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On Electric Power Supply Chain Model For Three Different Tariff Customers in South East Nigeria

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***Abstract* - The study proposed a new model of electric power supply chain networks in the Nigeria situation for three different tariff category customers. The network allows for multiple power generators, transmission, and distribution, retailing and demand customers. The supply chain network introduces retailing of power from distributors to demand customers. We derived the optimality conditions of the decision- makers and proved that the governing equilibrium conditions satisfy a variation inequality problem. The variation inequality problem for a single power generator, single transmission, single distributor, single retailer and a demand customer was used to illustrate the method. The multi start optimization method was used to solve the inequality using specified start up value obtained from the Nigerian Electricity Regulatory Commission (NERC) for residential customers with single phase supply with single meter with consumption on 50KWH and below (R1) tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators and the unit cost of power by distributors ; for customers with single phase supply with single phase meter with consumption above 50KWH (R2S) tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators and the unit cost of power by distributors ; for customers with three phase supply with three phase meter with consumption below 45KVA (R2T) tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators and the unit cost of power by distributors, The result of the analysis showed that the least cost of power regardless of the category of customer is ₦ 469.15 while the highest cost is ₦ 473.32. It found that the highest shadow price was ₦ 978.93 for R1 customers while the least was ₦ 928.61 for R2T customers.**

***Keywords*- Demand customers, Power generators, Supply chain network, Variation inequality, etc.**

I. INTRODUCTION

In the present global world especially the developing nations, the theory of electric power supply chain model analysis, computation, and management are of great interests because of the increasing trend in technological change and globalization of trade. Supply chain management plays a very vital role in companies' success and customer's satisfaction and royalty in terms of electricity supply, management, distribution and sales, the world over. It a search for evidence in order to answer questions on the computation of electric power flows as well as the prices associated with the various transactions between the tiers of decision makers in the electric power supply chain network.

These concerns have simulated in the use of different techniques or algorithms to solve such equilibrium models. Led by early reformers such as Argentina, Norway and Texas, state owned monopolies were transformed into market places of competing power producer. The goals of these reforms were diverse - more efficiently governed utilities, prices reflecting the economically efficient cost of different generation technologies and the prevention of fixed prices and cartels as well as the mitigation of long- term industry relations. Speaking on the situation in Nigeria, at the moment, the electricity industry in Nigeria has moved into restructuring in which national electric power Authority (NEPA) was unbundled into 18 companies as follows: Six (6) generating companies, one (1) transmission company (ie Transmission Company of Nigeria (TCN), and eleven (11) distribution companies.

As noted in Awosope (2014), the federal government owns 100 percent of the transmission company, while its hold on the generating companies is 20 percent (with 80 percent of equity sold to private investors) and in the case of the distribution companies, government owns 40 percent, while private investors owns 60 percent. However, the management of Transmission Company of Nigeria (TCN) is handled by the Canadian company, Manitoba Hydro Company. The electri power sector reform act of 2005 was signed into law by the Nigerian government. This is to enable private companies to participate in electricity generation, transmission and distribution. In February 2005, the World Bank agreed to provide NEPA with $100 million to assist in its privatization efforts. Among these reforms is the setting up of the National electric Regulatory commission (NERC) as an independent regulatory agency for the Nigerian Electricity Supply Industry (NESI). The objectives of the reform as clearly stated in Electric Power Research Institute (EPRI, 2003) are briefly captured as follows: Lower the cost of reliable, safe, clean electric service; Attract capital for infrastructure development; Enable greater consumer choice; Level the competitive playing field. Although much of these objectives are yet to be realized after 13 years of reform.

It was observed that only 40 percent of the population has electricity, the majority of who are concentrated in urban areas. The implication of these supply shortfalls has resulted in unannounced load shedding, prolonged and intermittent power outages which tend to affect the economy adversely. Instances abound like many small scale businesses which are the agent of economic growth have wounded up due to epileptic power supply. Many manufacturing companies have closed down and moved to other countries like Ghana where the public electricity supply is constant and more stable.

Emphasizing on the usual baggage of inefficiency and poor service delivery on the electric power sector in Nigeria, it was noted that sectors' Weakness manifested in distribution system planning which for example, South Eastern States is controlled by the firm named Enugu Electricity Distribution Company (EEDC). This monopoly enjoy by this firm misallocates resources by contriving shortages and in turn produce less than the competitive output in order to create monopoly profits.

II. LITERTURE REVIEW

The electricity distribution situation in Nigeria is so poor and epileptic such that Uwaifo (1994) stressed that in Nigeria, the power distribution efficiency plummeted to 74 percent, while since 1969 the USA power distribution efficiency has been 95 percent. In a similar vein, Uwaifo (1994) stated that since 1991, the average number of consumers for each distribution transformer increased to 220 in Nigeria. This glaring reality compared with the United States of America where the number of consumers per distribution transformer is 10, becomes a source great concern, apart from the hydra-headed socio economic implications. Speaking on monopoly in the electricity industry, Leibenstein (2012), argues that while competitive firms are forced to be efficient by the market, these does not hold true for monopolies. Supported by this view, Ejumudo (2014) asserted that absence of competition and poor service culture have severely constrained the much desired adequate electricity generation capacity and effective delivery in Nigeria.

Speaking on supply chain network, Chun *et al.* (2012) noted that the term supply chain is a complicated network and that the precise definition of this concept has become difficult. However, Niki et al. (2013) also noted that supply chain network is not just a group of stages involved to satisfy a customer request, but rather a network of interrelated stages involved to do customer requests. Nagurney (2006) in her contribution on the concept, defined supply chain network as critical infrastructure for the production, distributions and consumption of goods as well as services in today's globalized network economy. Braido *et al.* (2016) remarked that this kind of design problem, to solve it, the network is broken down into sub problems. This growing interest in the design of a supply chain network begins with the identification of interesting sites that may support the skills needed for new installations.

III. MATERIALS AND METHODS

This section deals with the research materials and methods for the electric supply chain network equilibrium (ESCNE) model.The model proposed in this study are represented and solved by the VI problem approach. In general, a finite dimensional VI problem is defined as follows:

$VI\ (F,S)$: find a vector $x^* \in R^n$

Such that:

$$x^* \in S, \left\langle F(x^*), x - x^* \right\rangle \geq 0 \quad \forall\, x \in S \qquad (1)$$

Where F is a given continuous function from S to R^n and S is a non empty closed and convex set (Nagurney, 2005). In the special case where $S = R^n_+$, the nonnegative in R^n, problem (1) can be rewritten as the non linear complementarily problem

$$x^* \geq 0, F(x^*) \geq 0 \text{ and } \left\langle x^*, F(x^*) \right\rangle = 0 \qquad (2)$$

When $S = R^n$, problem (1) simply reduces to solving the system of nonlinear equalities

$$F(x) = 0 \qquad (3)$$

Hence, it is quite natural that many algorithms developed to solve the variation inequality problem are generalizations of the classical methods for systems of equations, such as Newton's method, SOR methods, projection methods and nonlinear Jacobi (diagonalization) methods.

1. Algorithms Method For Variation Inequalities

The algorithms for solving $VI\ (S,F)$ can be classified into several categories depending upon formulation a method exploits. There are methods based on KKT conditions, gap/merit functions, interior and smoothing methods, and projection based methods and their variant forms Wiener – Hopt equations, auxiliary principle, descent, Newton, decomposition techniques and multi –starts.

The algorithms for solving variation inequalities can also be categorized based on the sub-problems that are solved in each iteration. A general approach to solving $VI\ (S,F)$ consists of creating a sequence $\left\{x^k\right\} \subset S$ such that each

x^{k+1} solves $VI\ (S,F^k)$ so that

$$\left\langle F^k\left(x^{k+1}\right), y - x^{k+1}\right\rangle \geq 0 \quad (4)$$

$\forall y \in S$

where $F^k(.)$ is some approximation to $F(x).F^k$ can be linear or nonlinear.

2. Re-Formulating Electric Power Supply Chain Network Equilibrium Models

In this section, an alternative model modifications and extensions for the ESCNE model with deterministic demand case in the line with the work of Nagurney, (2006) are presented.

3.2.1 Electric supply chain network equilibrium models

In this sub section, the ESCNE model with an extension of (Nagurney, 2006 and Dong et al; 2004) are introduced. In this study we consider a five-tier decentralized electric supply chain network comprising power generators, the power transmitters, the power distributors, power retailers and end users. In the network, nodes in the top tier represent power generators, the second tier of the nodes represent power distributors. The retailers are at the third tie of nodes and the end users are located at bottom tie of nodes. In order for electricity to be transmitted from power generators to the power distributors, a transmission service is required.

Transmission service providers are those entities that own and operate the electric transmission and distribution systems. The link connecting a power generator and power distributor pair represents the decision-making connectivity in the supply chain. Assume that there are G power, T power transmitters, D power distributors, R power retailers and K end-consumers in the power supply chain. Without loss of generality, a typical power generator, power transmitter, power distributor, power retailer and end – consumer are denoted by notations g, t, d, r, k respectively. The aim of power generator g is to maximize his profit by determining the optimal production and allocation of the electric power to power distributor d via transmission provider t, denoted by q^t_{gd}. Cost for producing the product by power generator g can be in general described by function $f_g(q)$, where $q=(q_1,\cdots,q_g)$ is row vector of production outputs of all power generators in the supply chain.

This production cost function $f_g(q)$ for power generator g can be regarded as a function of vector.

$Q^1=(\cdots,q^t,\cdots),\ g=(1,\cdots,G)$

$d=(1,\cdots,D)$ and $t=(1,\cdots,T)$ where

$$f_g(q)=f_g(Q) \quad (5)$$

The transaction cost of the product between power generator g and power distributor d via transmission service provider t is characterized by function $C^t_g(q^t_{gd})$.

For a general case, we have that

$$C^t_{gd}=C^t_{gd}(q^t_{gd}),\ g=(1,\cdots,G);\ t=(1,\cdots,T);\ d=(1,\cdots,D) \quad (6)$$

It is assumed that the power generates as the profit maximizes in the electric supply chain compete in a non-cooperative manner (Nash game) and that supply price of the power output is identified according to the marginal cost pricing principle.

Now, for completeness and easy reference, we describe the behavior of the power generators and derived their correspond variation inequality.

3.2.2 Formulation of variational inequality of power generator

Assume that a typical power generator g is a profit maximize. Let p^t_{1gd} denote the price that a power generator g charges a power supplier d per unit of electricity through the transmission service provider t There is tendency that the power generator to set different prices for different power distributors. Hence, the optimization problem of the power generator g can be expressed as follows

Maximize $U_g(q_{gd}^t) =$

$$\sum_{d=1}^{D}\sum_{t=1}^{T}\rho_{1gd}^{t*}\, q_{gd}^{t} - f_g(Q^1)$$

$$-\sum_{d=1}^{D}\sum_{t=1}^{T} C_{1gd}^{t}\left(q_{gd}^{t}\right) \qquad (7)$$

subjet to $q_{gd}^{t} \geq 0, \forall\, d = (1, \cdots, D),$

$t = (1, \cdots, T)$

It is assumed that the power generators in the supply chain compete in a no cooperative manner following the concepts of Nash (1950, 1951). Furthermore, assumed that the generation cost function and the transaction cost function for each generator are continuously differentiable and convex. The optimality conditions of all power generators g; g = (1, ...,G), simultaneously, under the above assumptions, can be compactly expressed as :

Determine $Q^{*} \in R_{+}^{GTD}$ satisfying:

$$\sum_{g=1}^{G}\sum_{d=1}^{D}\sum_{t=1}^{T}\left[\frac{\partial f_g(Q^{1*})}{\partial q_{gd}^{t}} + \frac{\partial C_{gd}^{t}(q_{gd}^{t})}{\partial q_{gd}^{t}} - \rho_{1gd}^{t*}\right] \qquad (8)$$

$$\times\left[q_{gd}^{t} - q_{gd}^{t*}\right] \geq 0, \ \forall\ Q^1 \in R_{+}^{GTD}$$

The following interpretation can be deduced from inequality (8). The first half of formula (8) shows that the optimality, there is a positive flow of electric power between a generator/ distributor pair and so the price charged is equal to the sum of the marginal production cost plus the marginal transaction cost. On the other hand, if the sum exceeds the price, then there will be no electric power flow between the pair.

3.2.3 Formulation of variational inequality of power distributors

The term power distributor refers to power marketers, traders and brokers, who serve as load-serving entities. They play a fundamental role in our model since they are responsible for acquiring electricity from power generators through transmission service provider and delivering it to the power retailers. A power distributor d is faced with certain expenses, which may include, for example, the cost licensing and the costs of maintenance. These costs can be collectively referred as an operating cost and denote it by C_d. Let q_{dr} denote the amount of electricity being transacted between power distributor d and power retailer r, and let Q^2 be the DR- dimensional row vector of all the products flows between power distributors and power retailers, ie,

$$Q^2\{\cdots, q_{dr}, \cdots\}, d = (1, \cdots, D) \text{ and } r = (1, \cdots, R)$$

Furthermore, we assume that:

$$C_d = C_d\left(Q^1, Q^2\right),\ d = (1, \cdots, D) \qquad (9)$$

Equation (7) enhances the modeling of competition. In order to captive all possible scenarios, we denote transaction cost function $\hat{C}_{gd}^{t}$ as the cost associated with power distributor d acquiring electric power from power generator g, where

$$\hat{C}_{gd}^{t} = \hat{C}_{gd}^{t}(q_{gd}^{t}),\ g = (1, \cdots, G),\ d = (1, \cdots, D),\ t = (1, \cdots, T)$$
(10)

Similarly, let C_{dr} denote the transaction cost associated with power distributor d transmitting electric power to power retailer r, where:

$$C_{dr} = C_{dr}(q_{dr}),\ d = (1, \cdots, D),\ r = (1, \cdots, R)$$
(11)

These transaction cost functions are assumed to be convex and continuously differentiable.

In line with Nagurney and Matsypura (2005), the total amount of electricity sold by a power distributor is equal to the total electric power that he purchased from generators. The above assumption can be expressed as:

$$\sum_{r=1}^{R} q_{dr} \leq \sum_{g=1}^{G}\sum_{t=1}^{T} q_{gd}^{t},\ \forall\ d = (1, \cdots, D) \qquad (12)$$

Let ρ_{2dr} denote the price associated with the transaction from power distributor d to power retailer r.

Assuming that a typical power distributor d is a profit-maximizer, one can express the optimization problem of power distributor d as follows:

$$\text{maximize } U_d(q_{dr}) = \sum_{r=1}^{R}\rho_{2dr}^{*}\, q_{dr} - C_d(Q^1, Q^2)$$

$$\sum_{g=1}^{G}\sum_{t=1}^{T}\rho_{1gd}^{t*}\, q_{dr}^{t} - \sum_{g=1}^{G}\sum_{t=1}^{T}\hat{C}_{gd}^{t}\, q_{dr}^{t} - \sum_{r=1}^{R} C_{dr}\, q_{dr}$$

(13)

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Subject to constraint (12)

$$q_{dr}^{t} \geq 0,\ \forall\ g=(1,\cdots,G),\ \forall\ t=(1,\cdots,T) \quad (14)$$

$$q_{dr} \geq 0,\ \forall\ r=(1,\cdots,R) \quad (15)$$

The objective function (13) represents the profit of power distributor d with the first term denoting the revenue and the subsequent terms the various cost and payouts to the generators. As noted above, it is assumed that each power distributor seeks to maximize his own profit. Hence the optimality conditions of all power distributors $d; d=(1,\cdots,D)$ simultaneously, under the above assumptions, can be compactly expressed as:

determine $(Q^{1*}, Q^{2*}, \lambda^{*}) \in R_{+}^{D(GT+R+1)}$ satisfying:

$$\sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{g=1}^{G}\left[\frac{\partial \hat{C}_{gd}^{t*}(q_{gd}^{t*})}{\partial q_{gd}^{t}}+\frac{\partial C_{d}^{t}(Q^{1*},Q^{2*})}{\partial q_{gd}^{t}}+\rho_{gd}^{t*}-\lambda_{d}^{*}\right]$$

$$\times\left[q_{gd}^{t}-q_{gd}^{t*}\right]+\sum_{d=1}^{D}\sum_{r=1}^{R}\left[\frac{\partial C_{dr}(q_{dr}^{*})}{\partial q_{dr}}+\frac{\partial C_{d}(Q^{1*},Q^{2*})}{\partial q_{dr}}+\lambda_{d}^{*}-\rho_{2dr}^{*}\right]$$

$$\times\left[q_{dr}-q_{dr}^{*}\right]+$$

$$\sum_{d=1}^{D}\left[\sum_{g=1}^{G}\sum_{t=1}^{T}q_{gd}^{t*}-\sum_{r=1}^{R}q_{dr}^{*}\right]$$

$$\times\left[\lambda_{d}-\lambda_{d}^{*}\right]\geq 0,\ \forall (Q^{1},Q^{2},\lambda)\in R_{+}^{D(GT+R+1)}$$

(16)

Where λ_{d}^{*} is the optimal Lagrange multiplier associated with constraint (12) and λ is the corresponding D dimensional vector of Lagrange multipliers.

3.2.4 Formulation of Variation Inequality Of Power Retailers

In this subsection, we present the proposed description of the behavior of energy retailers. Electricity retailing involves the supply of electricity to residential, small commercial and industrial customers. Retailer r should simultaneously face with the power distributors and the end – consumers in the process of transmitting the product. Nevertheless, the quantity of product sold by power retailer r does not exceed the total products obtained from all of the power distributors, namely:

$$\sum_{k=1}^{K} q_{rk} \leq \sum_{d=1}^{D} q_{dr},\ \forall\ r=(1,\cdots,R) \quad (17)$$

$$C_{r}=C_{r}(Q^{1},Q^{2},Q^{3}),\ \forall\ r=(1,\cdots,R) \quad (18)$$

Let function ρ_{3rk} denote the price associated with transmitting power from retailer r to end – consumer k.

Similarly, let function C_{rk} denote the transaction cost associated with power retailer r transmitting electric power to end – consumer k, where

$$C_{rk}=C_{rk}(Q^{3}),\ \forall\ r=(1,\cdots,R),\ k=(1,\cdots,K) \quad (19)$$

Power retailer r aims to maximize its profit, which can be modeled as the optimization problem:

$$\text{maximize} \sum_{k=1}^{K}\rho_{3rk}^{*}q_{rk}-\sum_{k=1}^{K}C_{rk}(Q^{3})$$

$$-C_{r}(Q^{1},Q^{2},Q^{3})-\sum_{d=1}^{D}\rho_{2dr}^{*}q_{dr}-\sum_{d}^{D}\hat{C}_{dr}(q_{dr}) \quad (20)$$

$$\text{subject to: } \sum_{k=1}^{K}q_{rk}\leq\sum_{d=1}^{D}q_{dr} \quad (21)$$

$$q_{dr}\geq 0,\ \forall\ d=(1,\cdots,D) \quad (22)$$

$$q_{rk}\geq 0,\ \forall\ k=(1,\cdots,K) \quad (23)$$

Assume that all retailers compete in a non-cooperative manner in the retailing market of the product, and that the transaction cost function for each retailer is continuously differentiable and convex. The Nash equilibrium solution for the retailers is thus equivalent to solving the following variational inequality (Nagurney *et al.*, (2002):

$$\sum_{r=1}^{R}\sum_{k=1}^{K}\left[\frac{\partial C_{rk}(Q^{3})}{\partial q_{rk}}+\frac{\partial C_{r}(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{rk}}+\beta_{r}^{*}-\rho_{3rk}^{*}\right]$$

$$\times\left[q_{rk}-q_{rk}^{*}\right]+\sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{g=1}^{G}\left[\frac{\partial C_{r}(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{gd}^{t}}\right]$$

$$\times\left[q_{gd}^{t}-q_{gd}^{t*}\right]+\sum_{r=1}^{R}\sum_{d=1}^{D}\left[\frac{\partial C_{r}(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{dr}}+\rho_{2dr}^{*}+\frac{\partial \hat{C}_{dr}(q_{dr}^{*})}{\partial q_{rk}}-\beta_{r}^{*}\right]$$

$$\times\left[q_{dr}-q_{dr}^{*}\right]+\sum_{r=1}^{R}\left[\sum_{d=1}^{D}q_{dr}^{*}-\sum_{k=1}^{K}q_{rk}^{*}\right]\times\left[\beta_{r}-\beta_{r}^{*}\right]\geq 0,$$

$$\forall (Q^{1},Q^{2},Q^{3},\beta)\in R_{+}^{GTD+R(D+K+1)}$$

(24)

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3.2.5 Equilibrium Condition For The Demand Market

With regard to demand market k, the end – consumers' consumption behaviour for the product is assumed to be governed by deterministic demand function $d_k(\rho_3)$ where the $K-$ dimensional row vector $\rho_3=(\rho_{31},\rho_{32},\cdots,\rho_{3k})$ in which ρ_{3k} denotes unit price of the power output that end – consumers in demand market $K(k=1,\cdots,K)$ are willing to pay.

Let q_{rk} be the quantity of electricity bought from power retailer r by end – consumers in demand market k. Let function $\hat{C}_{rk}(Q^3)$ denote unit transaction cost between power retailer r and demand market k. The equilibrium conditions for end – consumers located at all demand markets in the electric power supply chain, thus can be governed by the following VI (Nagurney et al., 2002):

Find a vector $(Q^{3*},\rho_3) \in R_+^{K(R+1)}$ Such that

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Find a vector $(Q^{3*},\rho_3) \in R_+^{K(R+1)}$ Such that

$$\sum_{k=1}^{K}\sum_{r=1}^{R}\left[\rho^*_{3rk}+\hat{C}_{rk}(Q^{3*})-\rho^*_{3k}\right]\times\left[q_{rk}-q^*_{rk}\right]$$

$$+\sum_{k=1}^{K}\left[\sum_{r=1}^{R}q^*_{rk}-d_r(\rho^*_3)\right]\times\left[\rho_{3k}-\rho^*_{3k}\right]\geq 0, \qquad (25)$$

$$\forall (Q^3,\rho_3)\in R_+^{K(R+1)}$$

The electric power supply chain involves five kinds of decision – makers: power generators, transmission service provider, power distributors, power retailers and end – consumers and they are interacted and highly correlated in the electric power supply chain of the product, respectively.

Nagurney *et al*. (2002) proposed a novel equilibrium concept from the point of view of entire supply chain network. The SCNE model can be formulated by the following variational inequality formulation:

Determine a vector

$$(Q^{1*},Q^{2*},Q^{3*},\lambda^*,\beta^*,\rho^*_3)\in R_+^{GTD+DR+RK+D+R+K}$$

such that

$$\sum_{t=1}^{T}\sum_{g=1}^{G}\sum_{d=1}^{D}\left[\frac{\partial f_g(Q^{1*})}{\partial q^t_{gd}}+\frac{\partial C^t_{gd}(q^{t*}_{gd})}{\partial q^t_{gd}}+\frac{\partial C_d(Q^{1*},Q^{2*})}{\partial q^t_{gd}}+\frac{\partial \hat{C}^t_{gd}(q^{t*}_{gd})}{\partial q^t_{gd}}-\lambda^*_d+\frac{\partial C_r(Q^{1*},Q^{2*},Q^{3*})}{\partial q^t_{gd}}\right]$$

$$\times\left[q^t_{gd}-q^{t*}_{gd}\right]+\sum_{d=1}^{D}\sum_{r=1}^{R}\left[\frac{\partial C_r(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{dr}}+\frac{\partial C_{dr}(q^*_{dr})}{\partial q_{dr}}+\frac{\partial C_d(Q^{1*},Q^{2*})}{\partial q_{dr}}+\frac{\partial \hat{C}_{dr}(q^*_{dr})}{\partial q_{dr}}+\lambda^*_d-\beta^*_r\right]$$

$$\times\left[q_{dr}-q^*_{dr}\right]+\sum_{r=1}^{R}\sum_{k=1}^{K}\left[\frac{\partial C_r(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{rk}}+\frac{\partial C_{rk}(Q^{3*})}{\partial q_{rk}}+\hat{C}_{rk}(Q^{3*})+\beta^*_r-\rho^*_{3k}\right] \qquad (26)$$

$$\times\left[q_{rk}-q^*_{rk}\right]+\sum_{d=1}^{D}\left[\sum_{g=1}^{G}\sum_{t=1}^{T}q^{t*}_{gd}-\sum_{r=1}^{R}q^*_{dr}\right]\times\left[\lambda_d-\lambda^*_d\right]+$$

$$\sum_{r=1}^{R}\left[\sum_{d=1}^{D}q^*_{dr}-\sum_{k=1}^{K}q^*_{rk}\right]\times\left[\beta_r-\beta^*_r\right]+\sum_{k=1}^{K}\left[\sum_{r=1}^{R}q^*_{rk}-d_k(\rho^*_3)\right]$$

$$\times\left[\rho_{3k}-\rho^*_{3k}\right]\geq 0,$$

$$\forall (Q^1,Q^2,Q^3,\lambda,\beta,\rho_3)\in R_+^{GTD+DR+RK+D+R+K}$$

where $R_+^{GTD+DR+RK+D+R+K}$ is the nonnegative in the $GTD+DR+RK+D+R+K$-dimensional real space $R^{GTD+DR+RK+D+R+K}$.

Having obtained the solution for the VI (26), the relevant equilibrium prices for power output can be identified by the formulae below:

$$\rho^{t*}_{1gd}=\frac{\partial f(Q^{1*})}{\partial q^t_{gd}}+\frac{\partial C^t_{gd}(q^{t*}_{gd})}{\partial q^t_{gd}}, \text{ if } q^{t*}_{gd}>0 \qquad (27)$$

$$\rho^*_{2dr}=\frac{\partial C_{dr}(q^*_{dr})}{\partial q_{dr}}+\frac{\partial C_{dr}(Q^{1*},Q^{2*})}{\partial q_{dr}}+\lambda^*_d, \text{ if } q^*_{dr}>0 \qquad (28)$$

$$\rho^*_{3rk}=\frac{\partial C_{rk}(Q^{2*})}{\partial q_{rk}}+\frac{\partial C_{dr}(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{rk}}+\beta^*_r, \text{ if } q^*_{rk}>0 \qquad (29)$$

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IV. DATA ANAYSIS AND RESULT

Considering the Nigeria power distribution network for a single power generating station (G), a single transmission supplier (T), a single distribution supplier (D), a single retailer (R) and a single demand customer (K). Hence, G=1, T=1, D=1, R=1 and K=1. The data are as follows, the power generating cost function is given as

$$f_1(q) = 2.5\left(q_1^1\right)^2 + 2q_1^1 \qquad (30)$$

The transaction cost function is given as:

$$C_{11}^1\left(q_{11}^1\right) = 0.25\left(q_{11}^1\right)^2 + 3.5\, q_{11}^1 \qquad (31)$$

The operating cost function is given as:

$$C_1^1\left(Q^1, Q^2\right) = 0.25\left(q_{11}^1\right)^2 \qquad (32)$$

Other transaction cost function are :

$$\hat{C}_{11}^1\left(q_{11}^1\right) = 0.01\left(q_{11}^1\right)^2 + 0.01\, q_{11}^1 \qquad (33)$$

$$C_{11}\left(Q^2\right) = q_{11} + 5 \qquad (34)$$

$$\hat{C}_{11}\left(q_{11}\right) = 0.25\, q_{11}^2 \qquad (35)$$

$$C_1\left(Q^1, Q^2, Q^3\right) = 0.5\left(q_{11}^1\right)^2 + 0.5\, q_{11}^2 \qquad (36)$$

$$C_{11}\left(Q^3\right) = q_{11} + 5 \qquad (37)$$

$$\hat{C}_{11}\left(Q^3\right) = q_{11} + 5 \qquad (38)$$

$$d_1\left(\rho_3\right) = -2\rho_{31} + 1100 \qquad (39)$$

Inputting the above data into the varaitional inequality (28) to determine the following, q_{11}^*, q_{11}^{1*}, p_{31}^*, λ_1^* and β_1^*, all nonnegative and satisfying

$$\begin{bmatrix} (5\, q_{11}^1 + 2 + 0.5\, q_{11}^1 + 3.5 + 0.5\, q_{11}^1 + 0.02\, q_{11}^1 \\ + 0.01 - \lambda_1^*)(q_{11}^1 - q_{11}^{1*}) \\ + (q_{11} + 0.5q_{11} + \lambda_1^* - \beta_1^*)(q_{11} - q_{11}^*) + \\ (q_{11} + 1 + q_{11} + 5 + \beta_1^* - \rho_{31}^*)(q_{11} - q_{11}^*) + \\ (q_{11}^{1*} - q_{11}^*)(\lambda_{11} - \lambda_{11}^*) + (q_{11}^* - q_{11}^*)(\beta_{11} - \beta_{11}^*) \\ + (q_{11}^* + 2\rho_{31} - 1100)(\rho_{31} - \rho_{31}^*) \end{bmatrix} \geq 0 \qquad (40)$$

Due to network topology and the cost structure of the system, equation (42) after algebraic simplification, yields the inequality:

$$\begin{bmatrix} (6.02\, q_{11}^1 - \lambda_1^* + 5.5)(q_{11}^1 - q_{11}^{1*}) + \\ (3.5\, q_{11} + \lambda_1^* - \rho_{31}^* + 6)(q_{11} - q_{11}^*) + \\ (q_{11}^{1*} - q_{11}^*)(\lambda_{11} - \lambda_{11}^*) + \\ (q_{11}^* + 2\rho_{31} - 1100)(\rho_{31} - \rho_{31}^*) \end{bmatrix} \geq 0 \qquad (41)$$

Suppose we assume that the demand market price $\rho_{31}^* > 0$. Substitution of $q_{11}^1 = q_{11}^{1*}$, $q_{11} = q_{11}^*$, and $\lambda_{11} = \lambda_{11}^*$ into the inequality (41) will yield:

$$q_{11}^* + 2\rho_{31} - 1100 = 0 \qquad (42)$$

Equation (41) will be used to solve for three tariff category of demand customers (R1: which is the residential with single phase supply with single meter with consumption on 50KWH and below; R2S: which is single phase supply with single phase meter with consumption above 50KWH; and R2T: which is three phase supply with three phase meter with consumption below 45KVA).

The multi start optimization method was used to solve the inequality using specified start up value obtained from the Nigerian Electricity Regulatory Commission (NERC). For R1 tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators. ($q_{11} = 68.93$ GWH) and the unit cost of power by distributors to R1 customers ($p_{31} = 4$ Naira). For R2S tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators ($q_{11} = 68.93$ GWH) and the unit cost of power by distributors to R1 customers. ($p_{31} = 30.93$ Naira). For R2T tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators ($q_{11} = 68.93$ GWH) and the unit cost of power by distributors to R1 customers ($p_{31} = 34.28$ Naira).

The summary of the result obtained from the three categories of customers were presented in table 1.

The result of the analysis showed that the least cost of power regardless of the category of customer is ₦ 469.15 while the highest cost is ₦ 473.32. It found that the highest shadow price was ₦ 978.93 for R1 customers while the least was ₦ 928.61 for R2T customers.

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Table 1 Summary of parameters for the three categories of customer

Category of Demand Customers	Tariff ₦	q_{11}^{1*} GWH	q_{11}^{*} GWH	P_{31}^{*} ₦	λ_1^{*} ₦	β_1^{*} ₦
R1	4	161.70	161.70	469.15	978.93	0
R2S	30.93	154.36	154.36	472.82	934.75	0
R2T	34.28	153.35	153.35	473.32	928.61	0

V. CONCLUSION

The study proposed a new model of electric power supply chain networks in the Nigeria situation, which allows for multiple power generators, transmission, and distribution, retailing and demand customers. The supply chain network introduces retailing of power from distributors to demand customers. We derived the optimality conditions of the decision- makers and proved that the governing equilibrium conditions satisfy a variational inequality problem. The variational inequality problem for a single power generator, single transmission, single distributor, single retailer and a demand customer was used to illustrate the method. The multi start optimization method was used to solve the inequality using specified start up value obtained from the Nigerian Electricity Regulatory Commission (NERC). For R1 tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators ($q_{11} = 68.93$ GWH) and the unit cost of power by distributors to R1 customers ($p_{31} = 4$ Naira). For R2S tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators ($q_{11} = 68.93$ GWH) and the unit cost of power by distributors to R2S customers ($p_{31} = 30.93$ Naira). For R2T tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators($q_{11} = 68.93$ GWH) and the unit cost of power by distributors to R2T customers ($p_{31} = 34.28$ Naira). The result of the analysis showed that the least cost of power regardless of the category of customer is ₦ 469.15 while the highest cost is ₦ 473.32. It found that the highest shadow price was ₦ 978.93 for R1 customers while the least was ₦ 928.61 for R2T customers.

REFERENCES

[1].Awosope, C.A. (2014). Nigeria Electricity Industry: Issues Challenges and Solutions. 38th Public Lecture Presented to Covenant University, Ota, Ogun State, Vol.3, No.2.

[2].Braido, G.M., Borenstein, D., and Casalinho, G.O. (2016). Supply Chain Network Optimization using a Tabu Search based heuristic Gest. Prod., Sao Carlos, V. 23, No. 1, P 3-17.

[3].Chun-Hsiung, L., Hai-Ming, C; Chih –Wei, C; and Mei – Hsiu, C. (2012). A complicated Continuous Model of Supply Chain. Journal of Applied Science and Engineering, Vol. 15, No.2, P.177-186.

[4].Dong, J, Zhang, D. and Nagurney, A. (2004) A supply chain network equilibrium model with random demands. European Journal of operation Research, Vol. 156, P. 194 – 212.

[5].Ejumudo, T.F. (2014). The operations of the Power Holding Company of Nigeria and Discriminatory Monopoly. Journal of Energy Technologies and Policy, Vol. 4, No. 6:

[6].EPRI (Electric Power Research Institute) (2003). Electricity Sector Framework for the future. Vol. 1: Achieving a 21st Century Transformation.

[7].FGN (2005) Electric Power Sector Reform Act, 2005. Federal Republic of Nigeria Official Gazette Vol. 92, No. 77, Government Notice No. 150.

[8].Leibenstein, N. (2012). Monopoly Power and Costs. Toronto: MCGraw – Hill.

[9].Nagurney A. and Matsypura D. (2005). A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption. In: Advances in computational Economics, Finance and Management Science, Kontoghvorghes, E.J, Gatu, C. (Eds) Springer, Berlin, Germany.

[10]. Nagurney, A. (2006). Supply Chain Network Economics: Dynamics of Prices, Flows and Profits. Edward Elgar Publishing, Inc. Massachusetts 01060, USA.

[11]. Nagurney, A., Dong, J. and Zhang, D. (2002). A Supply chain network equilibrium model. Transportation research, Vol. 38E, p. 281-303.

[12]. Nash, J.F. (1950). Equilibrium Points in N- Person Games. Proceedings of the National Academy of Sciences, Vol. 36, P. 48-49.

[13]. Nash, J.F. (1951). Non-cooperative Games. Annals of Mathematics, Vol. 54, P. 285-298.

[14]. Niki, M., Emad, R., and Zarifeh, R. (2013). Supply chain network optimization: A review of classification, models, solution techniques and future research. Vol. 1, P. 1-24. www.GrowingSenece.com

[15]. Uwaifo, S.O. (1994). Electric Power distribution planning and development. Lagos Malt-House press.

Appendix

R code for solving the inequality using multistart for R1:

R> objective.functionq11 <- function(x) (x[1] + 2*x[2] - 1100)

R> multiStart(c(68.93, 4), objective.functionq11, gr=NULL)

Output:

Successful convergence.

$par [q11] [p31]

[1,] 161.6972 469.1514

$fvalue

[1] 7.27756e-08

$converged

[1] TRUE

R code for solving the inequality using multistart for R2S:

R> objective.functionq11 <- function(x) (x[1] + 2*x[2] - 1100)

R> multiStart(c(68.93, 30.93), objective.functionq11, gr=NULL)

Output:

Successful convergence.

$par [q11] [p31] [1,] 154.3587 472.8206

$fvalue

[1] 4.662434e-08

$converged

[1] TRUE

R code for solving the inequality using multistart R2T:

R> objective.functionq11 <- function(x) (x[1] + 2*x[2] - 1100)

R> multi Start(c(68.93, 34.28), objective.functionq11, gr=NULL)

Output:

Successful convergence.

$par [q11] [p31] [1,] 153.348 473.326

$f value[1] 4.373115e-08

$converged

[1] TRUE