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A Study on Only of its Kind of Real Number is an Integral Domain

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***Abstract*: We take a broad view the notion of "unique factorization domain" in the spirit of "half-factorial domain". We show that new generalization of UFD implies the now well known notion of half factorial domain. As a consequence, we discover that the one of the normal axioms for unique factorization domains is unconscientiously redundant. That is, we interested in factoring numbers in integral domains so we have to scrutinize distribution, and so this post will begin with a fairly cursory look at the properties of distribution. At that moment we willpower bring in the crucial ideas of units and acquaintances. (In the Real number, 1 are units and n, for any fixed n, are associates.)**

***Keywords:* Real Number, Unique Factorizations, Rational Number, Prime Numbers**

I. INTRODUCTION

The conception of only one of its kind factorization is individual that is middle in the study of commutative algebra. A only one of its category factorization domain (UFD) is an connected domain, R, where every nonzero non unit can be factored uniquely. More formally we record the following standard definition. Connected domain in broad-spectrum, but connected domains that are not only one of its kind factorization domains (UFDs) in particular. We interested in the outer ring of that diagram.

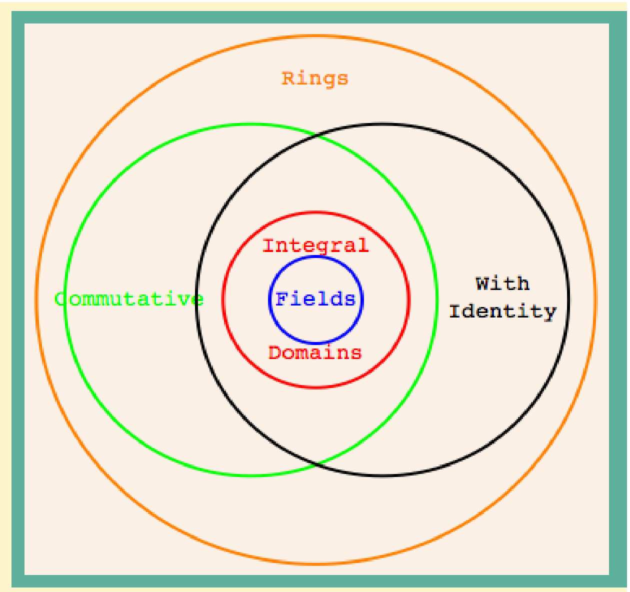


Fig 1. Ven Diagram Of Integral Domain

Definition 1

Integral domain R is irreducible over F field but reducible over ID

Suppose that D is an integral domain and F is a field containing ID. If r(x)∈ID[x] and r(x) is irreducible over R but reducible over ID. This doesn't make sense to me, how can ID⊆R and be irreducible in F but reducible in ID. The same polynomials in ID are in R, so any polynomial product representation of R is irreducible.

Example 1

We learning about Rings, commutative rings, IDs, UFDs, etc with each being a subset of the predecessor, and now we find an ID that is not a UFD Finally we understand R [1+√ (-7)] is an Integral domain, but not a UFD, A certain connected domain is not only one of its kind factorization domains We to prove the following: R is an ID and let F be its field of fractions. Suppose there exists a monic p(x) ∈R[x] such that p(x)= a(x)b(x) where both a, b are monic and non constant polynomials of F[x] but a ∉ R[x]. Then I need to show that R is not a UFD.

Theorem 1

Let R be an integral domain. Every prime element is irreducible.

Proof. Let p be a prime element. We assume that p is reducible and we want to get a contradiction. This mean that we can write p = ab where p is not associate to neither a, nor b. I notice that p/ab. Since p is a prime, this means that p/a or p/b. Without loss of generality I will assume that p/a. But now we have that p/a and also a/p. This means p and a is associate. Contradiction.

Example. 2

The converse is not true in general. As an example, consider the ring R[1+√(-7)].

The element 2 is irreducible in R. However,

2/6= (1+√((-7))i)(1-√(-7)i)

But 2 dose not divide [1+ √(-7)i] and 2 dose not divide [1-√(-7)i]. Hence 2 is not a prime.

Theorem 2

Let R be a connected domain in which each irreducible element is prime. Then the decomposition of an element as product of irreducible, if it exists, is unique.

(Notice that this is not enough to conclude that R is a UFD, since the decomposition as product of irreducible may not exist.)

Proof. Let's assume that we have two different decompositions where all $p_i$and q_j are irreducible. We want to prove that these two decompositions are the same, up to reordering and associates. Now we will proceed by induction on the maximum of n and m.

For the base case, if n = m = 1, we have P_1 = Q_1 and we do not need to do anything.

Theorem 3

Let R an integral domain. Assume that x contains an element that is not 0, not a unit, and cannot be written as product of irreducible. Then there exists an infinite sequence

X_0 ,X_1 …..X_N of elements in R such that $(X_0) \subset (X_1) \subset (X_2) \subset (X_3)$ ….. where all the inclusions are strict.

Proof: For elements in R, we know that x is a unit if f(x) = R. We know that x and y are associates. If f(a) = (b). Moreover, if we can factor a = bc non-trivially (so that b and c are neither units, nor associates to a), then (a) c (b) and (a) c(c).Pick an element a ∈ R which is not zero, not a unit, and not the product of irreducible. Call a_0= x. Since x is not a product of irreducible, in particular it is not irreducible, so we have a non-trivial factorization a = bc. At least one of b or c is also not a product of irreducible. Whichever it is, call itx_1. Repeat.

Theorem 4

R is a saturated multiplicatively closed set.

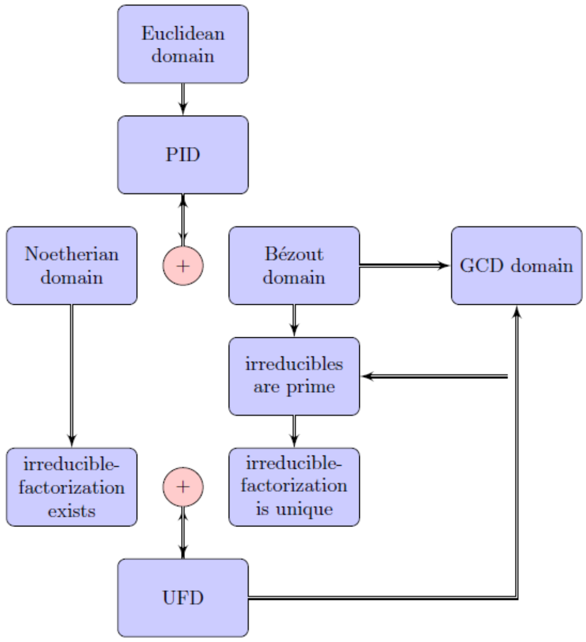
Proof: If a, b ∈R then we can write both a and b as product of primes, so a, b can be also written as product of primes. This shows that R is a multiplicatively closed set.

Now, to prove that R is saturated we have to show that if a∈ S, then every divisor of a is in S too. If we write x=$up_1, up_2 \ldots up_n$where u is a unit and thep_i's are prime, then it can be shown by induction on n that every divisor of x is in R. I let you to do this proof by induction.

Now, we argue by contradiction assuming that there is a nonzero element a ∈ R such that a ∉ R, so the ideal generated by a, ⟨a⟩, is disjoint from R, i.e., R∩⟨a⟩=∅, because if there were some ra ∈ S, then a would be in S (because a | ra and R is saturated by the lemma above), contradicting our hypothesis that a∉S.

Therefore the set A= {I non zero ideal of R:I∩R=∅} A={I non zero ideal is non-empty and then by Zorn's Lemma A has a maximal element P such that P is not only an ideal, but in fact a prime ideal. By our general hypothesis P contains a prime element, let's say pp, i.e., p∈P, but by the definition of R is clear that p∈R, so p∈ P∩S, which contradicts P∩R=∅. This contradiction comes from our assumption that a ∉ R.

Hence every nonzero a∈R belongs to R, i.e., R=R\{0} and this means that every nonzero, non unit element of R is expressible as a product of primes.



II. MOTIVATION

We know that every integer number is the product of prime numbers in a unique way. Sort of. We just believed our kinder garden teacher when she told us, and we omitted the fact that it needed to be proven. We want to prove that this is true, that something similar is true in the ring of polynomials over a field. More generally, in which domains is this true? In which domains does this fail?

References

[1]. D.D. Anderson, D.F. Anderson and M. Zafrullah, Factorization in integral domains. Journal of Pure and Applied Algebra 69 (1990), 1-19.

[2]. M.F. Atiyah and I.G. Macdonald, Introduction to commutative algebra. Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont. 1969.

[3]. M. Auslander and D.A. Buchsbaum, Unique factorization in regular local rings. Proc.Nat. Acad. Sci. U.S.A. 45 1959 733{734.

[4]. Baker, Linear forms in the logarithms of algebraic numbers. I, II, III. Mathematika13 (1966), 204-216; ibid. 14 (1967), 102-107; ibid. 14 1967 220{228.

[5]. S. Borofsky, Factorization of polynomials. Amer. Math. Monthly 57, (1950), 317{320.

[6]. Bouvier, Le groupe des classes de l'algbre ane d'une forme quadratique. Publ. Dep.Math. (Lyon) 15 (1978), no. 3, 53{62.

[7]. E.S. Barnes and H.P.F. Swinnerton-Dyer, The inhomogeneous minima of binary quadratic forms. I. Acta Math. 87, (1952). 259{323.

[8]. D.A. Buchsbaum, Some remarks on factorization in power series rings. J. Math. Mech. 10 1961 749{753.

[9]. E.D. Cashwell and C.J. Everett, The ring of number-theoretic functions. Pacific J. Math. 9 (1959) 975{985.
[10]. D.A. Clark, A quadratic field which is Euclidean but not norm-Euclidean. Manuscripta Math. 83 (1994), no. 3-4, 327{330
[11]. P.M. Cohn, Bfiezout rings and their subrings. Proc. Cambridge Philos. Soc. 64 (1968) 251{264.
[12]. P.M. Cohn, Unique factorization domains. Amer. Math. Monthly 80 (1973), 1{18.
[13]. D.A. Cox, Primes of the form x2 + ny2. Fermat, class field theory and complex multiplication. A Wiley-Inter science Publication. John Wiley & Sons, Inc., New York, 1989.
[14]. D. Eisenbud Commutative algebra. With a view toward algebraic geometry. Graduate Texts in Mathematics, 150. Springer-Verlag, New York, 1995
[15]. Euclid, The thirteen books of Euclid's Elements translated from the text of Heiberg. Vol. I: Introduction and Books I, II. Vol. II: Books III{IX. Vol. III: Books X{XIII and Appendix. Translated with introduction and commentary by Thomas L. Heath. 2nd ed. Dover Publications, Inc., New York, 1956.
[16]. R.M. Fossum, The divisor class group of a Krull domain. Ergebnisse der Mathematics und ihrer Grenzgebiete, Band 74. Springer-Verlag, New York-Heidelberg, 1973