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A Chaos Control Method with Analysis of Fractional Chaotic System

Ayub Khan [1], Pushali Trikha [1], Lone Seth Jahanzaib[1]
[1] Basic Sciences Research Centre, Imam Mohammad Ibn Saudi Islamic University, K.S.A.
[2] Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University, Punjab 144001, India.
[3] Department of Mathematical Sciences, Islamic University of Science and Technology, Awantipora Pulwama, Jammu and Kashmir, India.

Abstract- The paper introduces an effective way to control chaos of a fractional chaotic system in presence of uncertainties tc. Theoretical claims are verified numerically using MATLAB software.

Keywords: Chaos control; fractional order; Lyapunov diagrams; bifurcation diagrams. and disturbances using adaptive sliding method. The paper also analyzes the fractional system by using dynamical tools of phase portraits, bifurca- tion diagrams, Lyapunov diagrams.

I. INTRODUCTION

Many people are still impressed by the use of fractional operators in simulating real-world issues. Numerous studies have been conducted in the literature, including fractional oper- ators in science and engineering, mathematical physics, physics and numerous other fields [2, 6, 7].Nowadays, fractional operators are favored for modeling rather than integer-order derivatives since the literature notes that fractional operators take memory effects and extend the concept of the classical derivative.

Because it may also be applied to the classical begin-ning condition employed in physics, the Caputo derivative attracts increased attention.The initial circumstances of the model problem should be in integral form, which means that the Riemann-Liouville derivative is not a physical instrument. In physics, having a starting condi- tion in integral form is the biggest annoyance. Fractional-order derivative modeling of chaotic systems has garnered attention throughout the past ten years.Chaos theory start to receive attractions in a fractional environment. The rationale is that we can either preserve or lose the chaotic characteristics of the systems by varying the order of the employed derivative. As can be shown in [3, 4, 14, 15, 16], several novel attractors are also born with the frac- tional operators. The terms used to describe the chaos's characteristics are another factor. For instance, hyperchaotic systems with a single positive Lyapunov exponent can occur in a fractional setting.

We first provide a brief overview of the literature on modeling chaotic systems with and without fractional-order derivative, before moving on to the motives for the research discussed in this study. As can be seen in the following studies [17, 18], Sene et al. have recently intro- duced a number of fractional chaotic systems via Caputo derivative and studied their phase portraits, bifurcations diagrams, Lyapunov exponents, sensitivity to the initial conditions, and influence of the fractional orders in obtaining new types of attractors. Pacheco et al. introduce and study fractional chaotic systems with hidden and self-excited attractors in [20]. In [19], Akgul uses a fractional operator memcapacitor to analyze chaotic oscillations.

In re- cent publications [21], the fractional order chaotic systems with non-singular kernels have been proposed. Rajagopal et al. introduced a multi-scroll chaotic system with sigmoid nonlinearity in [22] and suggested the fractional context of their model; they specifically looked into the Caputo fractional derivative and the Adams-Bashforth-Moulton predictorcorrector method to get the phase portraits. Hyperchaotic behaviors is defined as having two positive Lyapunov exponents. However, this description does not appear appropriate in a fractional context, as there are fractional hyperchaotic systems that have one positive Lyapunov exponent.

In this study, we propose to use the fractional operator [10] to represent a chaotic system. We employ the Caputo derivative in our modeling, which is driven by the initial condition. The impact of the Caputo derivative on the dynamics of the chaotic system under

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consideration will be the main topic of this study. Following modeling, we will investigate [8, 13] the impact caused by the bifurcation diagrams [11] on the dynamics of our chaotic system to vary the model's parameters. By figuring out the Lyapunov exponents [5], we will validate the chaotic zone. It will be proven that the system we deem chaotic is also dissipative. Uncertainties and disturbances in the trajectories of the novel fractional order chaotic system are also estimated using adaptive sliding mode technique while controlling chaos in the system.

II. FRACTIONAL CHAOTIC SYSTEM

The fractional chaotic system motivated from [1]is given as:

$$
\begin{aligned}
D^{Q}Z_1 &= DZ_1 - Z_2Z_3 \\
D^{Q}Z_2 &= E - AZ_2 + Z_1Z_3 + CZ_2 \\
D^{Q}Z_3 &= -BZ_3 + Z_1Z_2
\end{aligned}
\qquad (1)
$$

where V = (Z1, Z2, Z3)T ∈ R3 are states of the system with A,B,C,D,E ∈ R as parameters. For A = 9, B = 4, C = 1.5, D = 2.85, E = 0.2 and initial states as (1, 1, 1) with Q as 0.97 the time series with phase portraits are displayed in Fig.1 and Fig.2 respectively. Phase portraits of the system for varying Q are shown in Fig.3.

III. DYNAMICAL ANALYSIS OF SYSTEM

Three dimensional fractional order system is studied with help of some dynamical tools. By varying Q between 0.8 to 1 the changes in system dynamics is observed.

3.1 Symmetry & Dissipative

The three dimensional chaotic system (1) shows rotational symmetry about Z2 axis as system remains invariant under the co-ordinate transformation Z1 → −Z1, Z2 → Z2, Z3 → −Z3.

The fractional order chaotic system is

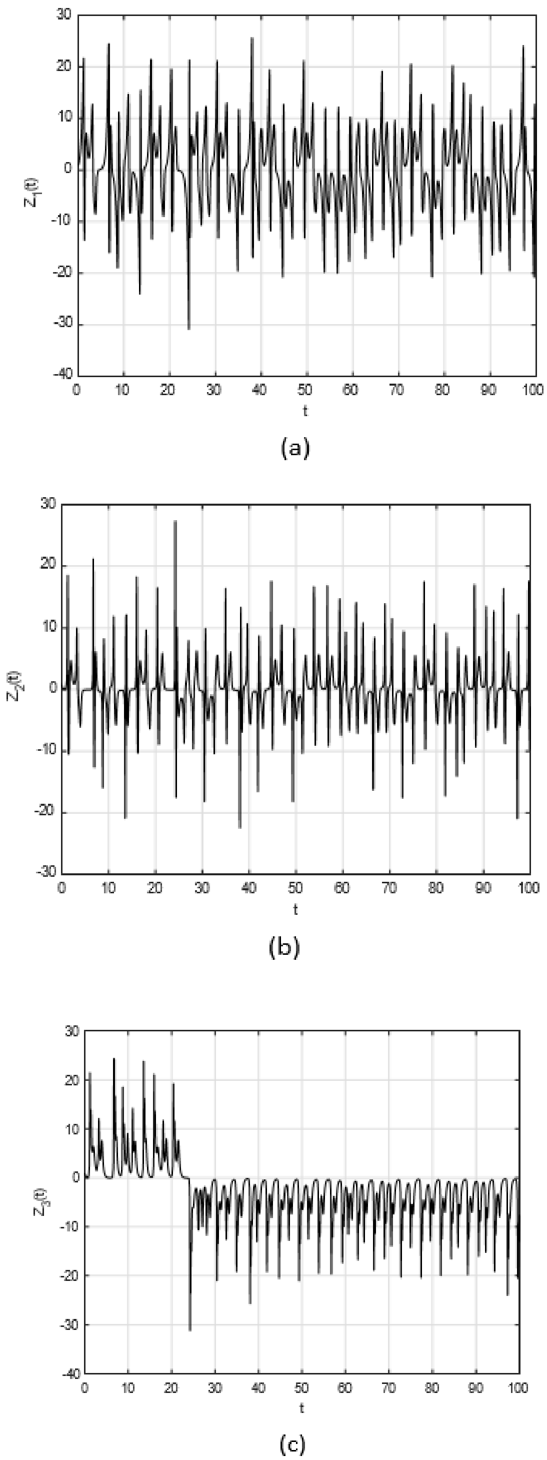

Fig. 1:State Trajectories of (1)

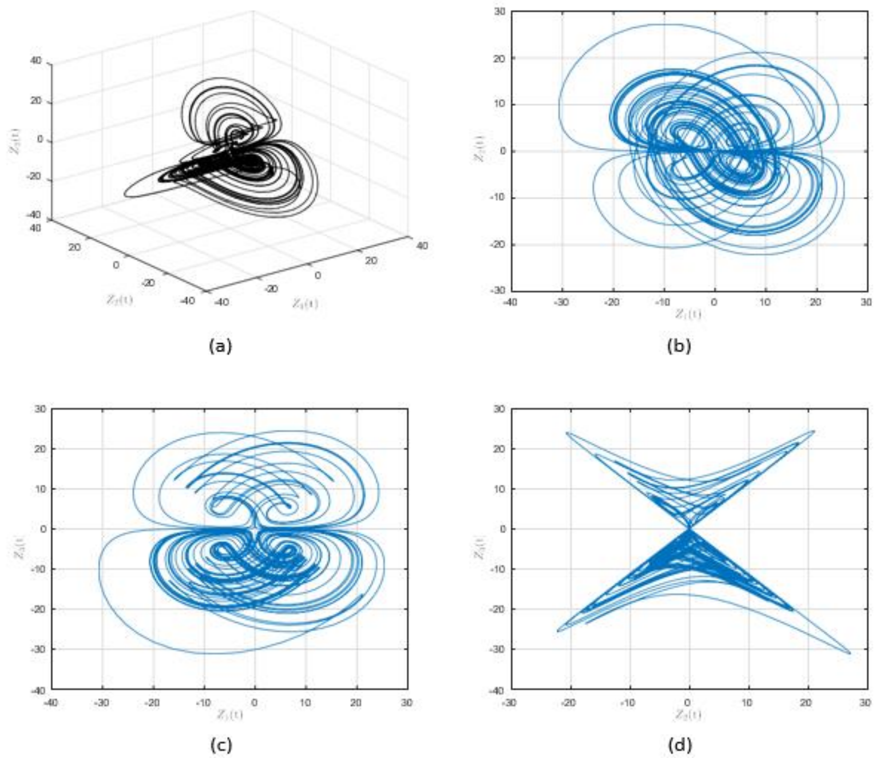

Fig. 2: Phase Portraits of (1)

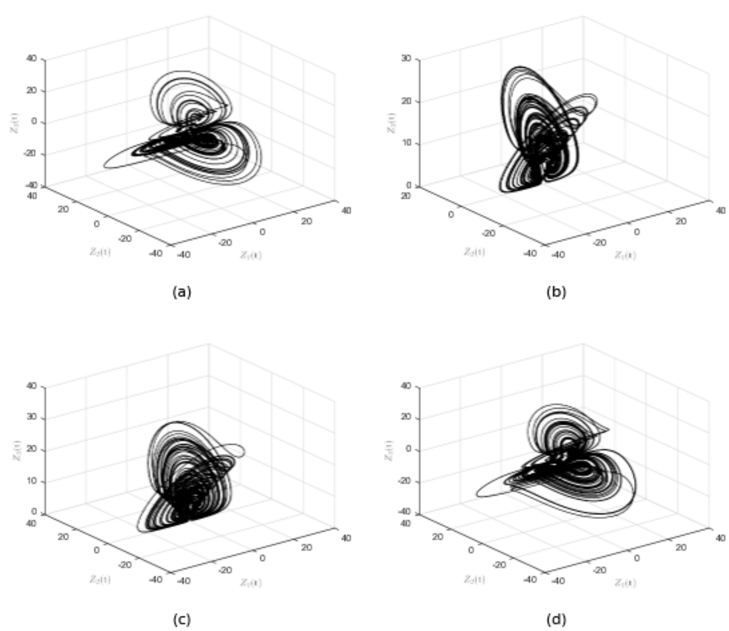

Fig. 3: Phase Portraits of (1) for Q (a)0.97 (b) 0.987 (c)0.99 (d)1

$$\begin{pmatrix} D^q Z_1 \\ D^q Z_2 \\ D^q Z_3 \end{pmatrix} = \begin{pmatrix} K_1(Z_1, Z_2, Z_3) \\ K_2(Z_1, Z_2, Z_3) \\ K_3(Z_1, Z_2, Z_3) \end{pmatrix} = \begin{pmatrix} DZ_1 - Z_2Z_3 \\ E - AZ_2 + Z_1Z_3 + CZ_2 \\ -BZ_3 + Z_1Z_2 \end{pmatrix}$$

Divergence of field K is

i.e. $$\frac{\partial(DZ_1 - Z_2Z_3)}{\partial Z_1} + \frac{\partial(E - AZ_2 + Z_1Z_3 + CZ_2)}{\partial Z_2} + \frac{\partial(-BZ_3 + Z_1Z_2)}{\partial Z_3}$$

$$\nabla K = D - A + C - B < 0$$

i.e. system (1) is dissipative as $\nabla K \neq 0$.

1.1 Solution of System

The chaotic system is

$$D^Q K(t) = \Psi(K(t))$$
$$K(0) = K_o$$

where $t \in (0, T]$.

Here

$$K = \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix}, K_o = \begin{pmatrix} Z_{1o} \\ Z_{2o} \\ Z_{3o} \end{pmatrix}$$

$$\Psi(K(t)) = \begin{pmatrix} DZ_1 - Z_2Z_3 \\ E - AZ_2 + Z_1Z_3 + CZ_2 \\ -BZ_3 + Z_1Z_2 \end{pmatrix}.$$

Solution of the fractional order chaotic system in $\omega \times I$, where I=(0,T] and $\omega = (V) : max|K| \leq P$ for $i = 1, 2, 3, P > 0$.

The I.V.P.

$$K(t) = K_o + \int_0^t \Psi(K(s))ds$$

Let $K_o + \int_0^t \Psi(K(s))ds$ be denoted by S(K) with $Z_1, Z_2, Z_3 \in R^3$

Then

$$S(K_1) - S(K_2) = \int_0^t (\Psi(K_1(s)) - \Psi(K_2(s)))ds$$

Hence

$$|S(K_1) - S(K_2)| = |\int_0^t (\Psi(K_1(s)) - \Psi(K_2(s)))ds|$$

Σ

Sup norm is chosen for $g(t) \in C(0,T]$ is $Sup_{t\in(0,T]}|g(t)|$. and $\|G\| =$

i,j $Sup_{t\in(0,T]}|g_{ij}(t)|$ is chosen for $G = [g_{ij}(t)]$ with $g_{ij}(t) \in C(0,T]$,

$$\Rightarrow \|S(V_1) - S(V_2)\| \leq P_1\|V_1 - V_2\|$$

where $P_1 = Tmax(P,P,P)$.
i.e. for $0 < P_1 < 1$ S(K) is a contraction map.

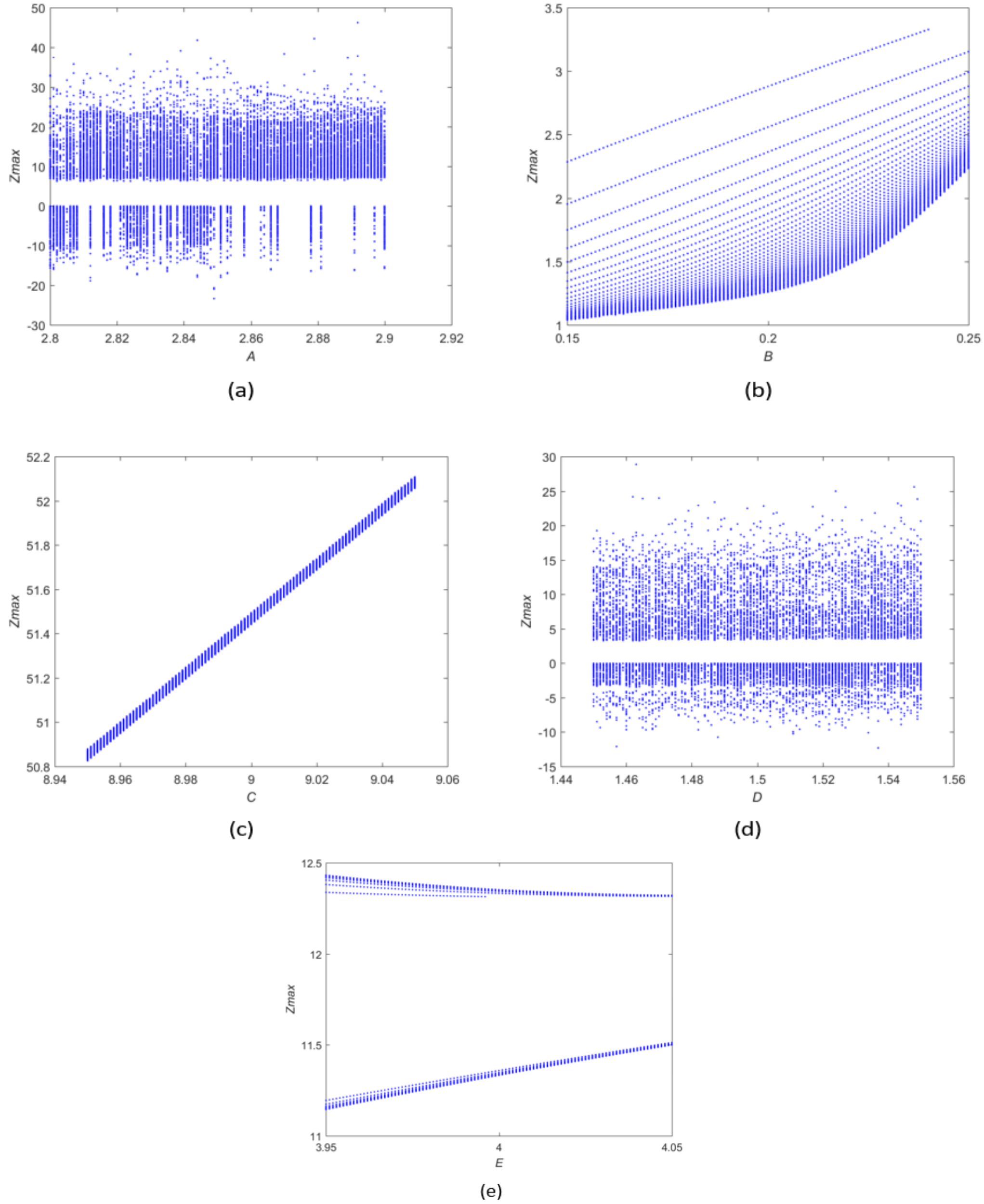

Fig. 4:Bifurcation diagram of (1) for different parameters.

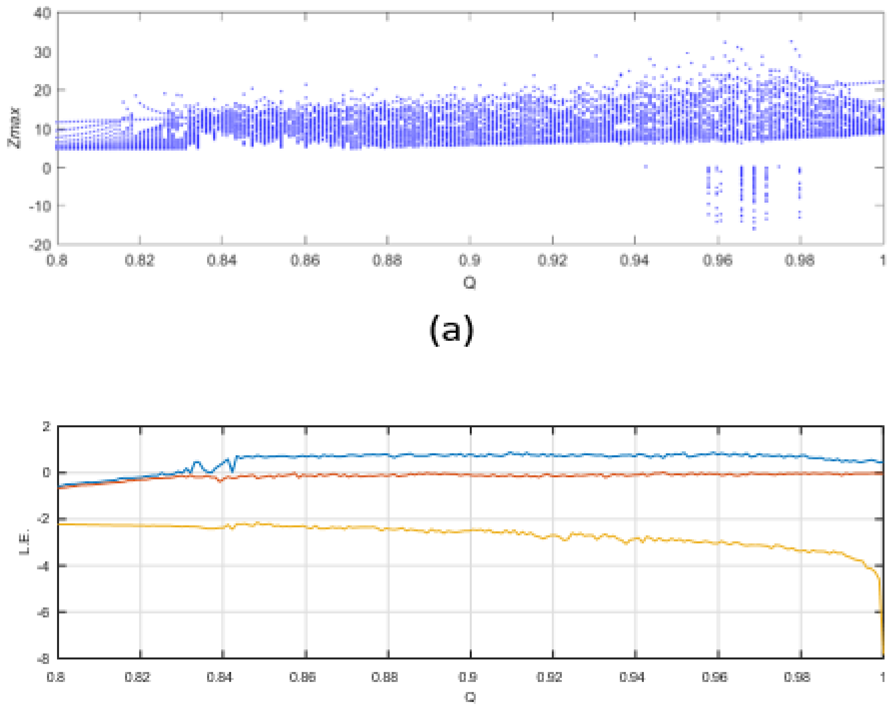

Fig. 5: Bifurcation and Lyapunov diagrams of (1) for different Q.

Lyapunov Exponents, Kaplan-Yorke dimension and Bifurcation Anal- ysis

To have the separation rate idea of infinitesimal close trajectories Lyapunov values are looked for. For A= 9, B=4, C=1.5, D=2.85, E=0.2 and initial conditions (1,1,1) the following Lyapunov values at fractional order 0.97 are obtained.

$$0.4969$$
$$0.0016 \approx 0$$
$$-10.2977$$

The first Lyapunov componentbeing positive confirms chaos in the system. The chaotic dimension (K.Y.) is:

$$D_{YK} = p + \frac{\sum_{s=1}^{p} L.E._s}{|L.E._{p+1}|}$$

where p is the greatest number satisfying $\sum_{s=1}^{p} L.E._s \geq 0$ and $\sum_{s=1}^{p+1} L.E._s < 0$. Using above values the K.Y. dimension is 2.04840886.

Fractional order has very sensitive dependence on system dynamics. A slight variation in value of system leads to entirely different dynamics. The effect of fractional order on the dynamics of the system and hence Lyapunov spectrum and bifurcation analysis is given in Fig. 4. From Fig. 4 it is evident that the novel fractional order system shows chaotic behavior for fractional order 0.85 and above.

Chaos Control

Chaos in trajectories of the chaotic system can be controlled using the adaptive sliding mode control technique. Chaos is controlled here considering uncertainties and

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disturbances in the chaotic system after designing suitable controllers. The disturbed fractional order chaotic system (2) is:

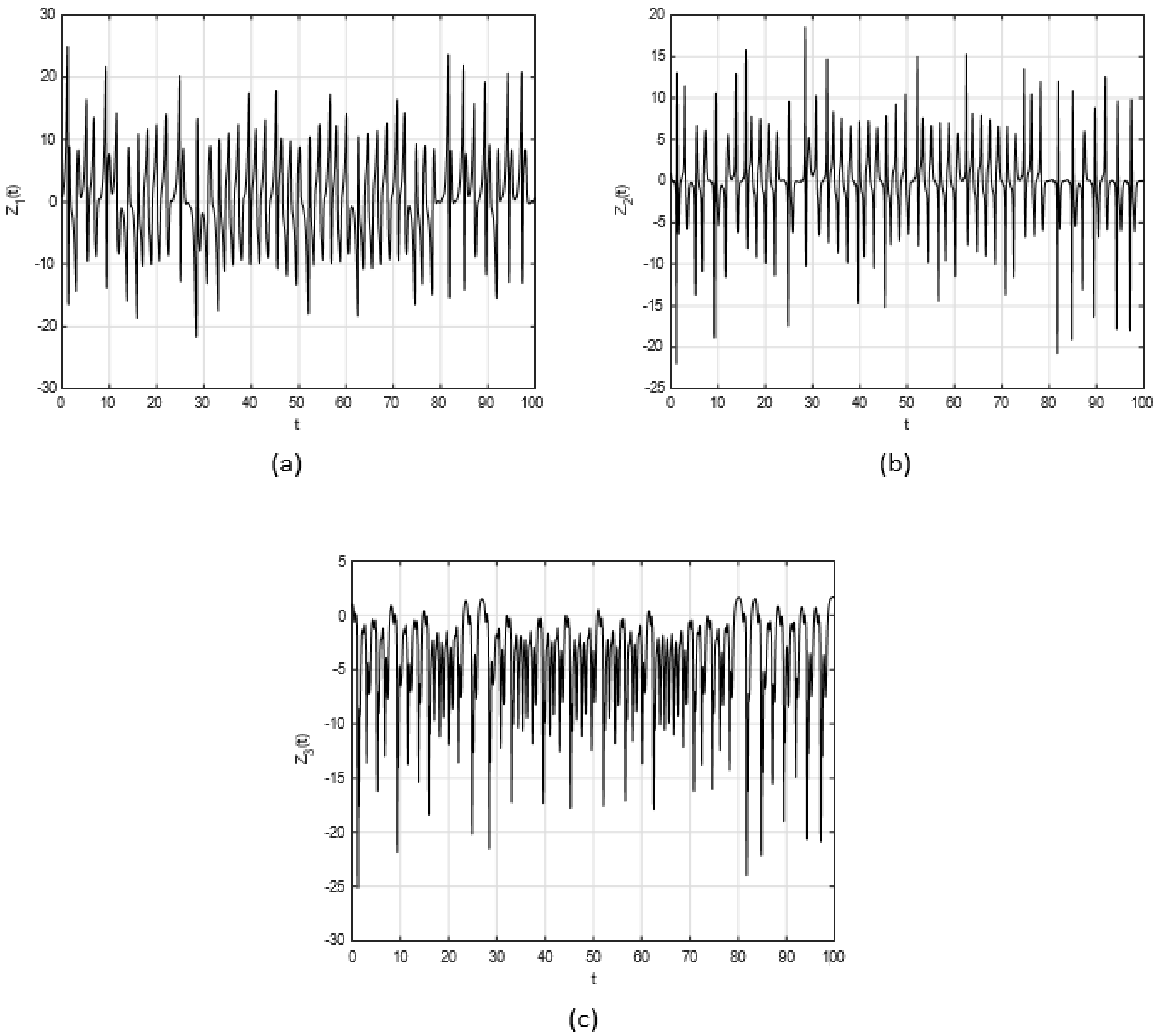

Fig. :6 State Trajectories of disturbed system (2)

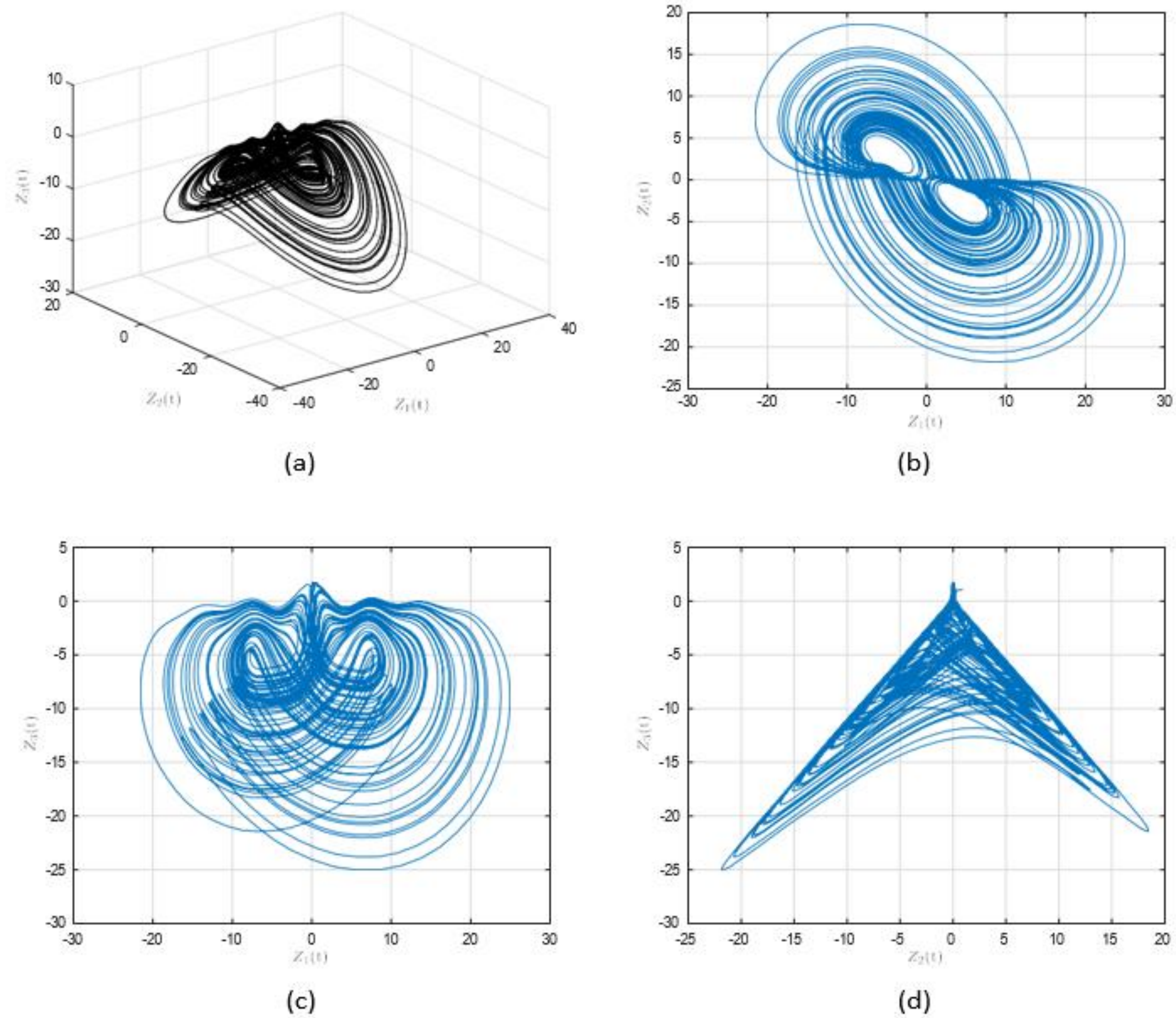

Fig. 7: Phase Portraits of disturbed system (2)

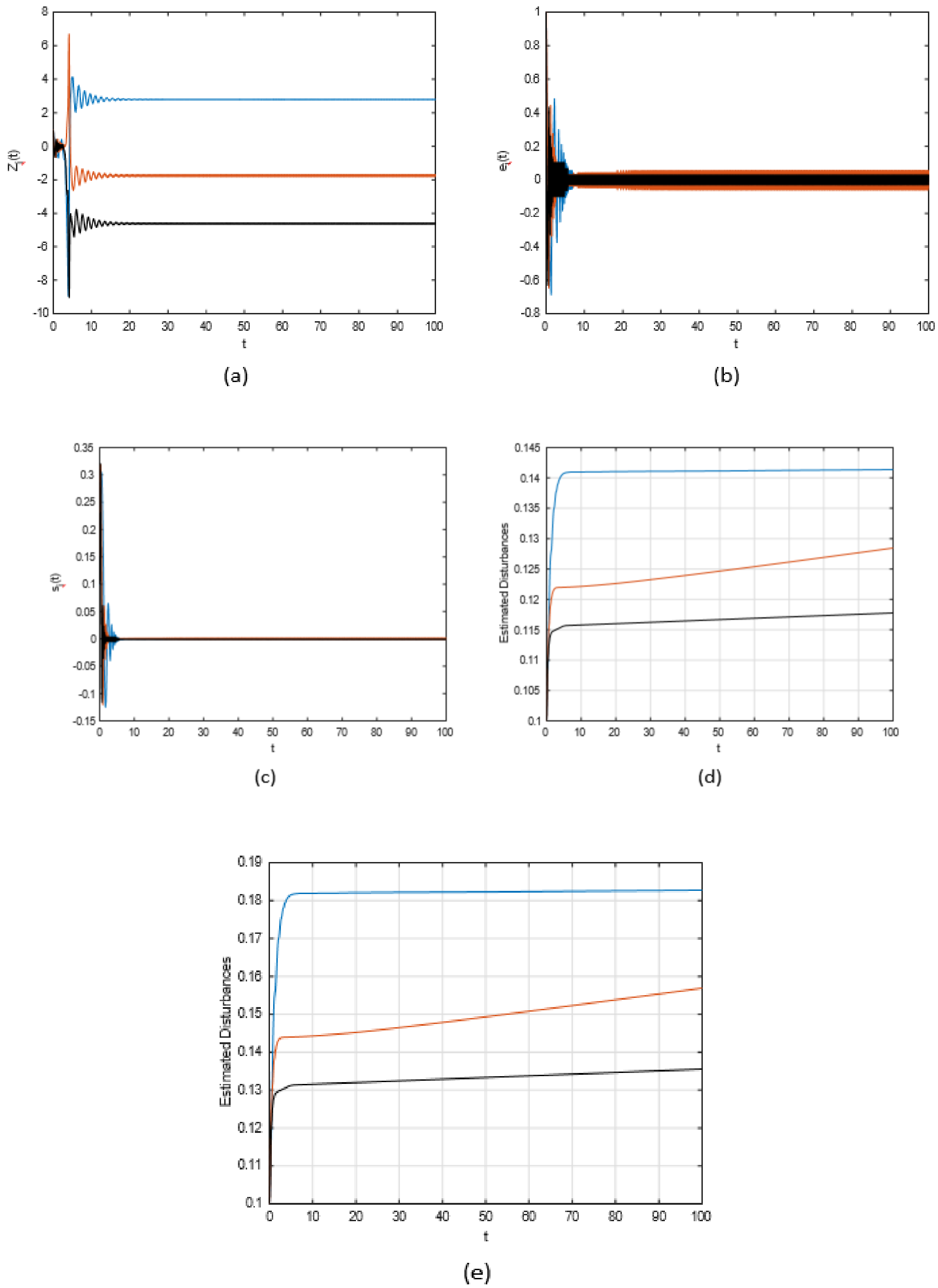

Fig. 8: (a) Controlled states (b) Error (c) Sliding surfaces (d) Disturbance estimates (e) Disturbance estimates.

$$\begin{aligned} D^{Q}Z_1 &= DZ_1 - Z_2Z_3 + \triangle H_1 + D_1 + v_1 \\ D^{Q}Z_2 &= E - AZ_2 + Z_1Z_3 + CZ_2 + \triangle H_2 + D_2 + v_2 \\ D^{Q}Z_3 &= -BZ_3 + Z_1Z_2 + \triangle H_3 + D_3 + v_3 \end{aligned} \tag{2}$$

where $\triangle$ Hiare uncertainties and Di are external disturbances, vi are controllers to be de- signed about the chosen point (a, b, c). Consider | $\triangle$ Hi| < Ci and Di < Fi where Ci, Fi > 0 for i=1,2,3. Let Cˆi and Fˆi be the estimates of Ci and Fi. State trajectories and phase portraits of disturbed system are shown in Fig. 5 and 6 respectively. Error about (a,b,c) is:

$$\begin{aligned} e_1 &= Z_1 - a \\ e_2 &= Z_2 - b \\ e_3 &= Z_3 - c \end{aligned} \tag{3}$$

Error dynamics is:

$$\begin{aligned} D^{Q}e_1 &= 2.85(e_1 + a) - (e_2 + b)(e_3 + c) + \triangle H_1 + D_1 + v_1 \\ D^{Q}e_2 &= 0.2 - 9(e_2 + b) + (e_1 + a)(e_3 + c) + 1.5(e_2 + b) + \triangle H_2 + D_2 + v_2 \\ D^{Q}e_3 &= -4(e_3 + c) + (e_1 + a)(e_2 + b) + \triangle H_3 + D_3 + v_3 \end{aligned}$$

Sliding surface in error space is:

$$s_i(t) = D^{q-1}e_i(t) + \lambda_i \int_0^t e_i(\xi)d\xi \tag{5}$$

To have (4) in sliding mode, we must have:

$$s_i(t) = 0, \dot{s}_i(t) = 0. \tag{6}$$

Differentiating (5):

$$\dot{s}_i(t) = D^{q}e_i(t) + \lambda_i e_i(t). \tag{7}$$

From (6):

$$D^{q}e_i(t) = -\lambda_i e_i(t). \tag{8}$$

From Matignon's Theorem [9], (8) is asymptotically stable. For ri > 0 design controllers based on SMC theory as:

$$\begin{aligned} v_1 &= -2.85(e_1 + a) + (e_2 + b)(e_3 + c) - sin(7t) - \lambda_1 e_1 - (\hat{C}_1 + \hat{F}_1 + r_1)sign(s_1) \\ v_2 &= -0.2 + 9(e_2 + b) - (e_1 + a)(e_3 + c) - 1.5(e_2 + b) - \lambda_2 e_2 - (\hat{C}_2 + \hat{F}_2 + r_2)sign(s_2) \\ v_3 &= 4(e_3 + c) - (e_1 + a)(e_2 + b) - 7cos(e_1 + a) - \lambda_3 e_3 - (\hat{C}_3 + \hat{F}_3 + r_3)sign(s_3) \end{aligned}$$

with sign(.) as signum function. Update laws for ci, fi > 0 are:

$$\begin{aligned} \dot{C}_i &= c_i \mid s_i \mid \\ E_i &= f_i \mid s_i \mid \end{aligned} \tag{10}$$

Theorem 8.1 Chaotic state trajectories of the disturbed fractional order chaotic system attain global and asymptotic stability using control and update laws as in (9)-(10).

Proof: Stability is proved based on Lyapunov's direct method [12]. Define Lyapunov function as:

Let.

$$V = V_1 + V_2 + V_3 \tag{11}$$

where

$$\begin{aligned} V_1 &= \frac{1}{2}s_1^2 + \frac{1}{2c_1}(\hat{C}_1 - C_1)^2 + \frac{1}{2f_1}(\hat{F}_1 - F_1)^2 \\ V_2 &= \frac{1}{2}s_2^2 + \frac{1}{2c_2}(\hat{C}_2 - C_2)^2 + \frac{1}{2f_2}(\hat{F}_2 - F_2)^2 \\ V_3 &= \frac{1}{2}s_3^2 + \frac{1}{2c_3}(\hat{C}_3 - C_3)^2 + \frac{1}{2f_3}(\hat{F}_3 - F_3)^2 \end{aligned} \tag{12}$$

Differentiating we obtain Lyapunov's dynamics:

$$\begin{aligned} \dot{V}_1 &= s_1\dot{s}_1 + \frac{1}{c_1}(\hat{C}_1 - C_1)\dot{\hat{C}}_1 + \frac{1}{f_1}(\hat{F}_1 - F_1)\dot{\hat{F}}_1 \\ \dot{V}_2 &= s_2\dot{s}_2 + \frac{1}{c_2}(\hat{C}_2 - C_2)\dot{\hat{C}}_2 + \frac{1}{f_2}(\hat{F}_2 - F_2)\dot{\hat{F}}_2 \\ \dot{V}_3 &= s_3\dot{s}_3 + \frac{1}{c_3}(\hat{C}_3 - C_3)\dot{\hat{C}}_3 + \frac{1}{f_3}(\hat{F}_3 - F_3)\dot{\hat{F}}_3 \end{aligned} \tag{13}$$

From (7), we have:

$$\begin{aligned} \dot{V}_1 &= s_1(D^{q}e_1 + \lambda_1 e_1) + \frac{1}{c_1}(\hat{C}_1 - C_1)\dot{\hat{C}}_1 + \frac{1}{f_1}(\hat{F}_1 - F_1)\dot{\hat{F}}_1 \\ \dot{V}_2 &= s_2(D^{q}e_2 + \lambda_2 e_2) + \frac{1}{c_2}(\hat{C}_2 - C_2)\dot{\hat{C}}_2 + \frac{1}{f_2}(\hat{F}_2 - F_2)\dot{\hat{F}}_1 \\ \dot{V}_3 &= s_3(D^{q}e_3 + \lambda_3 e_3) + \frac{1}{c_3}(\hat{C}_3 - C_3)\dot{\hat{C}}_3 + \frac{1}{f_3}(\hat{F}_3 - F_3)\dot{\hat{F}}_3 \end{aligned} \tag{14}$$

Substituting values in (14):

$$\begin{aligned} \dot{V}_i &= s_i[(\triangle H_i + D_i) - (\hat{C}_i + \hat{F}_i + r_i)signs_i] + (\hat{C}_i - F_i)|s_i| + (\hat{C}_i - F_i)|s_i| \\ &\leq (|\triangle H_i| + |D_i|)|s_i| + (\hat{C}_i - F_i)|s_i| + (\hat{C}_i - F_i)|s_i| \\ &< (C_i + F_i)|s_i| - (\hat{C}_i + \hat{F}_i + r_i)|signs_i| + (\hat{C}_i - F_i)|s_i| + (\hat{C}_i - F_i)|s_i| \\ &= -T_i|s_i| \end{aligned}$$

Finally

$$\dot{V} = \sum_{i=1}^{3} \dot{V}_i < -\sum_{i=1}^{3} (T_i|s_i|) \quad (15)$$

Thus ∃ a T ≥ 0 ∈ R such that

$$\sum_{i=1}^{3} T_i|s_i| > T$$

Then

$$\dot{V} < -T\sqrt{s_1^2+s_2^2+s_3^2} < 0 \quad (16)$$

From Lyapunov stability theory si → 0 as t → ∞, i.e. errors converge to si = 0 and stability is attained.

Simulation Results

For numerical simulations, we have A=9, B=4, C=1.5, D=2.85, E=0.2, Q= 0.97 and the I.C.(1, 1, 1) , △H1 = 0, D1 = sin(7t), △H2 = 0, D2 = 0, △H3 = 7cos(Z1), D3 = 0, λ1 = 1, λ2 = 2, λ3 = 3, r1 = 1, r2 = 2, r3 = 3. Controlled trajectories, errors, sliding surfaces, disturbance estimates are in Fig.7.

V. CONCLUSION

The paper studies the fractional order chaotic system, performs its thorough dynamical anal- ysis such as phase portraits, solution, Lyapunov spectrum, bifurcation analysis, effect of frac- tional order. The chaos in the trajectories of the novel fractional order chaotic system con- sidering uncertainties and disturbances is controlled about origin and are also estimated.

Future work comprises of exploring hidden attractors of the system.

Data Availability Statement

No data was used for this paper.

No Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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