International Journal of Scientific Research & Engineering Trends
Volume 5, Issue 2, Mar-Apr-2019, ISSN (Online): 2395-566X

Ψgs-Irresolute Functions in Topological Spaces

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Abstract - **The aim of this paper is to introduce a new class of irresolute functions called ψgs-irresolute functionsin topological spaces. Also we study some characterizations of ψgs- irresolute functions.**

Keywords- **ψgs - continuous functions, ψgs - closed sets , ψgs - open sets, irresolute functions and ψgs –irresolute functions.**

I. INTRODUCTION

Crossley and Hildebrand [2] introduced irresolute functions in topological spaces. Bhattacharyya and Lahiri [1] introduced sg - irresolute functions in topological spaces. Devi et al.[3] introduced gs - irresolute functions in topological spaces. Ramya and parvathi [9] introduced ψg- irresolute functions in topological spaces. Gowsalya and Balamani**[5]** introduced ψgs- continuous functions in topological spaces. In this paper we introduceψgs - irresolute functions in topological spaces, and obtain the interrelations between ψgs - irresolute functions and already existing various irresolute functions .

II.PRELIMINARIES

Definition 1 Let (X,τ) be a topological space.A Subset A of a topological space(X,τ) is called

- **Semi open** [7] ifA ⊆cl(int(A))and **semi - closed** ifint(cl(A)) ⊆ A .
- **generalized closed** [8] if cl(A)⊆U whenever A⊆ U and U is open in (X,τ).
- **Semi generalized closed [1]** if scl(A) ⊆ U whenever A ⊆ U and U is semi - open in (X,τ).
- **ψ - closed [13]** if scl(A)⊆U whenever A⊆ U and U is sg-open in (X,τ).
- **ψgs - closed[4]** if ψcl(A)⊆U whenever A⊆ U and U is semi - open in (X,τ).
- **ψgs - clopen[4]** if it is both ψgs - closed andψgs - openin (X,τ).

Result 2.

- Every closed (resp. open) subset in (X,τ) is ψgs- closed (resp. ψgs - open).
- Every clopen subset in (X,τ) is ψgs- clopen.

Definition 3 Let (X,τ) and (Y,σ) be two topological spaces.A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called

- **Continuous [8]** if $f^{-1}(V)$ is closed in (X,τ) for every closed set V of (Y,σ) .
- **Irresolute** [2] if $f^{-1}(V)$ is semi closed in (X,τ) for every semi closed set V of (Y,σ).
- **Totally continuous [6]** if $f^{-1}(V)$ is clopenin (X,τ) for every closed set V of (Y,σ) .
- **sg - irresolute** [1] if $f^{-1}(V)$ is sg - closed in (X,τ) for every sg - closed set V of (Y,σ) .
- **gs - irresolute** [3] if $f^{-1}(V)$ is gs - closed in (X,τ) for every gs - closed set V of (Y,σ) .
- **ψg - irresolute** [9] if $f^{-1}(V)$ isψg - closed in (X,τ) for every ψg - closed set V of (Y,σ).
- **ψgs- continuous [5]**if $f^{-1}(V)$ is ψgs- closed in (X,τ) for every closed set V of (Y,σ) .
- **Totally ψgs- continuous [10]** if $f^{-1}(V)$ is ψgs- clopen in (X,τ) for every open set V of (Y,σ).
- **ψgs- totally continuous[**10**]** if $f^{-1}(V)$ is clopen in (X,τ) for every ψgs - open set V of (Y,σ).
- **Contra ψgs- continuous**[11] if $f^{-1}(V)$ is ψgs- closed in (X,τ) for every open set V of (Y,σ).

III .ΨGS-IRRESOLUTE FUNCTIONS

Definition 1 A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is called **ψgs-irresolute** if $f^{-1}(V)$ is ψgs- closed in (X,τ) for every ψgs - closed V of (Y,σ).

Example- Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X}and σ = {ϕ,{a},{a,b}{a,c},Y}.
Let f:$(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f isψgs- irresolute.

Theorem 1 A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is ψgs-irresolute if and only if $f^{-1}(V)$ is ψgs-open in (X,τ) for every ψgs - open set V of (Y,σ).

Proof- Let V be any ψgs- open set in (Y,σ). Then Y -V isψgs- closed in (Y,σ).Since f is ψgs- irresolute, $f^{-1}(Y-V) = X - f^{-1}(V)$ is ψgs-closed in (X,τ) which implies that $f^{-1}(V)$ is ψgs-open in (X,τ).
Conversely, assume that U is anyψgs- closed set in (Y,σ). Then Y-U is ψgs- open in (Y,σ). By assumption, $f^{-1}(Y-U) = X - f^{-1}(U)$ is ψgs-open in (X,τ) which implies that $f^{-1}(U)$ is ψgs-closed in (X,τ). Hence f is ψgs - irresolute.

Proposition 1 If $f: (X,\tau) \rightarrow (Y,\sigma)$ is a ψgs- irresolute function, then it is a ψgs-continuous function but not conversely.

Proof Let V be any closed set in (Y,σ). By result 2.2, V is ψgs-closed in (Y,σ).Since f is ψgs-irresolute, $f^{-1}(V)$ is ψgs- closed in (X,τ).Hencef isψgs-continuous.

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Example- Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},{a,c},X}and σ = {ϕ,{a},{b,c},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) =b,f(b) = a,f(c) = c. Then f isψgs- continuous but not ψgs-irresolute, since for the ψgs-closed set{a,b} in(Y,σ), f^{-1}({a,b}) = {a,b} is not ψgs- closed in (X,τ).

Proposition 2 Every ψgs-totally continuous function isa ψgs-irresolute function but not conversely.

Proof Let V be any ψgs-closed set in (Y,σ).Since f is ψgs- totally continuous,f^{-1}(V) is clopen in (X,τ).By result 2.2,f^{-1}(V) isψgs-clopen in (X,τ) which implies that f^{-1}(V) isψgs-closed in (X,τ). Hencef isψgs-irresolute.

Example Let X = Y = {a,b,c},τ= {ϕ,{a},{b},{a,b},{a,c},X}and σ = {ϕ,{a},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f isψgs-irresolute but not ψgs-totally continuous, since for the ψgs - closedset {c} in(Y,σ), f^{-1}({c}) = {c} is not clopen in (X,τ).

Remark - Irresolute function is independent from ψgs-irresolute function as seen from the following examples.

Example- Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},{a,c},X}and σ = {ϕ,{a},{b,c},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) =b,f(b) = a,f(c) = c. Then f isirresolute but not ψgs-irresolute, since for the ψgs-closed set{b} in(Y,σ), f^{-1}({b}) = {a} is not ψgs- closed in (X,τ).

Example - Let X = Y = {a,b,c},τ = {ϕ,{a},{b,c},X}and σ = {ϕ,{a,b},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f isψgs - irresolute but not irresolute, since for the semi -closed set{c} in(Y,σ), f^{-1}({c}) = {c} is not semi - closed in (X,τ).

Remark - The following examples show that gs - irresolute function and ψgs-irresolute function are independent.

Example - Let X = Y = {a,b,c},τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f isgs - irresolute but not ψgs-irresolute, since for the ψgs-closed set{a,c} in(Y,σ), f^{-1}({a,c}) = {a,c} is not ψgs- closed in (X,τ).

Example - Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},{a,c},X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f isψgs-irresolute but not gs - irresolute, since for the gs - closed set{a,b} in(Y,σ), f^{-1}({a,b}) = {a,b} is not gs - closed in (X,τ).

Remark - ψg - irresolute function is independent from ψgs - irresolute function as seen from the following examples.

Example- Let X = Y = {a,b,c},τ = {ϕ,{a},X}and σ = {ϕ,{a},{b},{a,b},{a,c},Y}.Let f:(X,τ)→(Y,σ) be the identity function. Then f is ψg - irresolute but not ψgs - irresolute, since for the ψgs - closed set {a,c} in (Y,σ), f^{-1}({a,c}) = {a,c} is not ψgs - closed in (X,τ).

Example - Let X = Y = {a,b,c},τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is ψgs - irresolute but not ψg - irresolute, since for the ψg - closed set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψg - closed in (X,τ).

Remark - The following examples show that totally ψgs-continuous function and ψgs -irresolute function are independent.

Example- Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},{a,c},X}and σ = {ϕ,{a},{b,c},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) =b,f(b) = a,f(c) = c. Then f istotally ψgs-continuous but not ψgs-irresolute, since for the ψgs-closed set{a,b} in(Y,σ), f^{-1}({a,b}) = {a,b} is not ψgs- closed in (X,τ).

Example - Let X = Y = {a,b,c},τ = {ϕ,{a},{a,b},X}and σ = {ϕ,{a},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f isψgs-irresolute but not totally ψgs-continuous, since for the closed set{b,c} in(Y,σ), f^{-1}({b,c}) = {b,c} is not ψgs-clopen in (X,τ).

Remark- Contra ψgs-continuous function is independent from ψgs-irresolute function as seen from the following examples.

Example- Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X}and σ = {ϕ,{a},{b,c},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) =b,f(b) = a,f(c) = c. Then f iscontra ψgs-continuous but not ψgs-irresolute, since for the ψgs-closed set{a,b} in(Y,σ), f^{-1}({a,b}) = {a,b} is not ψgs- closed in (X,τ).

Example - Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X }and σ = {ϕ,{a},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f isψgs-irresolute but not contraψgs-continuous, since for the open set{a,b} in(Y,σ), f^{-1}({a,b}) = {a,b} is not ψgs- closed in (X,τ).

Proposition- If a function f:(X,τ)→(Y,σ) is ψgs - irresolute then for every subset A of (X,τ) such that f(A) is ψgs - closed in (Y,σ), f(ψgscl(A))⊆ ψgscl(f(A)).

Proof- Let A be a subset of (X,τ) such that f(A) is ψgs - closed in (Y,σ). Then ψgscl(f(A)) is ψgs- closed in (Y,σ). Since f is ψgs - irresolute,f^{-1}(ψgscl(f(A)) is ψgs - closed in (X,τ). Now A⊆f^{-1}(f(A)) ⊆f^{-1}((ψgscl(f(A))). Therefore ψgscl(A)⊆f^{-1}((ψgscl(f(A))) and hence f(ψgscl(A))⊆f(f $^{-1}$(ψgscl(f(A)))⊆ ψgscl(f(A)).

Proposition - If a function f:(X,τ)→(Y,σ) is ψgs - irresolute then for every ψgs - closed set B ⊆ Y,ψgscl(f^{-1}(B))⊆f^{-1}(ψgscl(B)).

Proof- Let B be a ψgs - closed set in (Y,σ). Then ψgscl(B) is ψgs - closed in (Y,σ). Since f is ψgs - irresolute,f^{-1}(ψgscl (B)) is ψgs - closed in (X,τ).Since B ⊆ψgscl(B) ,f $^{-1}$(B)⊆f $^{-1}$((ψgscl(B)). Therefore by definition of ψgs closure, ψgscl(f $^{-1}$(B))⊆f $^{-1}$((ψgscl(B)).

Proposition- If f:(X,τ)→(Y,σ) is a ψgs-irresolute function and g:(Y,σ)→(Z, η) is a ψgs-irresolute function, then g o f :(X,τ) →(Z,η) is a ψgs-irresolute function.

Proof- Let V be any ψgs - closed set in (Z, η).Since g is ψgs-irresolute, g^{-1}(V) isψgs - closed in (Y,σ).Since f is ψgs-irresolute,$(g \circ f)^{-1}$(V) = $f^{-1}(g^{-1}$(V)) is ψgs- closed in(X,τ).Hence g o f is a ψgs-irresolute function .

Proposition - If f:(X,τ)→(Y,σ) is a ψgs-irresolute function and g:(Y,σ)→(Z, η) is a ψgs- continuous function, then g o f :(X,τ) →(Z,η) is a ψgs- continuous function.

Proof- Let V be any closed set in (Z, η).Since g is ψgs-continuous, $g^{-1}(V)$ isψgs - closed in (Y,σ).Since f is ψgs-irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs- closed in(X,τ).Hence g o f is a ψgs- continuous function .

Proposition - If f:(X,τ)→(Y,σ) is a ψgs-irresolute function and g:(Y,σ)→(Z, η) is a contraψgs- continuous function, then g o f :(X,τ) →(Z,η) isa contra ψgs-continuous function.

Proof- Let V be any closed set in (Z, η).Since g is contra ψgs - continuous, $g^{-1}(V)$ isψgs - open in (Y,σ).Since f is ψgs-irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs-open in(X,τ). Hence g o f is a contra ψgs- continuous function.

Proposition - If f:(X,τ)→(Y,σ) is a ψgs-irresolute function and g:(Y,σ)→(Z, η) isa continuous function , then g o f :(X,τ) →(Z,η) isa ψgs - continuous function.

Proof- Let V be any closed set in (Z, η).Since g is continuous, $g^{-1}(V)$ isclosed in (Y,σ). By result 2.2,$g^{-1}(V)$ isψgs-closed in (Y,σ).Since f is ψgs-irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs-closed in(X,τ). Hence g o f is a ψgs- continuous function.

Proposition - If f:(X,τ)→(Y,σ) is a ψgs-irresolute function and g:(Y,σ)→(Z, η) is a totally ψgs-continuous function, then g o f :(X,τ) →(Z,η) isa ψgs-continuous function.

Proof- Let V be any closed set in (Z, η).Since g is totally ψgs- continuous, $g^{-1}(V)$ isψgs- clopen in (Y,σ) which implies that$g^{-1}(V)$ isψgs-closed in (Y,σ).Since f is ψgs-irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs-closed in(X,τ).Hence g o f is a ψgs- continuous function.

Proposition - If f:(X,τ)→(Y,σ) is a ψgs-irresolute function and g:(Y,σ)→(Z, η) isa totally continuous function, then g o f :(X,τ) →(Z,η) isa ψgs - continuous function.

Proof- Let V be any closed set in (Z, η).Since g is totally continuous, $g^{-1}(V)$ isclopenin (Y,σ).By result 2.2, $g^{-1}(V)$ is ψgs-clopen in (Y,σ) which implies that $g^{-1}(V)$ isψgs-closed in (Y,σ).Since f is ψgs-irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs-closed in(X,τ) . Hence g o f is aψgs - continuous function .

Proposition - If f:(X,τ)→(Y,σ) is a ψgs-irresolute function and g:(Y,σ)→(Z, η) isa ψgs- totally continuous function, then g o f :(X,τ) →(Z,η) isa ψgs- continuous function.

Proof- Let V be any closed set in (Z, η).By result 2.2, V isψgs-closed in (Z, η).Since g is ψgs-totally continuous, $g^{-1}(V)$ is clopenin (Y,σ).By result 2.2,$g^{-1}(V)$ is ψgs-clopen in (Y,σ)which implies that$g^{-1}(V)$ isψgs-closed in (Y,σ).Since f is ψgs-irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs-closed in(X,τ) .Hence g o f is a ψgs- continuous function .

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