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Perfect Semi Complete Graphs

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***Abstract* – As perfect semi- complete graphs play a vital role in tacking defense problem, a complete study of these graphs gives as overall view to apply them in our practical problems. In this paper a further study about perfect semi-complete graph is made. Path connector set, Edge path connector set, Perfect semi complete graphs are introduced and discussed.**

Keywords – **Complete Graph, Path connector set, semi complete graph etc.**

I. INTRODUCTION

In the earlier papersimple concept in semi complete graph the utility of perfect semi-complete graphs is mentioned. As there is wide application of these graphs in computer and defence problems further useful concepts, namely Path connector set, Edge path connector set, Path-critical edge, neighbourhood set with regard to these graphs are introduced and useful study about these is made.

II. PRELIMINARIES

We first give a few definitions, observation and results that are useful for development in the succeeding articles.

Definition 1. (i) A graph G is said to be sami-complete(SC) iff (if and only if) it is simple and for any two vertices $u,\ v$ of G there is a vertex w of G such that $\{u,\ w,\ v\}$ is a path in G.(ii) A graph G is said to be purely semi-complete if G is semi-complete but not complete.

Definition 2. A graph is perfect if no two vertices have the same degree

Example.1 This is does not satisfy simple graph.

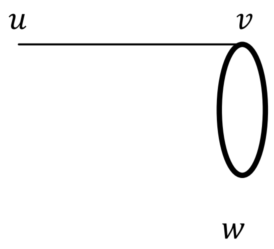

Fig.1 Simple graph.

Definition3. Perfect semi complete graph

Diagram

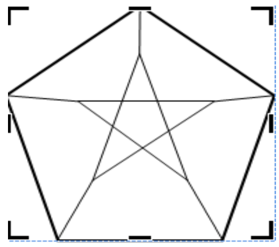

Fig.2 Complete Graph.

Theorem 1 G is a semi- complete graph. then there exists a unique path of length 2 between any two vertices of G if the edge set of G can be partitioned into edge disjoint triangles.

Proof Assume that, G is a semi complete graph then there exists a unique path of length 2 between any two vertices of G. We claim that the edge set of G can be portioned into edge disjoint triangles. Let the graph G is semi complete graph so contain $\{u,\ w,\ v\}$ path in G.Let any two vertices of G is a unique path of length is 2. So assume that four vertices in semi complete graph.Let $u,\ w$ is vertices is a path of length 2.So can be portioned in $\{u,\ w\}$ edge disjoint triangles.

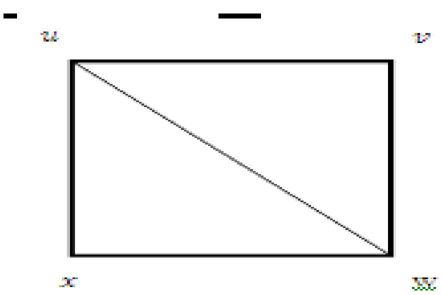

Fig.3 Disjoint triangle.

Conversely, Assume that, The edge set of G can be portioned into edge disjoint triangle. The disjoint triangles can we joined in common edge in graph. Hence that graph is semi-complete graph.

Theorem 2 G is a union of triangles such that no two triangles have a common edge; then all the triangles have a common vertex.

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Proof Let G is a union of triangle such that no two triangles have a common edges.Because G is have a three triangles. So must no two triangles have common edges. And three triangles is also connected. So G is a graph portioned into three triangles is must have a common vertex.

Example 2

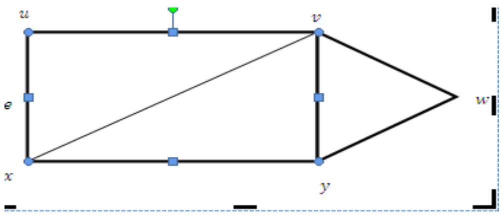

Fig.4 Semi complete graph.

Definition 4 A semi-complete (SC) graph G is said to be strong semi-complete (S.S.C) if there is at least one edge of G whose removal from G does not affect the semi-complete property (i.e it result in a semi-complete graph).A characterization result for a semi-complete graph to be strong semi-complete graph is the following:

Example 3

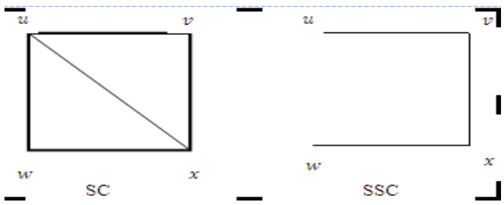

Fig.5 Strong Semi complete graph.

Theorem 3 A semi-complete graph G is strong semi-complete if there is an edge uv of G such that there there are at least two paths of length 2 from u to any point of N(v) - {u} and v to any point of N(u) - {v}.

E.Sampath Kumar [4] introduced the concept of neighbourhood sets as follows:

Definition 5

- A set S of vertices in a graph G is said to be a neighbourhood set of G iff$G = \cup_{v \in S} < N[v] >$, where $< N[v] >$ is the subgraph of G induced by "v" and all its neighbours (adjacent vertices) in G.For convenience, a neighbourhood set of G is referred as n-set of G.Since the vertex set of G is itself an n-set of G, there is no intrest to discuss about maximum n-set in a graph.
- The minimum amoung the cardinalities of all n-sets in a graph G is called the neighbourhood number of G and is denoted by n(G).

A characterization result for a subset of the vertex set of a graph to be an n-set is the following:

Result 1 A subset S of the vertex set V is an n-set of G iff each edge in $< V - S >$ (the subgraph induced by V-S inG) is in $< N[v] >$ for some $v \in S$.

To avoid trivialites, we consider only nonempty graphs. Now we introduce path connector set in a graphs.

III.PATH CONNECTOR SET

Definition 1

- A Path connector set(pc-set) in a graph G is a subset V' of the vertex set V of G such that for any distint pair of non-adjacent vertices in G there is shortest path whose internal vertices are from V'
- A path connector set in G is said to be a minimum path connector set (mpc-set) in G if (if and only if) it has the minimum cardinality amount all the pc-sets in G.

Example1 For the graph given in Figure {v_3, v_5, v6,v3, v5, v8 are mpc-sets.

Observations 1

- As there are no non-adjacent vertices in the complete graphK_n, it follows that any subset of the vertex set of K_n is a pc-set. In particular, the empty set is also a pc-set(infact mpc-set). So there is no interest in complete graphs with regards to this aspect.
- As there are at least two non-adjacent vertices in a disconnected graph such that there is no path between them it follows that pc-sets do not exist for such graphs. Clearly

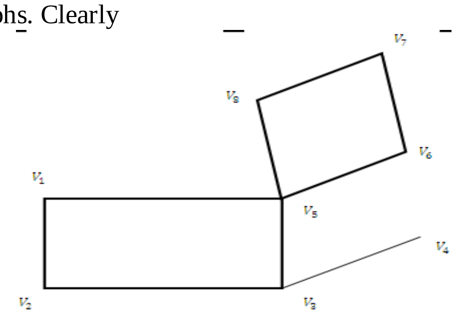

Fig.6 Non empty graph.

Result 1 A non empty graph is connected if it admits pc-sets.

Proof For, if G is such a graph its vertex set itself is a pc-set(so there is no interest to discuss about maximum pc-sets).

Conversely, if G admits pc-sets, then by definition, it follows that G is connected.

Note Any nonempty connected graph admits mpc-set. For, if V is the vertex set of G then ϱ. Hence ϱ admits an element S with minimum cardinality ⇒ S is ampc-set in G.

Theorem 1(Characterization Result) G is a purely semi-complete graph with vertex set V. Then $S \subseteq V$ is a pc-set in G iff for every distinct pair of non-adjacent vertices u and v in G there is a w ∈ S which is adjacent to both u and v in G.

Proof Since G is semi-complete there is a path of length two between any two vertices in G. Then the shortest path between any two non-adjacent vertices is of length two in such a graph. If S is a pc-set in G, by definition, follows the necessary part. Conversely, if S has the property stated then clearly S is a pc-set in G.

Theorem 2 G is a purely semi-complete graph with vertex set V. Then

- Any pc-set in G is a dominating set in G.
- Further, if $|S| \geq 2$ then S is a total dominating set in G.

Proof Since any semi-complete graph is connected, it follows that the graph G admits a nonempty pc-set, say S. If S is singleton say $\{v_0\}$, then since G is semi-complete follows that every vertex of G is adjacent with v_0. Thus S is a dominating set in G. Now, assume that $|S| \geq 2$. Let $u \in V$ and $v \in S - \{u\}$. If u is adjacent to v then we are through; otherwise since G is semi-complete, there is a $w \in S$ such that $\{u,\ w,\ v\}$ is a shortest path in G. Now u is adjacent to $w \in S \Rightarrow$ S is a total dominating set in G.

This completes the proof of the theorem.

Observation 2

- If S of the above theorem has exactly two elements (vertices) then they are adjacent in G.
- S of the above theorem is an independent set if$|S| = 1$.

Remark 1 The converse of theorem is true if the cardinality of the dominating set is 1.

For, that single vertex set is clearly a pc-set (in fact a mpc- set) in G.

If the cardinality of the dominating set is >1 then it need not be a pc-set in view of the following:

Example 2 Consider the following graph G in figure 2

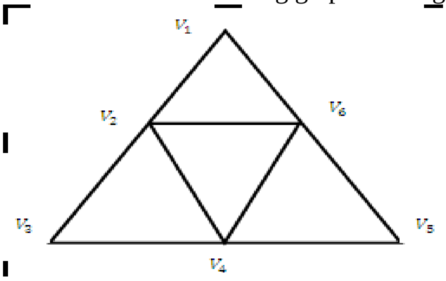

Fig.7 purely semi-complete graph.

Clearly $\{v_2, v_6\}$ is a (total) dominating set in G; but this is not a pc-set in G, since there is only one shortest path between $v_3$and v_5, namely $\{v_3,\ v_4,\ v_5\}$ and v_4 is not in $\{v_2, v_6\}$.

Infact, $\{v_2,\ v_4,\ v_6\}$ is a pc-set (further mpc-set) in G.

Theorem 3 G is purely semi-complete graph with n vertices. Then the domination number$\gamma(G) = 1 \Leftrightarrow |mpcs(G)| = 1$.

Proof Since G is purely semi-complete, it follows that $n \geq 4$. Let $\gamma(G) = 1$. So there is a $v_0 \in V(G)$ such that $d_G(v_0) = n - 1$. Denote $S = \{v_0\}$. Let $v_1, v_2 \in V(G)$ such that v_1 and v_2 are not adjacent in G. Now follows that $\{v_1,\ v_0,\ v_2\}$ is a shortest $v_1 - v_2$path in $G \Rightarrow S$ is a pc-set in G. Since $|S| = 1$, it follows that S is a mpc-set in $G \Rightarrow |mpcs(G)| = 1$

Conversely, assume that $|mpcs(G)| = 1$. So there is a pc-set S in G with $|S| = 1$. Now, by theorem, it follows that S is a dominating set in G $\Rightarrow \gamma(G) = 1$.

This completes the proof of the theorem.

Observations 3

- From Theorem and observation, it follows that for any graph G,$\gamma(G) = 1 \Leftrightarrow$ any mpc-set in G is an independent set in G.
- From Theorem, Remark and Theorem, we haveA purely semi-complete graph is a union of triangles, where all the triangles have a common vertex if$|mpcs(G)| = 1 \Leftrightarrow$ any mpc-set in G is an independent set in G.
- If G is a semi-complete graph such that there is a unique path of length twobetween every pair of non-adjacent vertices in G, then $|mpcs(G)| = 1 \Rightarrow$ there isa unique mpc-set and it is independent set in G.

For, by theorem, it follows that, the edge set of G is a union of edge disjoint triangles where all the triangles have a common vertex $\Rightarrow \gamma(G) = 1. \Leftrightarrow |mpcs(G)| = 1$.

The converse of (iii) is false in view of the following:

Example 3 Consider the following graph G in Fig. 3

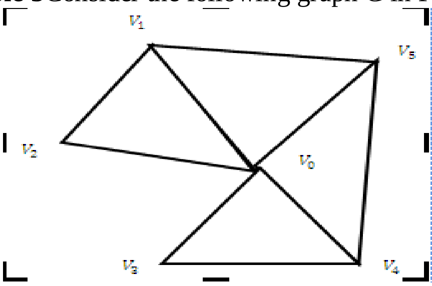

Fig.8 Two Paths.

$\{v_0\}$is a mpc-set of G. But there are two paths namely $\{v_2,\ v_0,\ v_5\}$, $\{v_2,\ v_1,\ v_5\}$ between the pair $v_2,\ v_5$ of non-adjacent vertices in G.

Theorem 4 G is a semi-complete graph such that $|mpcs(G)| = 2$. Then $\gamma(G) = 2$.

Proof Under the given hypothesis and Theorem, it follows that $\gamma(G) \geq 2$ by th follows that there is a dominating set with 2 elements; so $\gamma(G) \leq 2$. Hence $\gamma(G) = 2$.

The converse of the above theorem is false in view of the following:

Example 4 For the graph given in Remark, $\{v_2,\ v_6\}$ is a minimum dominating set and so $\gamma(G) = 2 \neq 3 = |mpcs(G)|$.

Theorem 5 G be a semi-complete graph which has a cut-vertex, say v_0. Then $\{v_0\}$ is a mpc-set in G($\Rightarrow |mpcs(G)| = 1$).

Proof By hypothesis follows that v_0 is adjacent to all the other vertices in G$\Rightarrow \{v_0\}$ is a pc-set in G $\Rightarrow$it is an mpc-set in G($|mpcs(G)| = 1$).

The converse of theorem is false in view of the following example.

IV. CONCLUSION

As perfect semi- complete graphs play a vital role in tacking defence problem, a complete study of these graphs gives as overall view to apply them in our practical problems. Thus a continuous study about these graphs is made.

REFERENCES

[1]. J.A.Bondy& U.S.R. Murthy, Graph theory with Applications, The Macmillan Press Ltd, (1976)
[2]. H.Naga Raja Rao, S.V.Siva Rama RajuSemi-complete Graphs, IJCC, vol.7(3) (2009), 50-54.
[3]. I.H.Naga Raja Rao, S.V.Siva Rama RajuSemi-complete Graphs – II, IJCC.vol.8(3) (2010), 61-66.
[4]. E.Sampathkumar, P.S.Neeralagi, On Neighbourhood Sets, Indian J.PureAPpl.Math., 16(2):126 (1985).
[5]. Douglas B. West, Introduction to Graph theory, Pearson Education, Singapore (2002