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A Study on Normality of Meromorphic Functions

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Abstract **– A heuristic principle attributed to Andr´e Bloch and P. Montel says that a family of holomorphic functions is likely to be normal if there is no nonconstant entire function with this property. We discuss this principle and survey recent results that have been obtained in connection with it. We pay special attention to properties related to exceptional values of derivatives and the existence of fixed points and periodic points, but we also discuss some other instances of the principle.**

***Key words*- Normal families, meromorphic functions, sharing of values**

I. INTRODUCTION

A family of meromorphic functions is called normal if every sequence in the family has a subsequence which converges (locally uniformly withrespect to the spherical metric). The concept of a normal family was introduced already in 1907 by P. Montel but there has been a lot of interest in normal families again in recent years, an important factor being their central role in complex dynamics.We assume that the reader is familiar with the theory of normal families of meromorphic functions on a domain $D \subset R$ one may refer to for more information.

The knowledge of the division of principles was introduced in the study of normality of families of meromorphic functions for the first time by Schwick in 1989.Twofold nonconstant meromorphic functions f and g are said to segment a charge$\alpha \subset R$ IM (ignoringmultiplicities) if f and g have the same α-points counted with ignoring multiplicities. If multiplicities ofα-points of f and g are counted, then f and g are said to share the valueαCM. For deeper insight into thesharing of values by meromorphic functions.

In this paper all meromorphic functions are considered on $D = \{z:|z| < R, 0 < R < \infty\}$ exceptingTheorems. Where the domain is the whole complex plane. A meromorphic function α(z) is g.
said to be a small function of a meromorphic function f(z) if T(r;α) = (T(r; f)) as r →R. further way that a meromorphic function f shares a small function αpartially with a meromorphic function g.

II. NORMALITY AND MEROMORPHIC FUNCTIONS

Definition 1
Let F be a family of meromorphic functions in adomain. If F is not normal at a point z_0∈R, there exist a sequence of points $\{z_n\}$ ∈R with $z_n \to z_0$ a sequence of positivenumbers $\rho_0 \to 0$ and a sequence of functions f_0∈ Fsuch that

$$g_n(z) = f_n(z_n(z) + \rho_n(z))$$

Converges locally uniformly with respect to the spherical metric to g(z), where g(z) is anon-constant meromorphic function on C.

Definition 2
Let F be a family of meromorphic functions in a domain D and let a and b be distinct functions holomorphic on D. Suppose that, for any f ∈ F and any z ∈ D, f(z) ≠ α(z) and f(z) ≠ α_0(z). If F is normal in D − {0}, then F is normal in D.

Theorem.1 MARTY'S THEOREM.
A family F of functions meromorphic on D is normal on D if and only if for each compact subset K ⊂D there exists a constant M(K) such that

$$\alpha(z) \leq M(k) \text{for all } z \in F$$

Here αdenotes the spherical derivative

$$\alpha(z = \lim_{c \to 0} \frac{x(f(z+c), f(z))}{|c|}$$
$$= \frac{|f'(z)|}{1+|f(z)|^2}, f(z) \neq \infty$$

Since x(z, w) = x(1/z, 1/w), α = 1/f,which provides a convenient formulafor αat poles off.

Theorem.2. Let f be a transcendental entire function and P(z) be a polynomialof degree at least 2. Then

$$\lim_{r\to\infty} sub \frac{(r, 1/(P°f - z))}{T(r, f)} \geq 1$$

J.H. Zheng and C.C. Yang and M. Fang and W. Yuan proved the followingtwo results on the value distribution of composite meromorphic function.

Theorem.3 Let f be a transcendental entire function, P(z) a polynomial of degree at least 2, and α(z) a nonconstant meromorphic function satisfying T(r; α) =S(r; f). Then

$$T(r,f)\leq k\bar{N}\left(r,\frac{1}{p0f}\right)+S(r,f).$$

Here k $=2/(deg\,p-1)$ if $p'(z)$ has only one zero, otherwise k = 2.

Theorem.4 Let f be a transcendental entire function, P(z) a polynomial of degree at least 2, and α(z) a meromorphic function satisfying T(r; α) = S(r; f). Ifα (z) is a constant and there exists a constant $\alpha_0 \neq \alpha$ such that P(z) -$\alpha_0$has azero of multiplicity at least 2. Then

$$T(r,f)\leq k\bar{N}\left(r,\frac{1}{p0f}\right)+S(r,f).$$

Here k = $1/(deg\,p-1)$if $p'(z)$ has only one zero, otherwise k = 1.

Theorem.5 If G is a family of holomorphic functions in a domain D, P(z) is apolynomial of degree at least 2, α(z) is a holomorphic function such that P(z)-α(z)has at least two distinct zeros, and if P°f(z) ≠ α(z) for each f ∈ G, then G isnormal in D.

Proof If G is a family of holomorphic functions in a domain D, R(z) is a rational function of degree at least 2, α(z) is a nonconstant meromorphic function, and if R°f(z) ≠ α(z) for each f ∈ G, then G is normal in D. W. Yuan, Z. Li and B. Xiao further better-quality this result and proved: If α(z) is a nonconstant meromorphic function, R(z) is a rational function of degree at least 2, and R° f and R°g share α(z) IM for all f(z),g(z) ∈G, then G is normal in D if one of the following conditions holds:

- R(z) -α(z0) has at least two distinct zeros or poles for any z_0 ∈D

There exists z_0 ∈D such that R(z) -α (z0)= P(z)/Q(z) has only one distinctzero (or pole) β_0 and suppose that the multiplicities l and k of zeros of f(z)-$\beta_0$andp(z) -α (z0) at z0, respectively, satisfy k ≠lp (or k ≠ lq), for each f ∈ F,where P and Q are two co-prime polynomials with degree p and q respectively.

Theorem.6 Let α(z) be a holomorphic function, G be a family of meromorphicfunctions in a domain D and P(z) be a polynomial of degree at least 3. If P°f(z)and P°g(z) share α(z) IM for each pair f; g∈ G and one of the following conditionsholds:

- R(z) -α (z_0) has at least two distinct zeros or poles for any z_0 ∈D:
- There exists z_0 ∈D such that P(z) -α (z_0) has at most two distinct zeros and α(z) is nonconstant.

Assume thatβ_0 is the zero of P(z) -α(z_0) with multiplicity p and that the multipliers l and k of zeroes of f(z) - (β_0) and f(z)-$\beta_0$and α(z) -α (z_0) at z_0, respectively, satisfy k ≠lp for all f∈ G.

Theorem.7 Let α(z) be a holomorphic function and G a family of holomorphicfunctions in a domain D. Supposes further Pw= P(z, αw) = (w - a1(z))(w - a2(z)) ….(w -p(z)),

wherea_i(z); i = 1, 2…. p with p ≥ 2, are holomorphic functions in domain D.If p_α°f(z) and p_α°g(z) share α(z) IM for each pair f,g ∈G and one of thefollowing conditions holds

- P(z_0,w) -α(z_0) has at least two distinct zeros for any z_0 ∈ D.
- There exists z_0 ∈ D such that P(z_0,w) -(z_0) has only one distinct zeroand α(z) is non constant

Assume that $\beta_0$is the zero of P(z_0,w) -α(z_0) andthat the multiplicities l and k of zeros of f(z) - β_0 and α(z) -α (z_0) at $z_0$respectively, satisfy k≠lp, for all f∈G,then G the example is given fellow.

Example1.

Consider the family F={f_n:$n \in N$}, where

$$f_n(z) = nz^{k-1}, k \geq 2$$

in the unit disk D, P(z, w) = (w + e^z)(w -e^z) and α(z) ≡0. Then, for everyfn; fm ∈F, Pw°M[fn](z) and Pw °M[fm](z) share α(z) IM. However, the familyF is not normal in D. Thus, the condition that every f ∈F has zeros of multiplicityat least k.

Example 2

Consider the family F={f_n:$n \in N$}, where

$$f_n(z) = \frac{1}{nz}$$

in the unit disk D, P(z, w) = (w + ize^z)(w-ize^z) and α(z) = z^2e^{2z}. Then

$$P_w{}^\circ M[f_n(z)] = \frac{\prod_{r=1}^{k} r^{2n_r}}{z^{2\gamma m}\,n^{2\gamma m}}+z^2e^{2z}.$$ Then

Clearly, for every fn; fm ∈ F, (Pw °M)[fn](z) and (Pw ° M)[fm](z) share α(z) IMHowever, the family F is not normal in D. Thus, the condition that P(z_0,w)-α(C)has at least two distinct zeros for any z_0 ∈D.

III. CONCLUSIONS

Though our results do generalize and improve the results of Andr´e Bloch and P. Montel. When thedomain $D = \{z{:}|z| < R, 0 < R < \infty\}$there seems no way of proving our results on an arbitrary domain since the idea of a small function on an arbitrary domain is not available, as far as we know. However, by making certain modifications in the proofs of results of Andr´e Bloch and P. Montel, one can easily extend and improve these results on an arbitrary domain with a shared value being a nonzero complex value.

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