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Factorization in Integral Domains

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***Abstract*: The M-evaluated spaces R = M Rm, which are nearly Schreier are classified under the suspicion that the fundamental conclusion R of R is a root augmentation of R, where M is a without torsion, commutative, cancellative monoid. For the situation that D[M] is a commutative monoid area it is demonstrated that if M funnel shaped and D[M] ⊆ D[M] is a root augmentation, at that point D[M] is nearly Schreier if and just if M and √D are nearly Schreier. On the off chance that R = Æ[nx] is a request in a quadratic augmentation field $(d) of $, it is demonstrated that the conditions; R[X] is IDPF; R[X] is inside factorial; R[X] is nearly Schreier; R[X] ⊆ R[X] is a root expansion; and each prime divisor of n likewise partitions the discriminant of the augmentation K/$; are equal conditions.**

Keywords: **Almost Schreier; Atomic domain; Inside factorial; Pre-Schreier domain.**

I. INTRODUCTION

A vital space R is said to be nuclear if each nonzero non-unit of R has a factorization into a result of a finite number of final components (iot as). A fundamental area R is said to be an IDF space if each nonzero component of R has at most a finite number of non-relate unchangeable divisors. These were first contemplated by Grams and Warner in their 1975 article [10] and have gotten very piece of consideration from that point forward. In [13], Malcolmson and Okoh talk about a thought of factorization that is a more grounded condition on an indispensable space than the IDF property, however weaker than the special factorization property. Given a nonzero, non-unit component an every a necessary space R, let$D_n(a)$ mean the arrangement of non-relate final divisors of an. The area R is said to be an IDPF space (for unchangeable divisors of forces finite) if for each nonzero, non-unit a ∈ R, the association $*n_1$ $D_n(a)$ is finite, up to partners. Krull areas are IDPF [13, Corollary 3.3]. Recall that a ring augmentation R ⊆ T is a root expansion if for every t ∈ T, there exists n ∈ IN with the end goal that t_n ∈ R. By [8, Theorem 2.8], a Noetherian space R with vital conclusion R and nonzero conductor (R) is IDPF if and just if the remainder of unit bunches U(R)/U(R) is a finite gathering and R is a root augmentation.

In [5], P. M. Cohn defined a nonzero component p of an essential area R to be primal if at whatever point p xy, x, y R, there existp_1, p_2 R to such an extent that p_1 p_2 with p_1(x) and p_2 (y), and defined a space to be Schreier in the event that it is vitally shut and each nonzero component of R is primal. In [16], M. Zafrullah presented the helpful wording of pre-Schreier for an area R in which each nonzero component of R is prim

In their 2010 paper [6], Dumitrescu and Khalid define a nonzero element *p* of an integral domain *R* to be almost primal if whenever | p_{xy} *x*, *y* ∈*R*, there exists an integer *k* ≥1 and p_1, p_2∈*R* such that p^k= p_1p_2 with p_1 | x^k and p_2 | y^k. They define *R* to be **almost Schreier** if every nonzero element of *R* is almost primal.

A monoid homomorphism *$: D H* → is called a **divisor homomorphism** if for any *a*, *b D*∈, *$(a) $(b)* | in *H* implies *a b* | in *D*. A monoid *H* is called **inside factorial** if there exists a divisor homomorphism *$: D H* → from a factorial monoid *D* such that for every *x H* ∈there exists some *n* ∈Æ such that x^n∈ *$(D)*. An integral domain *R* is called **inside factorial** if its multiplicative monoid *R*=* *R {0}* is inside factorial. (See for example [4], [12] for information on this class of rings and monoid.)

A framework of this article is as per the following. In Section 2, a classification is given of when a M-evaluated area R is nearly Schreier, under the presumption that R is a root augmentation (see Theorem 2.5), where M is a without torsion, cancellative monoid. By practicing to the case that R = D[M] is a monoid space, it is appeared in Theorem 2.8 that if D[M] is a root augmentation and M is funnel shaped, at that point D[M] is nearly Schreier if and just if D and M are nearly Schreier. This is a partner to an aftereffect of Krause [12, Theorem 3.2], which gives the comparing result with "nearly

Schreier" supplanted by "inside factorial." In Section 3, a classification is given of when an evaluated space is inside factorial, as far as the relatively primal property (see Theorem 3.1). In Section 4, a sufficient condition is given on the requests R in a quadratic number ring for R[X] to be an IDPF ring, by means of the discriminate &$(√d) of $(d). In Section 5, it is demonstrated that if R = Æ[ns] is a request in a quadratic augmentation field $(d) of $, at that point the conditions R[X] is IDPF; R[X] is inside factorial; R[X] is nearly Schreier; R[X] is a root augmentation; and each prime divisor of n additionally separates the discriminate of the expansion K/$; are proportional conditions. This gives a valuable class of cases of rings fulfilling the conditions considered here, including illustrations demonstrating that the root expansion condition is fundamental in the previously mentioned Theorems 2.5 and 2.8.

II. ALMOST SCHREIER GRADED DOMAINS

In this area, let R m M Rm be a M-reviewed space, where M is a without torsion, cancellative, commutative monoid, and let S mean the arrangement of nonzero homogeneous components of R. We say that R is gr-nearly Schreier if at whatever point s xy, where s, x, y S, there exists a whole number k1 to such an extent that sk s1s2 with s1 x^k and s^2 y^k. Let x x_1 x_2 xn be the novel portrayal of x as a whole of homogeneous components. At that point, the substance of x is the homogeneous perfect C(x) (x_1,..., x_n)R of R.

In Theorem 2.5, we demonstrate a halfway simple of an outcome [3, Theorem 2.1] on pre-Schreier reviewed areas for when evaluated spaces are nearly Schreier. Obviously the greater part of the factorization properties of basic areas considered in this article has partners for abelian sans torsion cancellative monoid. See for instance [7]. Since our monoid are generally composed additively, we frequently find it advantageous utilize the characteristic preorder on M which is defined by x y if x z y for some z M. It is anything but difficult to see that the normal preorder on M is ant symmetric, and in this manner a halfway request on M if and just if M is tapered; that is, x y = 0 in M suggests x = y = 0.

On the off chance that M is a multiplicative monoid, Y M and x M, we compose x Y if x y for every Y . On the off chance that Y_1, Y_2 M, Y_1Y_2 means {y_1y_2 Y_1, y_2 Y_2}:

(†): For any nonempty finite subsets $Y_1, Y_2 \subseteq M$ and $x \in M$ such that $x \mid Y_1Y_2$, there exist $z_1, z_2 \in M$ and an integer $k \geq 1$ such that $x^k = z_1z_2$ with $z_1 \mid Y^k$ and $z_{12} \mid Y^k$. (If M_2 an additive monoid, then in terms of the natural preorder, this becomes as follows: if $x \leq Y_1 + Y_2$, there are $z_1, z_2 \in M$ and an integer $k \geq 1$ such that $kx = z_1 + z_2$ with $z_1 \leq kY_1$ and $z_2 \leq kY_2$.)

Clearly, for any monoid *M*, condition *(†)* implies *M* is almost Schreier. In the next lemma, we see that the converse is true for cancellative monoid.

Lemma 2.1. *If M is almost Schreier, then M satisfies (†).*

Proof. The proof is by induction on $n = |Y_1| |Y_2|$ the case $n = 2$, when $|Y_1| |Y_2|$ is true by definition of almost Schreier.

Suppose the claim is true for all $n \leq N$ for some $N \in$ Æ. To prove the claim for $n = N+1$, we will suppose that there are nonempty subsets Y_1, Y_2 $M \subseteq$with $N = |Y_1| +|Y_2$and $| x, y_1 \in M$ such that $x (Y\leq_1 \{y_1\})\cup+ Y_2$, and show that there exists a positive integer k such that $kx = z_1 + z_2$ for some $z_1, z_2 \in M$ such that $z_1 \leq kY_1 \{\cup ky_1\}$ and $z_2 \leq kY_2$.

By hypothesis, we have the two inequalities,

$$x \leq Y_1 + Y_2 \quad \text{and } x \leq y_1 + Y_2. \quad (1)$$

By the induction hypothesis, the first of these inequalities implies that there are $w_1, w_2 \in M$ and an integer $k \geq 1$ such that

$$kx = w_1 + w_2 \quad \text{with } w_1 \leq kY_1 \quad \text{and } w_2 \leq kY_2. \quad (2)$$

In particular, $kY_2 = w_2 + Z_2$ for some subset Z_2 $M\subseteq$ zwith $| Z_2 |= | Y_2|$. From the equality in (2) and the second inequality in (1), we now get

$$w_1 + w_2 = kx \leq ky_1 + kY_2 = ky_1 + w_2 + Z_2. \quad (3)$$

Since the monoid *M* is cancellative, (3) implies $w_1 \leq ky_1$ Z_2.+We have $\{ky_1\}|$ $| Z_2 |= 1+ | Y_1 |\leq Y_2\ N$, so by the induction hypothesis there are $t \geq 1$ and z_1 and $w_3$such that

$$tw_1 = z_1 + w_3 \quad \text{with } z_1 \leq tky_1 \quad \text{and } w_3 \leq tZ_2. \quad (4)$$

Setting $z_2 = tw_2 + w_3$, the first equality in (3) and the equality in (4) give

$$ktx = tw_1 + tw_2 = z_1 + w_3 + tw_2 = z_1 + z_2. \quad (5)$$

From (4) we have $z_1 \leq tky_1$. Also from the equality in (4) and the first inequality in (2), we have $z_1 \leq tw_1 \leq tkY_1$. Thus $z_1 \leq tkY_1 \cup\{tky_1\}$. Finally, from the second inequality in (4), $z_2 = tw_2 + w_3 \leq tw_2 + tZ_2 = t(w_2 + Z_2) = tkY_2$, as required.

We also use the following property:

(††): If $x, y \in R$, $s \in S$ and $y \in (s) :_R (x)$, then $C(y^k) \subseteq (s^k) :_R C(x^k)$ for some positive integer *k*.

Lemma 2.2. *If $R = {}_{m\in M} R_m$ is almost Schreier, then R satisfies (††).*

Proof. Let $y \in (s) :_R (x)$, where $x, y \in R$, $s \in S$. Then $xy \in (s)$ and so $s \mid xy$. Thus since *R* is almost Schreier, $s^k = s_1s_2$ with $s_1 \mid x^k$ and $s_2 \mid y^k$ for some integer $k \geq 1$. Clearly s_1, s_2 are in *S*. Then $y^k \in (s_2)$, a homogeneous ideal. Thus $C(y^k) \subseteq (s_2)$. Similarly, $C(x^k) \subseteq (s_1)$. So we have $C(x^k) \subseteq (s_1)$, $C(y^k) \subseteq (s_2)$, and therefore, $C(x^k)C(y^k) \subseteq (s_1s_2) = (s^k)$. So $C(y^k) \subseteq (s^k) :_R C(x^k)$.

We use the following result from [15].

Lemma 2.3 ([15, Corollary 3.3])**.** *If D is an integrally closed domain, d, a_1,..., $a_r \in^n$ for i = 1,...,r, then $d \mid a^{n_1} a_1^{n_2}$ $_2 a^{n_r}$ whenever $n_r \geq 0_i$and Σ n = $n._{ii}$ D, and d | a_i*

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Lemma 2.4. *Suppose $R = {}_{mM} R_{\in m}$ is gr-almost-Schreier, that R satisfies (††) and that $R \subseteq R$ is a˜ root extension. Then every $s \in S$ is almost primal in R.*

Proof. Let $s \mid xy$. Then $xy = sr$ for some $r \in R$. Thus $y \in (s) :_R (x)$. So, by *(††)*, $C(y^k) \subseteq (s^k) :_R C(x^k)$ for some integer $k \geq 1$. Thus $C(x^k)C(y^k) \subseteq (s^k)$. So s^k divides every member of $C(x^k)C(y^k)$. Let $x = x_1 + x_2 + \cdots + x_n$, $x_i \in R_a$, $a_i = a_j$ if and only if $i = j$, and $y = y_1 + y_2 + \cdots + y_m$, $y_j \in R_þ$, $þ_i = þ_j$ if and only if $i = j$. Now, $x^k = \Sigma\, x^j$ and $y^k = \Sigma\, y_j^j$, where each monomial x^j_i is of the form $x\, x^{l_n}{}_n$, where s of the form $y^{b_1} y^{b_2} \cdots y^{b_m}$, where $\Sigma\, b = k$. Since R is k, and each monomial ygr-almost-Schreier, then by Lemma 2.1 there exists an integer $^{kt} = s_1 s_2$ with $s_1 \mid x^{jt}$ and $s_2 \mid y^{jt}$ for every i and j. Thus by Lemma 2.3,$t \geq 1$ and s_1, $s_2 \in S$ such that $s\, \Sigma a_j{}^1{}_t =_1 x_j t_t{}^2. {}_2 \cdot\cdot$So in $\cdot\, x_{jt}{}^n{}_n R$ ˜in we have that ˜R, where $\Sigma\, t\, s_1{}^i$ divides each element of $= t$, and$_j$ similarly, $s^2 \mid y^j{}_a{}^1{}_1 C(xy^j{}_a{}^2{}_2{}^{kt} \cdot\cdot)\cdot$and $y^j{}_a{}^m{}_m s$in$_2$ divides each ˜R, where $s^1 \mid x$ element of $C(y^{kt})$. Thus we have $s^{kt} = s_1 s_2$ and in R, ˜$s_1 \mid x^{kt}$ and $s_2 \mid y^{kt}$. As $R \subseteq R$ is a˜ root extension, there exists an integer $w \geq 1$ such that $s^{ktw} = s^w s^w$ and ${}_{1\,2}\, s^w| \, x^{ktw}{}_1$and $s_2{}^w| \, y^{ktw}$ in R.

We now have the following theorem.

Theorem 2.5. *Consider the following conditions on $R = {}_{m\in M} R_m$.*

(1) R is almost Schreier.

(2) Every element s S is almost primal in∈ R.

(3) R is gr-almost-Schreier and R satisfies (††).

Then (1) ⇔ (2) ⇒(3). If $R \subseteq R$ is a root extension, these conditions are equivalent.˜

Proof. (1) ⇒ (2): As R is almost Schreier, every element in the saturated multiplicative set S is almost primal in R.

(2) (1):⇒ By [1, Proposition 2.1], R_S is a GCD-domain, and hence it is almost Schreier [6, Proposition 2.2(a)]. As every $s \in S$ is almost primal in R, it follows by the Nagata type result for almost Schreier domains [6, Theorem 4.3], that R is almost Schreier.

(1) ⇒ (3): As R is almost Schreier, R is gr-almost-Schreier. The fact that R satisfies *(††)* follows from Lemma 2.2.

(3) ⇒ (2): This follows from Lemma 2.4.

In the remainder of this section, we specialize to the case that $R = {}_{m\ \ M} R_m$ is the commutative monoid ring *D[M]* where D is an integral domain. To do this, we $_\in$ use the following lemmas. We continue to assume that M is a cancellative, torsion- free commutative monoid.

Lemma 2.6. *If M is conical, D and M are almost Schreier, and D[M] ˜⊆D[M] is a root extension, then D[M] is gr-almost-Schreier and satisfies (††).*

Proof. Suppose $r_1 X^{a_1} \mid r_2 X^{a_2} r_3 X^{a_3}$ in *D[M]*. Then, $r_1 \mid r_2 r_3$ in D and $a_1 \leq a_2 + a_3$ in M, where ≤ is the natural order on M. Since D is almost Schreier, there exists an integer $k_1 \geq 1$ and w_2, w_3 in D such that

$r_{1k_1} = w\, w_{23}$ with $w_2 \mid r_{k_21}$ and $w_3 \mid r_{k_31}$ in D, (6)

and since M is almost Schreier, there exists an integer k_2 1 and ≥ $þ_2, þ_3$ M such ∈that

$k_2 a_1 = þ_2 + þ_3$ with $þ_2 \leq k_2 a_2$ and $þ_3 \leq k_2 a_3$. (7)

From Eqs. (6) and (7), respectively, we get

$r_1^{k_1 k_2} = w^{k_2}{}_2 w^{k_2}{}_3$with $w_{k_2 2} \mid r_{k_2 1 k_2}$ and $w_{3k_2} \mid r_{k_3 1 k_2}$, and (8)

$k_1 k_2 a_1 = k_1 þ_2 + k_1 þ_3$ with $k_1 þ_2 \leq k_1 k_2 a_2$ and $k_1 þ_3 \leq k_1 k_2 a_3$. (9)

So $(r_1 X^{a_1})^{k_1 k_2} = r_1^{k_1 k_2} X^{k_1 k_2 a_1} = w_2^{k_2} w_3^{k_2} X^{k_1 þ} \ (r\, X^{a_3}{}_3)^{k_1 k_2}$in *D[M]*. So $D[M]$, where by Equations (8)$k\ k\ a$ and (9), , and similarly $w_{k_2 2} X_{k_1 þ_2} w$divides ${}_{k_2} X^k{}_{1 þ_3}$ divides $(r\, X_{2\ a_2})_{k_1 k_2} = r_{k_1 k_2} X_{k_1 k_2 a_2}$

We first show that D ˜ and M ˜ are almost Schreier. Let K be the quotient field of D, and suppose $r\ K \in$is integral over D. Then, $r\ \ D\ \in$ ˜ ⊆*D[M]* ˜ . So, $r^k \in$ *D[M]*, for some positive integer k, and hence $r^k \in D$. So D D ⊆ ˜is a root extension. As D is almost Schreier, it follows that D˜is almost Schreier [6, Proposition 2.2(d)].

Let $a \in M$˜. Then, $X^a \in D\ [M]$˜ . Thus, there exists an integer $k \geq 1$ such that $X^{ak} \in D[M]$. So $ak \in M$. So $M \subseteq M$˜is a root extension. As M is almost Schreier, it ˜follows that M is almost Schreier [6, Proposition 2.2(d)].

We now show *D[M]* satisfies *(††)*. Let $g \in (rX^a) :_{D[M]} (f)$, where f, $g \in D[M]$ and $rX^a$is a homogeneous element of *D[M]*. So, $rX^a \mid fg$ in *D[M]*, and hence in *D[M]*˜ . So rX^a divides every element of *C(fg)* in *D[M]*˜ . As *D[M]* ˜ is integrally closed, $(C(fg))_v = (C(f)C(g))_v$ [2, Theorem 3.5(1)]. So rX^a divides every element of *C(f)C(g)* in *D[M]*˜ . Let $f = r_1 X^{a_1} + \cdots + r_n X^{a_n}$ where $a_i = a_j$ if and only if $i = j$ and $g = s_1 X^{þ_1} + \cdots + s_m X^{þ_m}$ where $þ_i = þ_j$ if and only $i = j$. Then $r \mid r_i s_j$ for all i and j in D. As D is almost Schreier, Lemma 2.1 implies that there exists a positive ˜ ˜ integer $k_1 \geq 1$ and z, $w \in D$˜ such that $r^{k_1} = zw$ with $z \mid r^{k_1}{}_i$ and $w \mid s^{k_1}_j$for all for all i and j.

Also, $X^a \mid X^{a_i} X^{þ_j}$ for all i and j. So $a \leq a_i + þ_j$ *for* all i and j. As M˜ is almost Schreier, Lemma 2.1 implies that there exists an integer k_2 1˜≥ and y_1, $y \in_2 M$ such that $k_2 a = y_1 + y_2$ with $y_1 \leq k_2 a_i$ and $y_2 \leq k_2 þ_j$ for all i and j. Then $r_{k_1 k_2} = z_{k_2} w_{k_2}$ with $z_{k_2} \mid r_{k_1 k_2 i}$and $w_{k_2} \mid s_{k_1 k_2}$ for all $_j$ i and j. Also

$k_1 k_2 a = k_1 y_1 + k_1 y_2$ with $k_1 y_1 \leq k_1 k_2 a_i$ and $k_1 y_2 \leq k_1 k_2 þ_j$, for all i and j. Thus

${}_{k_2} = .z_{k_2} X_{k_1 y_1} \Sigma\ .w_{k_2} X_{k_1 y_2} \Sigma$ with $z_{k_2} X_{k_1 y_1} \mid (r\ {}_i X_{a_i})_{k_1 k_2}$

and $w_{k_2} X_{k_1 y_2} \mid .s\ X_{jþ\,j} \Sigma_{k_1 k_2}$, $(rX_a)_{k_1}$ in D [M]. As

D [M] is integrally closed, $z^{k_2} X^{k_1 y_1} \mid (r_1 X^{a_1})^{l_1} \cdots (r\ \ X^{a_n})^{l_n}$ ˜[M] whenever $l_i \geq 0$ and $\Sigma\, l_i = k_1 k_2$, by Lemma 2.3. $_n$Also, in D 1 m j j 1 2 by Lemma 2.3.

So, $w^{k_2} X^{k_1 y_2} | .s\, X^{p_1} \Sigma^{b_1} \cdots .s\ X^{p_m} \Sigma^{b_m}$ in $\tilde{D}\,[M]$ whenever $b \geq 0$ and $\Sigma\, b = k\,k$ in $\tilde{D}\,[M]$

$_2^k X^{ak}{}_1{}^k{}_2 = .z^k{}_2 X^{k}{}_1^{y}{}_1 \Sigma\, .w^k{}_2 X^{k}{}_1^{y}{}_2 \Sigma$ with $z^k{}_2 X^{k}{}_1^{y}{}_1 | f^k{}_1{}^k{}_2$ and $w^k{}_2 X^{k}{}_1^{y}{}_2 | g^k{}_1{}^k{}_2$ in $D[M]$. $\tilde{}$ $r^k{}_1$

Since $D[M] \subseteq D[M]\tilde{}$ is a root extension, there exists an integer $T \geq 1$ such that

$(rX_a)_{k_1k_2T} = z_{k_2T} X_{k_1y_1T}\, w_{k_2T} \Sigma X\, \dot{}_{k_1y_2T}$ with $z_k \Sigma_{2T} X_{k_1y_1T} | f_{k_1k_2T}$ and $w_{k_2T} X_{k_1y_2T} | g_{k_1k_2T}$ in $D[M]$. So, $z^{k_2T} X^{k_1y_1T}$ divides every element of $C(f^{k_1k_2T})$ in $D[M]$ and $w^{k_2T} X^{k_1y_2T}$ divides every element of $C(g^{k_1k_2T})$ in $D[M]$. Thus, $(rX^a)^{k_1k_2T} = (z^{k_2T} X^{k_1y_1T})(w^{k_2T} X^{k_1y_2T})$ divides every element of $C(f^{k_1k_2T})C(g^{k_1k_2T})$ in $D[M]$.So,$C(f_{k_1k_2T})C(g_{k_1k_2T})((rX_a)_{k_1k_2T})$, and hence $C(g_{k_1k_2T}) \subseteq ((rX_a)_{k_1k_2T}) :_{D[M]} C(f_{k_2k_2T})$. So, $Ds[M]$ satisfies *(††)*.

Lemma 2.7. *If M is conical and D[M] is almost-Schreier, then D and M are almost Schreier.*

Proof. Let $r\ xy$ | in D. Then, there exists an integer $k \geq 1$ and $r_1, r_2 \in D[M]$ such that $r^k\ r{=}_1 r_2,\ r_1 x^k, | r_2 y^k$. Since the degree of | r is 0, it follows that r_1 and r_2 have degree 0 since M is conical. Thus $r_1, r_2 \in D$. So D is almost Schreier. Now, suppose $a \leq a_1 + a_2$. Then, $X^a\ X^a| {}^1 X^{a_2}$. As $D[M]$ is gr-almost-Schreier, there exists an integer $k \geq 1$, and $f_1, f_2\ D[M] \in$such that $X^{ak} = f_1 f_2$ where $f_1\ X^{a_1k}$ and | $f_2\ X^{a_2k}$. By [9, | Theorem 11.1], f_1 and f_2 are monomials. So $f_1\ = uX^{p_1}$ and $f_2\ \ u{=}^{-1} X^{p_2}$. Thus $X^{ak}\ uX^p{=}^1\ u_{-1}X_{p_2},\ uX_{p_1}\ X_{a_1k}$, |and $u_{-1}X_{p_2}\ X_{a_2k}$. So| $ak = p_1\ p{+}_2$and $p_1\ a_1k \leq$and$p_2 \leq a_2k$. So M is almost Schreier.

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