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A Study on Unique Factorization an Integral Domain

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***Abstract* – We take a broad view the notion of "unique factorization domain" in the spirit of "half-factorial domain". We show that new generalization of UFD implies the now well known notion of half factorial domain. As a consequence, we discover that the one of the normal axioms for unique factorization domains is union scientiouslyred undant.That is, we interested in factoring numbers in integral domains so we have to scrutinize distribution, and so this post will begin with a fairly cursory look at the properties of distribution. Then we will introduce the crucial ideas of units and associates. (In the integers, ±1 are units and ±n, for any fixed n, are associates.)**

***Key words*- UFD, Integral Domain, integer number etc.**

I. INTRODUCTION

The notion of unique factorization is one that is central in the study of commutative algebra. A unique factorization domain (UFD) is an integral domain, R, where every nonzero non unit can be factored uniquely. More formally we record the following standard definition. Integral domains in general, but integral domains that are not unique factorization domains (UFDs) in particular.We interested in the outer ring of that diagram.

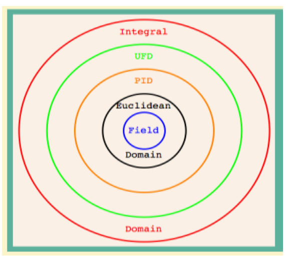

Fig.1Ven Diagram of Integral Domain

Definition 1
Integral domain is irreducible over F field but reducible over ID Suppose that D is an integral domain and F is a field containing ID. If g(x)∈ID[x] and g(x) is irreducible over G but reducible over ID.This doesn't make sense to me, how can ID⊆G and be irreducible in F but reducible in ID. The same polynomials in ID are in G, so any polynomial product representation of G is irreducible.

Example 1 We learning about Rings, commutative rings, IDs, UFDs, etc with each being a subset of the predecessor, and now we find an ID that is not a UFD

A certain integral domain is not a unique factorization domain We to prove the following: R is an ID and let F be its field of fractions. Suppose there exists a monic p(x)∈R[x] such that p(x)=a(x)b(x) where both a,b are monic and non constant polynomials of F[x] but a∉R[x]. Then I need to show that R is not a UFD**.**

Theorem 1 Let R be an integral domain. Every prime element is irreducible.

Proof. Let p be a prime element. We assume that p is reducible and we want to get a contradiction.This mean that we can write p = ab where p is not associate to neither a, nor b. I notice that p/ab. Since p is a prime, this means that p/a or p/b. Without loss of generality I will assume that pja. But now we have that p/a and also a/p. This means p and a are associate. Contradiction.

Example. 2 The converse is not true in general. As an example, consider the ring z[√(-5)].The element 2 is irreducible in R. However,

2/6=(1+√((-5)) i)(1-√(-5) i)

But 2 dose not divide [1+ √(-5)i] and dose not divide [1-√(-5)i]. Hence 2 is not a prime.

Theorem 2 Let R be a domain in which every irreducible element is prime. Then the decompositionof an element as product of irreducible, if it exists, is unique.(Notice that this is not enough to conclude that R is a UFD, since the decomposition as productof irreducible may not exist.)

Proof Let's assume that we have two different decompositionswhere all $p_i$and $q_j$are irreducible. We want to prove that these two decompositions are the same, upto reordering and associates. Now we will proceed by induction on the maximum of n and m.For the base case, if n = m = 1, we have P_1 = Q_1 and we do not need to do anything.

Theorem 3 Let R an integral domain. Assume that x contains an element that is not 0, nota unit, and cannot be written as product of irreducible. Then there exists an infinite sequence

X_0 ,X_1 …..X_N of elements in R such that(X_0) ⊂ (X_1) ⊂(X_2) ⊂(X_3) …..where all the inclusions are strict.

Proof For elements in R, we know that x is a unit iff (x) = R. We know that x and y are associates. Iff (x) = (y).

Moreover, if we can factor x = yz non-trivially (so that y and z are neither units, nor associates to x), then (x) ⊂ (y) and (x) ⊂(z).Pick an element x ∈ R which is not zero, not a unit, and not the product of irreducible. Callx_0= x. Since x is not a product of irreducible, in particular it is not irreducible, so we havea non-trivial factorization x = yz. At least one of y or z is also not a product of irreducible.

Whichever it is, call it x_1. Repeat

Theorem4. S is a saturated multiplicatively closed set.

Proof If x,y∈S then we can write both x and y as product of primes, so x,y can be also written as product of primes. This shows that S is a multiplicatively closed set.

Now, to prove that S is saturated we have to show that if x∈S, then every divisor of x is in S too. If we write x=$up_1, up_2 \ldots up_n$where u is a unit and thep_i's are prime, then it can be shown by induction on n

$$\Longrightarrow (A \cap X_i) \subseteq X_i$$

that every divisor of x is in S. I let you to do this proof by induction.

Now, we argue by contradiction assuming that there is a nonzero element a∈R such that a∉S, so the ideal generated by a, ⟨a⟩, is disjoint from S, i.e., S∩⟨a⟩=∅, because if there were some ra∈S, then a would be in S (because a|ra and S is saturated by the lemma above), contradicting our hypothesis that a∉S.

Therefore the set A={I non zero ideal of R:I∩S=∅}A={I non zero ideal is non-empty and then by Zorn's Lemma A has a maximal element P such that P is not only an ideal, but in fact a prime ideal. By our general hypothesis P contains a prime element, let's say pp, i.e., p∈P, but by the definition of S is clear that p∈S, so p∈P∩S, which contradicts P∩S=∅.

This contradiction comes from our assumption that a∉S.Hence every nonzero a∈R belongs to S, i.e., S=R\{0} and this means that every nonzero, non unit element of R is expressible as a product of primes.

II.SUMMARY

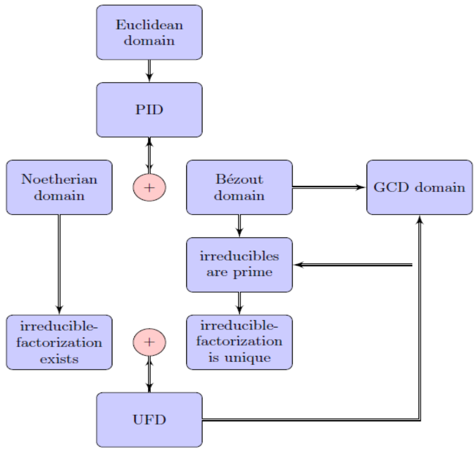

Fig.2 Function Diagram

III. MOTIVATION

We know that every integer number is the product of prime numbers in a unique way. Sort of. We just believed our kinder garden teacher when she told us,and we omitted the fact that it needed to be proven. We want to prove that this is true,that something similar is true in the ring of polynomials over a field. More generally, inwhich domains is this true? In which domains does this fail?

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