition 1

A function f: (X, τ, τf) → (Y, σ, σf) is said to be fine semi totally continuous if inverse image every fine semi open subset of Y is fine clopen in X.

Theorem1

Every fine semi totally continuous function is fine totally continuous function.

Proof

Suppose f: (X, τ, τf) → (Y, σ, σf) is a fine semi totally continuous function.
Let A be any fine open set in Y.
Since every fine open set is fine semi open set, A is fine semi open in Y. Since f is fine semi totally continuous, f-1(A) is fine clopen in X. Thus f is fine totally continuous function.

Theorem 2

Let f: (X, τ, τf) → (Y, σ, σf) is fine semi totally continuous and g : (Y, σ, σf) → (Z, μ, μf) is fine semi continuous. Then g∘f is fine totally continuous.

Proof

Let G be fine open in Z.
Since g is fine semi continuous, g-1 (G) is fine semi open in Y.
Also f is fine semi totally continuous, f-1g-1 (G) is fine clopen in X.
i.e.) (g∘f)-1 is fine clopen in X.
Therefore, g∘f is fine totally continuous.

Definition 2

If f : (X, τ, τf) → (Y, σ, σf) and A0 is a subset of (X, τ, τf). We define the fine restriction of f to A0 be the function mapping A0 into (Y, σ, σf) whose rule is {(a, f (a)) / a ∈A0}. It is denoted by f|A0 which is read f is fine restricted to A0.

Theorem 3

If f: (X, τ, τf) → (Y, σ, σf) is fine semi totally continuous and A is fine clopen subset of (X, τ, τf) then the fine restriction f|A : A →(Y, σ, σf) is fine semi totally continuous.

Proof

Consider the function f|A and V be fine semi open in Y.
Since f is fine semi totally continuous, f-1(V) is fine clopen in X.
Since A is fine clopen subset of U and (f|A)-1 (V) = A∩ f-1(V) is fine clopen in X, it follow that
(f|A)-1 (V) is fine clopen in X.
Hence, f|A is fine totally continuous.

Theorem 4

The composition of two fine semi totally continuous functions is fine semi totally continuous:.

Proof

Let f: (X, τ, τf) → (Y, σ, σf) and g: (Y, σ, σf) → (Z, μ, μf) be any two fine semi totally continuous functions.
Let V be fine semi open in Z.
Since g is fine semi totally continuous, g-1(V) is fine clopen in Y.
⇒ g--1(V) is fine open in Y.
Since f is fine semi totally continuous, f-1g-1(V) is fine clopen in X.
i.e.) (g∘f)-1 is fine clopen in X.
Hence g∘f is fine semi totally continuous.

Theorem 5

Every fine semi totally continuous function is fine semi continuous function.

Proof

Suppose f: (X, τ, τf) → (Y, σ, σf) is a fine semi totally continuous function.
Let A be any fine open set in Y.
Since f is fine semi totally continuous, f-1(A) is fine clopen in X.
⇒ f-1(A) is fine open in X.
Thus f is fine semi continuous function.

V.FINE TOTALLY SEMI CONTINUOUS FUNCTIONS AND THEIR SUBSTRATAL QUALITY

Definition 1

A function f: (X, τ, τf) → (Y, σ, σf) is said to be fine totally semi continuous if inverse image every fine open subset of Y is fine semi clopen in X.

Lemma: 5.2

A function f: (X, τ, τf) → (Y, σ, σf) is fine totally semi continuous iff inverse image of every fine closed subset of Y is fine semi clopen in X.

Proof:

Suppose f: (X, τ, τf) → (Y, σ, σf) is fine totally semi continuous.

Let F be fine closed in Y.

Then FC is fine open in Y.

Since f is fine totally semi continuous, f-1 (FC) is fine semi clopen in X.

⇒ f-1 (F) is fine semi clopen in X.

Conversely,

Suppose inverse image of every fine closed subset of Y is fine semi clopen in X.

Let G be fine open in Y.

Then GC is fine closed set in Y.

By hypothesis, f-1 (GC) = (f-1(G))C is fine semi clopen in X.

i.e.) f-1 (G) is fine semi clopen in U. Hence, f is fine totally semi continuous.

Theorem 1

Every fine totally semi continuous function is fine semi continuous function.

Proof

Suppose f: (X, τ, τf) → (Y, σ, σf) is fine totally semi continuous.

Let A be a fine open set in Y.

Sine f is fine totally semi continuous, f-1(A) is fine semi clopen in X.

⇒ f-1(A) is fine semi open in X.

Thus f is fine semi continuous.

Theorem 2

Every fine semi totally continuous function is a fine totally semi continuous function.

Proof

Suppose f: (X, τ, τf) → (Y, σ, σf) is fine semi totally continuous function.

Let A be a fine open set in Y.

Since every fine open set is a fine semi open set and f is fine semi totally continuous function it follows that f-1(A) is fine clopen in X.

Hence, f is a fine totally semi continuous function.

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