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A Study on Normality of Meromorphic Functions and Discrete Exceptional Sets

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Abstract - **We consider a class of Normality of meromorphic functions, denoted by K in this paper, which are meromorphic outside a compact and countable set Q (f), investigated by P. Bolschin his thesis in 1997. The set Q (f) is the finish of isolated indispensable singularities. We review main definitions and properties of the Fatou and Julia sets of functions in class K. It is studied the role of Q (f) in this context. Following Eremenko it is defined escaping sets and we prove some results related to them. For instance, the dynamics of a function is extended to its singularities using escaping hairs. We give an example of an escaping hair with a wandering singular end point, where the hair is contained in a wandering domain of f ∈ K.**

Keywords – **Meromorphic, Function Normal Family, Shared Values Locally, Uniformly Discrete Sets, etc.**

I. INTRODUCTION

Mathematical models for phenomena in the natural sciences often lead to iteration. An often-quoted example comes from population biology. Assuming that the size of a generation of a population depends solely on the size of the previous generation and may thus be expressed as a function of it, questions concerning the further development of the population reduce to iteration of this function.

More often, a phenomenon from physics or other sciences is described by a differential equation. In certain cases, for example, if there is a periodic solution, this differential equation may be studied by looking at a Poincaré return map and again we are led to iteration. If we solve the differential equation numerically, we are also likely to use a method based on iteration

Definition 1.

The basic objects studied in iteration theory are the Fatou set F = K (f) and the Julia set J =J (f) of a meromorphic function f. Roughly speaking, the Fatou set is the set where the iterative behavior is relatively tame in the sense that points close to each other behave similarly, while the Julia set is the set where chaotic phenomena take place. The formal definitions are F = {f ∈C: $\{f^n$: n ∈ N} is defined and normal in some neighborhood of f} and

J = C\F

Meromorphic functions with exactly one pole if this pole is an omitted value. A complex number f_0 is called an omitted value of the meromorphic function f, if k (f) $\neq f_0$ for all f∈C. In this case, if the pole of f is denoted byf_0 , we have $\{f_0, \infty\} \in J$, and $k^n(f)$ is defined for all f_0 C\{f_0,∞}. It is not difficult to show that f has the form

$$k(f) = f_0 + \frac{e^{g(f)}}{(f-f_0)^m}\text{........}(1)$$

For some positive integer m and some entire function g in this case. It is no loss of generality to

$$\text{k (f)}=\frac{e^{g(f)}}{f^m}\ \text{..................}(2)$$

According to the on top of comments we shall separate the class of transcendental meromorphic functions for further reference into three subclasses:

- E = {k : k is transcendental entire}
- P = {k : k is transcendental meromorphic, has exactly one pole, and this pole is an omitted value }.
- M = {k: k is transcendental meromorphic and has either at least two poles or exactly one pole which is not an omitted value}.

Here E and P consideration of as m for entire and meromorphic functions, while k stands for one pole As already mentioned, we may assume that functions in k have the form (1).

Definition 2.

Let R be an integral domain. Then R is a unique factorization domain if and only if the above conditions below are both satisfied:

- For all m∈ R such that m≠R and a is not a unit, there is a factorisation of a in to irreducible in R.
- Whenever $p_1, p_2, ...p_n = q_1, q_2 ... q_n$ for irreducibles p_i, q_j ∈ R, then n = m and, possiblyafter reordering the qj, pi and qi are associates for every 1 ≤i ≤ n.

Theorem 1

If f is rational, f € k, or f ∈l, then K(f) = K(f^n) and J(f) = J(f^n) for all n>2.

Proof Here we have to exclude f∈K, because then f^n is not meromorphic in C so that F(f") and J(f") are not defined. There is, of course, a natural way to define K(f") for f∈ K and n > 2, or, more generally, to define

K(f) for functions f meromorphic in C except for accountably many points.

Theorem 2

K and J are completely invariant.

Proof Here, by definition, a set S is called completely invariant if z € K implies that k (f) ∈ K, unless k (f) is undefined, and that w € K for all w satisfying K (w) = f.

Theorem 3

An entire transcendental function has infinitely many periodic points of period n for all n>2.

Proof The idea of the proof is similar. in its place of Picard theorem, nevertheless, Rosen bloom used something stronger, namely, Nevanlinna's theory on the distribution of values, which may be considered as a quantitative version of Picard's theorem. Since it is the only place in this paper where we use Nevanlinna's theory. To prove Theorem 3, we suppose that k^n and hence k have only finitely many fixed points and consider the auxiliary function h defined by

$$h\,(f)=\frac{k^n(f)-f}{k^{n-1}(f)-f}$$

Then $$N(t,h)=0(T(t,k^{n-1}))$$

$$N(t,\tfrac{1}{h})=0(\log t)$$

And $$N(t,\tfrac{1}{h-1})=0(T(t,k^{n-1}))$$

As, t→ ∞. Also, it is not difficult to prove that T(t, k^{n-1}) = o{T(t,f^n)) as t → ∞ outside some exceptional set of finite measure. In fact, this last result even holds without exceptional set. We deduce that T(t, h) ~0(T(t, f^n,) and hence that

$$N\,(t,h) + N(r,\tfrac{1}{h}) + N(t,\tfrac{1}{h-1}) \equiv 0(T(t,h))$$

As, t→ ∞ outside the exceptional set. This contradicts Nevanlinna's second fundamental theorem. Thus the proof of Theorem 3 is complete.

One may ask whether there is some quantitative version of Theorem 3 in the sense that there is a lower bound for N (t, l/(k^n(f) - f)) in terms of T(t, k^n) if n > 2 and f ∈L. Denote by δ(a, h) the deficiency of a meromorphic h with respect to the value a ∈ C.

Example 1.

Consider the family

$$F=\{\tfrac{1}{2mf};f\in N\}$$

On the open unit disk D, and let φ(f) = 1/f and ψ(f)≡0. Then clearly, for every f, g∈ F,f and g share φ(f), and k(f)and g(f)share ψ(f) in D. However, the family F is not normal in D. This shows that φ cannot be taken meromorphic in Theorem 3.

Further, for the same family F, if we take φ(f)≡0 and ψ(f) = $\frac{1}{f^{k+1}}$, then for every f, g∈ F ,f and g share φ(f), and k(f)and g(f)share ψ(f) in D. But, the family F is not normal in D.This shows that the condition in Theorem 3 is essential

Example 2.

Consider the family

F=$\{f^{k+1}+\frac{1}{mf}:m\in N\}$F is not on the open unit disk D, and let φ(f) = $f^{k+1}$and ψ(f) = (k+ 1)!f. Then clearly, for every f, g∈ F,f and g share φ(f), and k(f)and g(f)share ψ(f) in D. However the family normal in D.

II.MOTIVATION

We know that every integer number is the product of prime numbersin a unique way. Sort of. We just believed our primary teacher when she told us, and we omitted the fact that it needed to be proven. We want to prove that this is true, that something similar is true in the ring of polynomials over a field. More generally, in which domain is this true? In which domains does this fail?

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