International Journal of Scientific Research & Engineering Trends
Volume 5, Issue 3, May-Jun-2019, ISSN (Online): 2395-566X

On Regular Fuzzy Soft Graphs

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***Abstract*- In this paper, regular fuzzy soft graphs, and totally regular fuzzy soft graphs are examined. Total degree of a fuzzy soft graph is introduced. Theorems for Regular fuzzy soft graphs and totally regular fuzzy soft graphs are introduced. A necessary condition under which they are equivalent is provided. Some properties of regular fuzzy soft graphs and totally regular fuzzy soft graphs are studied.**

***Keywords* - regular fuzzy soft graph, total degree of a fuzzy soft graph, totally regular fuzzy soft graph.**

I. INTRODUCTION

In 1736, Euler first introduced the concept of graph theory. In the history of mathematics, the solution given by Euler of the well-known Konigsberg bridge problem is considered to be the first theorem of graph theory. The graph theory is a very useful tool for solving combinatorial problems in different areas such as operations research, optimization, topology, geometry, number theory, algebra and computer science. Fuzzy set theory, introduced by Zadeh in 1965 is a mathematical tool for handling uncertainties like vagueness, ambiguity and imprecision in linguistic variables [12]. Research on theory of fuzzy sets has been witnessing an exponential growth; both within mathematics and in its application. Fuzzy set theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest. The first definition of fuzzy graph was introduced by.

Haufmann in 1973, based on Zadeh's fuzzy relations in 1971. In 1975, Rosenfeld introduced the concept of fuzzy graphs [9]. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs using fuzzy relations, obtaining analogs of several graph theoretical concepts. During the same time Yeh and Bang have also introduced various connectedness concepts in fuzzy graph [11]. Now, fuzzy graphs have been witnessing a tremendous growth and finds application in many branches of engineering and technology.

A. NagoorGani and K. Radha introduced the concept of regular fuzzy graphs in 2008 [5]. In 1999, D.Molodtsov[12] introduced the notion of soft set theory to solve imprecise problems in economics, engineering and environment. He has shown several applications of this theory in solving many practical problems. There are many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets, etc. which can be considered as mathematical tools to deal with uncertainties. But all these theories have their own inherent difficulties. The theory of probabilities can deal only with possibilities. The most appropriate theory to deal with uncertainties is the theory of fuzzy sets, developed by Zadeh[9] in 1965. But it has an inherent difficulty to set the membership function in each particular rcases. Also the theory of intuitionistic fuzzy set is more generalized concept than the theory of fuzzy set, but also there have same difficulties. The soft set theory is free from above difficulties.

In 2001, P.K.Maji, A.R.Roy,R.Biswas [20, 21] initiated the concept of fuzzy soft sets which is a combination of fuzzy set and soft set. In fact, the notion of fuzzy soft set is more generalized than that of fuzzy set and soft set. Muhammad Akram and Saira Nawaz [25] introduced more concepts on fuzzy soft graphs.

II.PRELIMINARIES

Definition 1: A fuzzy graph G is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy
Sub set of a non empty set V and μ is a symmetric fuzzy relation on σ . The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*(V,E)$ where $E \subseteq V\times V$. A fuzzy graph G is complete if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where uv denotes the edge between u and v.

Definition 2:A fuzzy graph H : (t, u) is called a partial fuzzy sub graph of G : (s, μ) if $t(u) \leq s(u)$ for every u and $u(u, v) \leq \mu(u, v)$ for every u and v . In particular we call a partial fuzzy sub graph H : (t, u) a fuzzy sub graph of G : (s, μ) if $t(u) = s(u)$ for every u in t * and $u(u, v) = \mu(u, v)$ for every arc (u, v) in u*.

Definition 3: Let U be a nonempty finite set of objects called Universe and let E be a nonempty set called parameters. An ordered pair (F,E) is said to be a Soft set over U, where F is a mapping from E into the set of all subsets of the set U. That is
$F: E \rightarrow P(U)$. The set of all Soft sets over U is denoted by S (U)

Definition 4: Let (F, A) be a soft set over V. Then (F, A) is said to be a Soft graph of G if the sub graph induced by F(x) in G, F%(x) is a connected sub graph of G for all x belongs to A. The set of all soft graph of G is denoted by SG(G).

Definition 5 : Let V be a non empty set of vertices , E be the set of parameters and $A \subseteq E$. Also let

$\rho : A \to F(V)$ (collection of all fuzzy subsets in V)

$e \mapsto \rho(e) = \rho_e$.(say) and

$\rho_e : V \to [0,1]$

$x_i \mapsto \rho_e(x_i)$

(A, ρ) is a fuzzy soft vertex.

$\mu : A \to F(V \times V)$ (collection of all fuzzy subsets in E)

$e \mapsto \mu(e) = \mu_e$.(say) and

$\mu_e : V \times V \to [0,1]$

$(x_i, x_j) \mapsto \mu_e(x_i, x_j)$

(A, μ) is a fuzzy soft edge.

Then ((A, ρ),(A,μ)) is called fuzzy soft graph if and only if $\mu_e(x_i, x_j) \le \rho_e(x_i) \wedge \rho_e(x_j)$ for all e belongs to A.

Definition 6 : Let $G_{A,V} = ((A, \rho),(A,\mu))$ be a fuzzy soft graph, then the order of $G_{A,V}$ is defined as: $O(G_{A,V}) = \sum_{e \in A} (\sum_{x_i \in A} \rho_e(x_i))$.

Definition 7: Let $G_{A,V} = ((A, \rho),(A,\mu))$ be a fuzzy soft graph. Then the size of $G_{A,V}$ is defined as:

$S(G_{A,V}) = \sum_{e \in A} (\sum_{u \neq v} \mu_e(x_i, x_j))$

Definition 8 : Let $G_{A,V} = ((A, \rho),(A,\mu))$be a fuzzy soft graph. The degree of a vertex u is defined as

$dG_{A,V}(u) = \sum_{e \in A} (\sum_{u \neq v} \mu_e(u, v))$

III. REGULAR AND TOTALLY REGULAR FUZZY SOFT GRAPH

Definition 1 : Let G* = (V ,E) be a crisp graph and GA,Vbe a fuzzy soft graph of G* .Then GA,V is said to be a regular fuzzy soft graph if HA,V(e) is a regular fuzzy graph for all e ∈A. If HA,V(e)is a regular fuzzy graph of degree r for all e∈A; then GA,Vis a r-regular fuzzy soft graph.

Definition 2 : Let G* = (V ,E) be a crisp graph and GA,Vbe a fuzzy soft graph of G* .Then GA,V is said to be a totally regular fuzzy soft graph if HA,V(e) is a totally regular fuzzy graph for all e ∈A. If HA,V(e)is a totally regular fuzzy graph of degree r for all e∈A. Then GA,V is a r- totally regular fuzzy soft graph.

Definition 3 : Let GA,V = ((A, ρ),(A,μ))be a fuzzy soft graph. The total degree of a vertex u is defined as

$tdG_{A,V}(u) = \sum_{e \in A} (\sum_{u \neq v} \mu_e(u, v) + \rho_e(u))$

which is equivalent to

$tdG_{A,V}(u) = dG_{A,V}(u) + \sum_{e \in A} \rho_e(u)$

Theorem 1:

Let GA,V = ((A, ρ),(A,μ)) be a fuzzy soft graph on G* = (V ,E). Then $\sum_{u \in V} dG_{A,V}(u) = 2S(G_{A,V})$

Theorem 2 :

Let GA,V = ((A, ρ),(A,μ)) be a fuzzy soft graph on G* = (V ,E)

Then $\sum_{u \in V} tdG_{A,V}(u) = 2S(G_{A,V}) + O(G_{A,V})$

Proof:

$tdG_{A,V}(u) = (\sum_{e \in A} (\sum_{u \in V} \mu_e(u, v) + \rho_e(u)))$

$\Rightarrow$

$\sum_{u \in V} tdG_{A,V}(u)$

$= \sum_{u \in V} \{\sum_{e \in A}((\sum_{u \in V} \mu_e(u, v) + \rho_e(u))\}$

$\Rightarrow$

$\sum_{u \in V} tdG_{A,V}(u)$

$= \sum_{u \in V} \{dG_{A,V}(u) + \sum_{e \in A} \rho_e(u)\}$

$= \sum_{u \in V} (dG_{A,V}(u)) + \sum_{u \in V} (\sum_{e \in A} \rho_e(u))$

$= 2S(G_{A,V}) + O(G_{A,V})$.

Theorem 3 :

Let GA,V = ((A, ρ),(A,μ)) be a fuzzy soft graph on G* = (V , E) . If ρeis a constant function then the following cases are equivalent

- GA,V is a k1 regular fuzzy soft graph.
- GA,V is a k2 totally regular fuzzy soft graph.

Proof:

Suppose that ρe is a constant function.

Let $\rho_e(u) = c$,a constant for all $u \in V$

Assume that GA,V is a k1 regular fuzzy soft graph.

Then $d(u) = k1$ for all $u \in V$

$\Rightarrow tdG_{A,V}(u) = dG_{A,V}(u) + \sum_{e \in A} \rho_e(u)$

$= dG_{A,V}(u) + \sum_{e \in A} c$

$= dG_{A,V} + c\sum_{e \in A} 1$

$= dG_{A,V}(u) + cp$ where $p = |A|$.

$\Rightarrow tdG_{A,V}(u) = k1 + cp = k2$ for all $u \in V$

GA,V is a k2 totally regular fuzzy soft graph

Thus (1) ⇒ (2) is proved

Now , suppose that GA,V is a k2 totally regular fuzzy soft graph.

Then ,

$tdG_{A,V}(u) = k2$ for all $u \in V$

$\Rightarrow dG_{A,V}(u) + \sum_{e \in A} \rho_e(u) = k2$ for all $u \in V$

$\Rightarrow dG_{A,V}(u) + cp = k2$ for all $u \in V$

$\Rightarrow dG_{A,V}(u) = k2 - cp = k1$ for all $u \in V$

GA,V is a K1 regular fuzzy soft graph.

Thus (2) ⇒ (1) is proved.

Theorem 4:

If a fuzzy soft graph GA,V is both k1-regular and k2-totally regular , then $\sum_{e \in A} \rho_e(u)$ is constant for all $u \in V$.

Proof :

Let GA,V is a k1 regular and k2 totally regular fuzzy soft graph.

Then $d(u) = k1$ for all $u \in V$ and $tdG_{A,V}(u) = k2$ for all $u \in V$.

$tdG_{A,V}(u) = k2$ for all $u \in V$.

$\Rightarrow dG_{A,V}(u) + \sum_{e \in A} \rho_e(u) = k2$ for all $u \in V$

$\Rightarrow k1 + \sum_{e \in A} \rho_e(u) = k2$ for all $u \in V$

$\Rightarrow \sum_{e \in A} \rho_e(u) = k2 - k1 = c$ for all $u \in V$

$\Rightarrow \sum_{e \in A} \rho_e(u) = c$ for all $u \in V$

Hence $\sum_{e \in A} \rho_e(u)$ is constant for all $u \in V$.

IV.PROPERTIES OF REGULAR AND TOTALLY REGULAR FUZZY SOFT GRAPHS

Theorem 1:

The size of the k – regular fuzzy soft graph GA,V on G* = (V , E) is pk / 2 where p = |V|.

Proof:

Then the size of GA,V ,

S (GA,V) = ∑e∈A(u∈V μe (u , v)))

Since GA,V is a k - regular fuzzy soft graph,

d(u) = k for all u ∈ V

Now , S (GA,V) = ∑e∈A ((∑u≠v μe (u , v)))

={∑u∈V (dGA,V (u)} / 2 (by theorem 1 of III)

={∑u∈V k} / 2

= pk / 2.

Hence The size of the k – regular fuzzy soft graph GA,V on G* = (V , E) is pk / 2 where p = |V|.

Hence The size of the k – regular fuzzy soft graph GA,V on G* = (V , E) is pk / 2 where

p = |V|.

Theorem 2:

If GA,V = ((A, ρ),(A,μ)) be a fuzzy soft graph on G* = (V , E) is a r – totally regular fuzzy soft graph. Then 2 S (GA,V) + O(GA,V) = pr where p = |V|.

Proof:

Since GA,V is a r – totally regular fuzzy soft graph,

td(u) = r for all u ∈ V

⇒ dGA,V (u) + ∑e∈A ρe (u) = r for all u ∈ V

⇒ ∑u∈V dGA,V (u) + ∑u∈V∑e∈A ρe (u) = ∑u∈V r

⇒2 S (GA,V) + O(GA,V) = pr where p = |V|.

Theorem 3:

If GA,V = ((A, ρ),(A,μ)) be a fuzzy soft graph on G* = (V , E) is a k-regular and r – totally regular fuzzy soft graph. Then O(GA,V) = n (r – k) where n = |V |.

Proof:

Since GA,V is a k - regular fuzzy soft graph,

d(u) = k for all u ∈ V

Since GA,V is a r – totally regular fuzzy soft graph,

td(u) = r for all u ∈ V

Now, ∑u∈V tdGA,V (u) = 2S(GA,V) + O(GA,V)

⇒ ∑u∈V r = ∑u∈V dGA,V (u) + O(GA,V)

⇒ ∑u∈V r = ∑u∈V k + O(GA,V)

⇒ n r = n k + O(GA,V)

⇒O(GA,V) = n k - n r

⇒O(GA,V) = n(k - r)

Hence the result.

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