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A Study on Distance Pattern of Vertices in a Graph

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***Abstract* – This paper discuss about the distance pattern of vertices in a graph. The distance between two vertices is the basis of the definition of this graph parameter. There are several distance related concepts and parameters. We also present several fundamental results on open distance pattern uniform graphs and open distance pattern uniform number of a graph.**

***Keywords* – All basic definitions, Distance, Metric, Geodesic, Eccentricity, Radius. Self-centered graph, Coloring, Labeling. etc**

I. INTRODUCTION

In recent years, there is an increased demand for the application of mathematics. Graph theory has proven to be particularly useful toa large number of rather diverse fields. Graph theory is considered to have begun in 1736 with the publication of Euler's solution of the Konigsberg bridge problem.

A Graph theory gives mathematical operations and ideas with which many of these properties can be quantified and measured. Given this vocabulary and these mathematics, graph theory gives the ability to prove theorems and hence about representation of social structure and networking.

Chromatic theory goes back to a problem, posed some 140 years ago, relating to the coloring of maps, either real or imaginary.A graph G is called a Smarandachely uniform k-graph if there exist subsets $M_1, M_2, \ldots\ldots\ldots\ M_K$for an integer k ≥ 1 such tat ${f^0}_{M_i}(u) = {f^0}_{M_j}(u)$ and ${f^0}_{M_i}(u) = {f^0}_{M_i}(v)$ for 1 ≤ i, j ≤ k and ∀u,v∈V(G).

Such subsets$M_1, M_2, \ldots\ldots\ldots\ M_K$f are called a k-family of open distance-pattern uniform (odpu) set of G and the minimum cardinality of odpu-sets in G, if they exist, is called the Smarandachely odpu-number of G, denoted by ${od^s}_k$ (G). Usually, a Smarandachely uniform 1-graph G is called an open distance-pattern uniform (odpu-) graph. In this case, its odpu-number ${od^s}_k$ (G) of G is abbreviated to odd (G).

II. BASIC DEFINITIONS

1. Graph- A graph G is a finite nonempty set V of objects called vertices together with a set E of 2-element subsets of V called edges. Each edge {u, v} of V is commonly denoted by uv or vu.G (V; E) has vertex set V and edge set E. The number of vertices in a graph G is called its order, and the number of edges its size.

2.Subgraphs -Let G be a graph, then a graph H is said to be a subgraph of G if V (H) ⊆ V (G) and E(H) ⊆ E(G). If H is a sub graph of G, then we write H⊆*G*.

3.Cycle - A cycleis a closed walk of at least three vertices in which all edges are distinct and all vertices, except the beginning and ending vertex are distinct.

4.A cyclic - A graph contains no cycle is termed acyclic.

5.Circuit - A circuitin a graph *G* is a closed trail of length of 3 or more.

6.Eulerian circuit - A circuit *C* in a graph *G* is called an Eulerian circuitif *C* contains every edge of G.

7.Eulerian graph - A connected graph that contains an Eulerian circuit is called an Eulerian graph.

8.Hamiltonian cycle- A cycle *C* in a graph *G* that contains every vertex of *G* is called a Hamiltonian cycleof G.

9.Hamiltonian graph - A Hamiltonian graph is a graph that contains a Hamiltonian cycle.

10.Connected graphs and components -A graph is connectedif there is a path between every pair of vertices of *G*. If a graph is not connected, then it is disconnected.

11.Distance- The Distance $d(u;v)$between two vertices *u* and *v* in *G* is the length of the shortest path joining them if any other wised (u;v)=*1*.

12.Metric - Ina connected graph, the distance is a metric.

13.Geodesic - A shortest *u* - *v* path is called a *u* - *v* geodesic.

14.Eccentricity - The eccentricity, $e(v)$ of a vertex *v* in a connected graph *G* is the distance between *v* and a vertex farthest from *v* in *G*.

15.Radius- The radius,$r(G)$ ofa graph *G* is the minimum eccentricity of the vertices of *G*.

16.Self-centered graph -A vertex *v* is a central vertex of *G* if $e(v) = r(G)$ and the center of *G* is the sub graph induced by the set of all central vertices. If every vertex of *G*is a central vertex, then *G* is called a self-centered graph. The diameter, $d(G)$ of a connected graph *G* is the maximum eccentricity among the vertices in *G*

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17. Coloring Many of the concepts, theorems, and problems of coloring hairy lies in the shadows of the Four Color Problem. The problems in graph colorings that have received the most attention involve coloring they entices of a graph.

Furthermore, the problems in vertex colorings that have been studied most often are those referred to as proper vertex colorings. Beginning with the origin of the Four Color Problem in1852, research on graph colorings has developed into one of the most popular are of graph theory. A proper vertex coloring of a graph *G* is an assignment of colors to the vertices of *G;* one color to each vertex, so that adjacent vertices are colored differently. In a k-coloring, we may assume that it is the colors1being used.

While all *k* colors are typically New Directions in Theory of Set libeling of Graphs and related topics used in *k*-coloring of a graph, there are occasions when only some of the *k*-colors are used. The chromatic number of a graph *G;* denoted byχ (*G*)*;* is the minimum *k* for which G has *k*-coloring. We list below some important theorems on vertex coloring of graphs.

18.Labeling Formally, given a graph *G*, a vertex labeling is a function, mapping vertices of *G* to a set of labels. A graph with such a function defined New Directions in Theory of Set libeling of Graphs and related topics is called a vertex-labeled graph. Likewise, an edge labeling is a function, mapping edges of *G* to a set of labels.

III.DISTANCE PATTERNS OF VERTICES IN A GRAPH

Several authors have developed fundamental results on these concepts. The relevance of distance labeling schemes in the context of communication networks has been pointed out in and illustrated by presenting an application of such labeling schemes to distributed connection setup procedures in circuit-switched networks. It seems very plausible that distance labeling schemes may be use full also in the design of memory free routing schemes, which are routing schemes geared towards supporting architectures based on very fast and simple switches, allowed to store very little data locally. Some other problems where distance labeling schemes may play an active role are bounded broadcast protocols and topology update mechanisms.

The concept of open distance-pattern and open distance pattern uniform graphs was studied in given an arbitrary nonempty subset M of vertices in a graph G= (V, E) the open M-distance pattern of a vertex u in Gis defined to be the set $f^0{}_M(u) = \{d(u,v): v \in M, u \neq v\}$ $, where\ d(x,y)$Denotes the distance between the vertices x and y in G. If there exists a nonempty set $M \subseteq v(G)\ such\ that\ f^0{}_M(u)$ is independent of the choice of u, then G is called open distance-pattern uniform (odpu) graph and the set M is called an open distance pattern uniform.

1.Dpd-Graph

Definition- Let *G* (*V; E*) be a given connected simple (*p; q*) graph with diameter d_G, $\emptyset \neq$ M $\subseteq V(G)$and $u \in V$ (*G*). Then, the M-distance-pattern of *u* is the set$f_M(u) = \{\, d(u,v) : v\ \in M\,\}$If $f_M : u \mapsto f_M\,(M)$ is an injective function then the set *M* is a distance-pattern distinguishing *set* (or, a 'dpd-set' in short) of *G*. A graph *G* with a dpd-set is called a distance-pattern distinguishing graph (dpd - graph).

Theorem 1.A graph G has a dpd set of cardinality 1 if and only i

Fig.1 G is a path.

2.Odpu-graph

Definition- Associate with each vertex u of a graph *G* =(*V; E*), its open A-distance pattern(or, 'odp' in short) $f^0{}_A(u) = \{\, d(u,v): v\ \in A, u \neq v\,\}$such that every vertex has the same open distance pattern; such graphs are called odp-uniform graphs(or, simply, 'odpugraphs'), where the set-valued function (or, set-valuation) $f^0{}_A(u)$is called the open distance pattern uniform (or, a odpu)- function and *A* is called an odpu-set of G.

3.Odpc-Graph

Definition- Given a connected (*p; q*)-graph *G*(*V; E*) of diameter(*G*)*;* let *X* = {1, 2, 3, … *d*(*G*)} and $\emptyset \neq$ M $\subseteq V(G)$, u∈ *M*a nonempty set of colors of cardinality *d*(*G*). Let $f^0{}_M$be an assignmentof subsets of *X* to the vertices of *G;* such that $f^0{}_M(u) = \{\, d(u,v): v \in M, u \neq v\,\}$where *d*(*u; v*) is the usual distance between *u* and *v:* We call$f^0{}_M$an *M* open distance pattern coloring of *G*, if no two adjacent vertices have same $f^0{}_M$and if such an *M* exists for a graph *G* ,then *G*is called an *M* open distance pattern colorable graph (in short odpcgraph).The main aim of this chapter is to present an account of odpc graphs and their corresponding odpc labeling

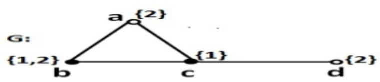

Fig.2 An odpc graph

Theorem 2. Trees are open distance pattern colorable.

Proof-Let *T* be a tree. Then, the center of *T* is either K_1 or K_2.

Case 1. Center of *T*isK_1.Let *x* be the center of *T:* Consider *T* as a rooted tree with root vertex as *x:* Let n_i ; $i = 1, 2, \ldots, h(T)$ (where *h*(*T*), is the maximum distance of a vertex from the root), denote the number of vertices in the i^{th} level. Let v_{ij}, $j = 1, 2, \ldots n_i$ denote the vertices in the i^{th} level. Let $M = \{v_{11}, v_{12}\}$. Then $f^0{}_M(x) = \{1\}$ *and* $f^0{}_M(v_{ij}) = \{2\}$ for $j = 1, 2, 3, \ldots n_1$. For $i \geq 2$, and for each $j = 1, 2, 3, \ldots n_1$, $f^0{}_M(v_{ij}) = \{(i-1), (i+1)\}$, if v_{ij} is a descendent vertex from a vertex of M, and $f^0{}_M(v_{ij}) = \{(i+1)\}$, otherwise.

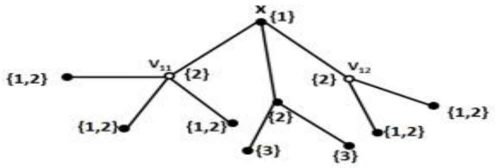

Fig.3 An odpc graph tree T, with center K_1

Case 2. Center of T is K_2. Let $V(K_2) = \{x, y\}$. Choose x as the root vertex of T and $M = \{y, v\}$ for some vertex v such that $xv \in E(T)$. $f^0{}_M(x) = \{1\}$, $f^0{}_M(y) = \{2\}$, $f^0{}_M(v) = \{2\}$, $f^0{}_M(w) = \{d(y,w), d(y,w)+2\}$ if w is a descendent from y, $f^0{}_M(w) = \{d(v,w), d(v,w)+2\}$ if w is a descendent from v, and $f^0{}_M(w) = \{d(x,w), +2\}$ if w is a descendent from x. Hence M is an open distance pattern color set of T.

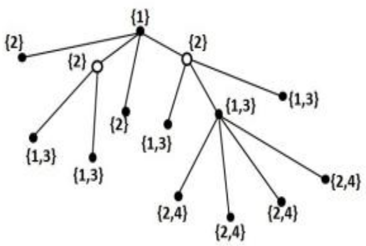

Fig.4 An odpc graph tree T, with center K_2

4. Odpu-Number of a Graph

Definition- The Odpu-number of a graph G, denoted by odd (G), is the minimum cardinality of an odpu-set in G.

Theorem 3. There is no graph with odpu-number three.

Proof: Suppose there exists a graph G with od(G) = 3 and let M = {x, y, z} be an odpu-set in G. Since G is connected, $1 \in f^0{}_M(x) \cap f^0{}_M(y) \cap f^0{}_M(z)$. We claim that x, y, z form a triangle in G. Since $1 \in f^0{}_M(x)$, and $1 \in f^0{}_M(z)$, we may assume that xy, yz $\in$ E(G).

Now if xz $\notin$ E(G), then d(x, z) = 2 and hence $2 \in f^0{}_M(x) \cap f^0{}_M(z)$ and $f^0{}_M(y) = \{1\}$, which is not possible. Thus xz $\in$ E(G) and x, y, z forms a triangle in G. Now $f^0{}_M(w) = \{1\}$ for any w $\in$ V (G) -M and hence w is adjacent to all the vertices of M. Thus G is complete and od(G) = 2, which is again a contradiction. Hence there is no graph G with od(G) = 3. Next we prove that the existence of graph with odpu-numbers $k \neq 1, 3$. We need the following definition.

IV. CONCLUSION

The fundamental definition concerning graphs has also been introduced. We define open distance pattern coloring of a graph. We discuss cycle-related odpc graphs. Open distance pattern edge coloring of a graph. Let G be a connected graph with diameter $d(G)$, The characterization of odpu-graphs leads to an interesting for many important classes of graphs such as chordal graphs, interval graphs, split graphs, strongly chordal graphs, self-complementary graphs. we define open distance pattern edge coloring of a graph. Let G be a connected graph with diameter $d(G)$, $X = \{1, 2, 3, \ldots, d(G)\}$ be a non-empty set of colors of cardinality(G).

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