

# Investigation of Dependent Rikitake System to Initiation Point

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**Abstract** – In this paper we investigate depending of the Rikitake system to initiation point, and monitor changing behavior of this system. We will have 4 initiation points in Cartesian system. We at 4 positions, will monitor behavior of this system, while holding constant other values, and after per position, will draw operation of system on axes of x, y, z and 3-D plot. We want to know, what is the effect of initiation point on Rikitake system? Numerical simulations to illustrate the effect of initiation point are presented, and at the end conclusions and comparing the states together are obtained.

**Keywords** — Chaos, Rikitake System, Sinusoid, Stability, Initiation Point, Dependent.

## I. INTRODUCTION

At the first, I want to explain, the physics rules behind of Rikitake system for more and better understanding behavior of Rikitake system.

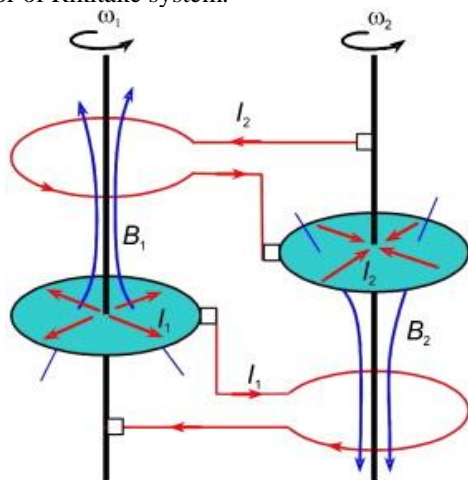


Fig.1. The Rikitake dynamo is composed of two disk dynamos coupled to one another.

This system is a mathematical model obtained from a simple mechanical system used by Rikitake [1] to study the reversals of the Earth's magnetic field. It is a common torque of magnitude G is applied to both conducting disks D1 and D2. Both disks rotate in the same sense, one with an angular velocity of  $\omega_1$  around the axis of rotation A1 and the other with angular velocity  $\omega_2$  around the axis of rotation A2. A current  $I_1$  circulates the current loop L1 where the current loop is coaxial with D2 with axis A2 and L1 is located below D2, and causes a magnetic field B2 to pass through the rotating disk D2. Loop L1 is connected to Disk D1 and its axis of rotation A1 by conducting brushes

and the current along A1 is upward and in D1 it is radial outward. Similarly a current  $I_2$  circulates the current loop L2 where the current loop is coaxial with D1 with axis A1 and L2 is located above D1, and causes a magnetic field B1 to pass through the rotating disk D1. Loop L2 is connected to Disk D2 and its axis of rotation A2 by conducting brushes and the current along A2 is upward and in D2 it is radial inward. Since D1 is rotating B1 caused by L2 will create an induced emf (electromagnetic field) between the center of D1 and its rim causing an induced inward current  $I_1$  to occur in the opposite direction to  $I_1$  where the total current becomes less than the original current  $I_1$ . Similarly since D2 is rotating B2 caused by L1 will create an induced emf between the center of D2 and its rim causing an induced outward current  $I_2$  to occur in the opposite direction to  $I_2$  where the total current becomes less than the original current  $I_2$ . This process continues until we achieve current reversal in both loops which causes a reversal in the total magnetic field. Under particular initial conditions this process becomes chaotic. [3]

Many researchers have discussed the dynamics of Rikitake system. Historically E. C. Bullard has extensively discussed the behavior of earth's magnetic field and its simulation with dynamos. He first discussed the magnetic field within the earth, then the similar behavior between a set of homogeneous dynamos and terrestrial magnetism and in 1955 discussed the stability of a homopolar dynamo. Liu Xiao-Jun, et.al. [5] Analyzed the dynamics of Rikitake two disk dynamo to explain the reversals of the Earth's magnetic field. They concluded that the chaotic behavior of the system can be used to simulate the reversals of the geomagnetic field. The Rikitake chaotic attractor was studied by several authors. T. McMillen [6] and Mohammad Javidi, et.al. [4] has studied the shape and dynamics of the Rikitake attractor. J. Llibre .et.al [7] used

the Poincare compactification to study the dynamics of the Rikitake system at infinity. Chien- Chih Chen et al [10] have studied the stochastic resonance in the periodically forced Rikitake dynamo. In the past decade, many researchers start working on controlling the chaotic behaviors. Harb and Harb [11] have designed a nonlinear controller to control the chaotic behavior in the phase-locked loop by means of nonlinear control. Ahmad Harb[3] have designed a controller to control the unstable chaotic oscillations by means of back stepping method. The synchronization for chaotic of Rikitake system was studied by several authors. Mohamad Ali khan [12] and Carlos Aguilar-Ibañez[9] and U.E. Vincent[8]. Yousof Gholipour and Mahmood moula [13] Investigated stability of Rikitake system with changing the resistance wires of system.

In this paper we suppose that the all situations and values are constant, and we change initiation point  $E(x_0, y_0, z_0)$ , and compare states of system in different points.

## II. MODELING AND ANALYSIS

The system mathematical model can be written as follows:

$$\begin{cases} \dot{x} = -ux + zy \\ \dot{y} = -uy + (z-a)x \\ \dot{z} = 1 - x^2 - y^2 \end{cases} \quad (1)$$

Where  $(x, y, z) \in R^3$  are the state variables and  $a > 0, u > 0$  are parameters. Note that system (1) is a quadratic system in  $R^3$ . The choice of the parameters  $a > 0$  and  $u > 0$  reflects a physical meaning in the Rikitake model.

It is well known that system (1) has two equilibrium points  $E^+ = (x_0, y_0, z_0)$ ;  $E^- = (-x_0, -y_0, z_0)$  In order to study the stability of  $E_{\pm}$ , it is only sufficient to study the stability of the equilibrium point  $E^+$ .

$$\begin{cases} X = \sqrt{\frac{a + \sqrt{a^2 + 4u^2}}{2u}} \\ Y = \sqrt{\frac{2u}{a + \sqrt{a^2 + 4u^2}}} \\ Z = \frac{a + \sqrt{a^2 + 4u^2}}{2u} \end{cases} \quad (2)$$

Where:

$$a = R \sqrt{\frac{LC}{GM}} \quad \& \quad u = (\omega_1 - \omega_2) \sqrt{\frac{CM}{GL}} \quad (3)$$

## III. SIMULATION RESULTS AT 4 POINTS

In this section we will investigate dependent Rikitake system to initiation point. While all the situations and values are constant and suppose chaos situation for

Rikitake system [13]. When  $a=3$  and  $u=1.2$ , change initiation point  $E(x_0, y_0, z_0)$  and observe amount of dependent and change of Rikitake system.

We consider 4 positions:

$$E^+(x_0, y_0, z_0) = (1, 1, 1)$$

$$E^+(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$$

$$E^+(x_0, y_0, z_0) = (3, 2, 1)$$

$$E^-(x_0, y_0, z_0) = (-1, -2, -3)$$

Now display the numerical solution of Rikitake system.

For all positions the situations and values are constant, and set  $u=1.2, a=3$  and steps  $h=0.01$ . We plot the system behavior for four positions around X, Y, Z axes and 3-D plot.

$$E^+(x_0, y_0, z_0) = (1, 1, 1)$$

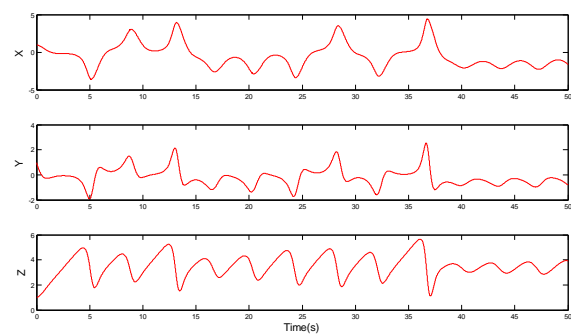


Fig.2. Rikitake system behavior for  $E = (1, 1, 1)$

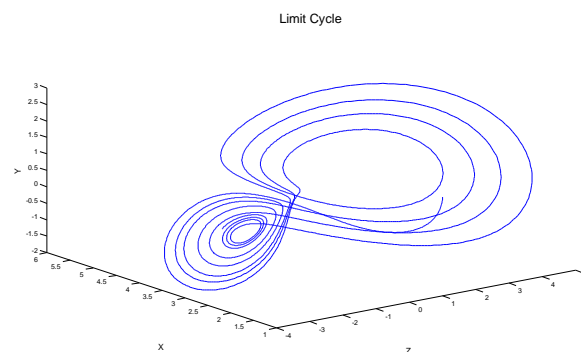


Fig.3. The Limit cycle. Amount of Stability system when  $E = (1, 1, 1)$

$$E^+(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$$

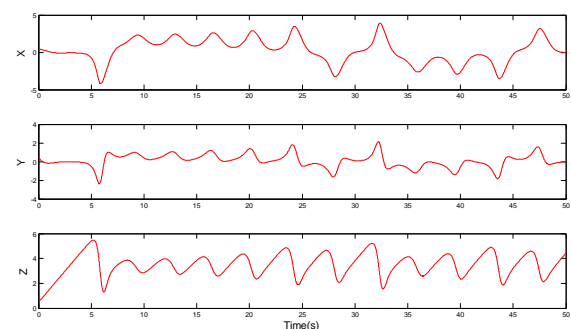


Fig.4. Rikitake system behavior for  $E = (0.5, 0.5, 0.5)$

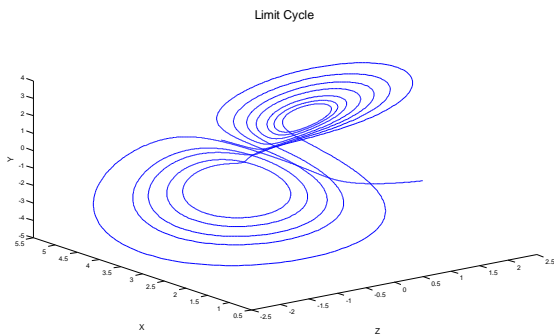


Fig.5. The Limit cycle. Amount of Stability system when  $E = (0.5, 0.5, 0.5)$

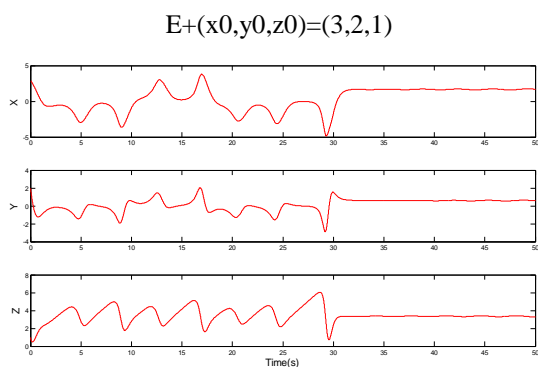


Fig. 6 Rikitake system behavior for  $E = (3, 2, 1)$

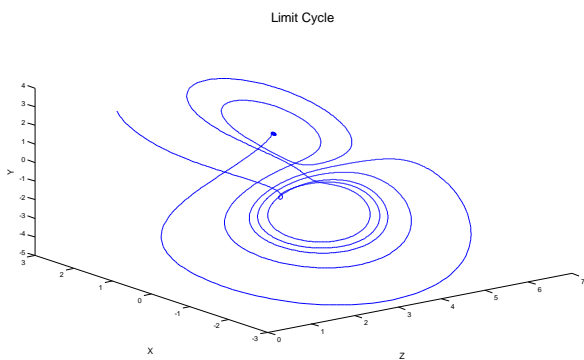


Fig. 7 The Limit cycle. Amount of Stability system when  $E = (3, 2, 1)$

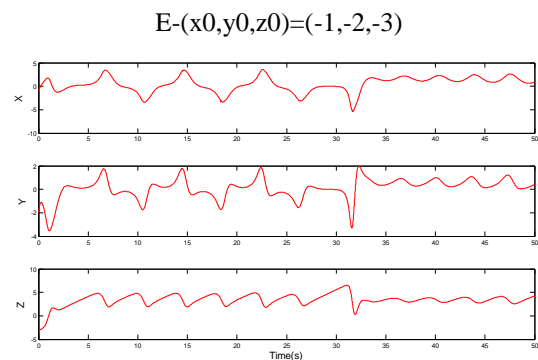


Fig. 8 Rikitake system behavior for  $E = (-1, -2, -3)$

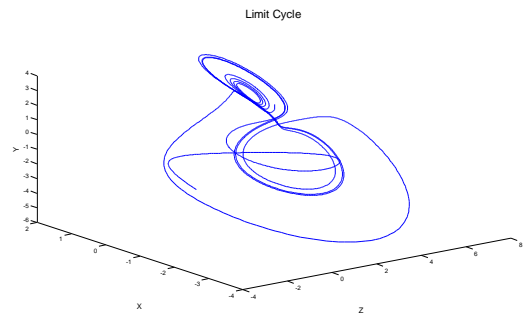


Fig.9. The Limit cycle. Amount of Stability system when  $E = (-1, -2, -3)$

In above simulations numerical we see that the initial point has effect on behavior of system, the effect of initial point on this system is too deep, and all behavior of the system changed when initial point changed.

#### IV. CONCLUSION

In this paper we, Investigated of dependent Rikitake system to initiation point. Firstly, discussed the physics rules behind Rikitake system and, last works about this system, after that presented the Rikitake model and at the behavior of system at four positions, investigated amount of dependent system to initiation point, and plotted axes  $x$ ,  $y$ ,  $z$  and 3-D plot, while all the situations and values was constant and suppose chaos situation for Rikitake system and  $a=3$ ,  $u=1.2$ . We concluded from numerical simulations that, the Rikitake system is sharply dependent to initiation point. By note that, the studies showed that this system has intrinsic chaotic behavior, change in initiation point, will change behavior of system. Because of this Intrinsic properties we should adjust  $a$  &  $u$  (Equation 3) of this system carefully for have a system with fix behavior.

#### ACKNOWLEDGMENT

We acknowledge our friend, Mehdi fatemi and the Associate Editor and anonymous reviewers for their valuable comments and suggestions that have helped us to improving the paper.

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