International Journal of Scientific Research and Engineering Trends
Volume 1, Issue 2, March-2015, ISSN (Online): 2395-566X

On Complement of Distinct Approximations of Fuzzy Number

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***Abstract* – In this paper, we introduce and investigate the concept of Distinct Approximation of complement of trapezoidal fuzzy numbers. A fuzzy arithmetic involving trapezoidal fuzzy numbers using α-cut method is applied in the complement of distinct approximation of fuzzy numbers. Some properties of the complement of fuzzy numbers are also studied the above discussed arithmetic. A relevant numerical example is included to illustrate our result.**

***Keywords* – Fuzzy Numbers, Trapezoidal Fuzzy Numbers, α-cut Method, Distinct Approximation Of Trapezoidal Fuzzy**

I. Introduction

Fuzzy numbers are often used in practice. An interesting problem is to approximate trapezoidal fuzzy numbers by distinct approximation of complement of trapezoidal fuzzy numbers, as to simplify the calculations.

Recently, there have been many papers investigating the approximation of [1, 8, 9, 14]. In this paper, we introduce the distinct approximation of complement of trapezoidal fuzzy numbers by using α-cut method [13], the α-cut method is a standard method for performing different arithmetic operations like addition, subtraction, multiplication and division.

In [3] and [11] the authors proposed a method for construction of membership function without using α-cut. They claim that the standard method of α-cut [2] fails in certain situation viz in determining square root of a fuzzy number. In this paper, we are going to show that α-cut method can be used for finding nth root of fuzzy number.

In [12] Mahmoud Taheri introduced and investigate the concept complement fuzzy numbers. A good overview of fuzzy number was proposed by Dubois et al [4]. A specialized book on fuzzy arithmetic with fuzzy numbers was written by Kaufmann and Gupta[10]. Some theoretical details and applications of fuzzy quantities and specially fuzzy numbers can be found in Dubois and Prade [5]. As well as in Fuller and Mesiar [6]. In addition, Ma et al [9] present fuzzy numbers with a new parametric forms and provide a fuzzy arithmetic based on this representation.

This paper organized as follows. In section 1, introduction is introductory in nature. In section 2, we introduce some basic definitions. In section 3, we introduce the definition of Distinct Approximation of Trapezoidal Fuzzy Number (DATFN) and derivation of addition, subtraction, multiplication and division of the Distinct Approximation of Complement of Trapezoidal Fuzzy Number in terms of membership function. In section 4, some special operations on Distinct Approximation of Complement of Trapezoidal Fuzzy Numbers are also studied.

II. Preliminaries

2.1. Fuzzy set

Let x denotes a universal set ie, X={x} then the characteristic function which assigns certain values or a membership grade to the elements of this universal set within a specified range [0,1] is known as the membership function and the set thus defined is called a fuzzy set. The membership grades correspond to the degrees to which are element is compatible with the concept represent by the fuzzy set. If μ_A is the membership function defining a fuzzy set A.

Then $\mu_A: X \to [0, 1]$

Where [0, 1] denotes the interval of real numbers from 0 to 1.

2.2. α-Cut

An α – cut of a fuzzy set A is a crisp set A_α that contains all the elements of the universal set X that have a membership grade in A greater than or equal to the specified value of α.

Thus, $A_\alpha = \{x \in X; \mu_A(x) \geq \alpha, 0 \leq x \leq 1\}$

2.3. Fuzzy Number

A convex and normalized fuzzy set defined on R. whose membership function is piecewise continuous is called fuzzy number. A fuzzy set is called normal when at least one of its elements attains the maximum possible membership grade.

ie., $\forall x \in R \quad \max_x \mu_A(x) = 1$

2.4. Complement of Fuzzy Number

A fuzzy set of R is called to be a complement fuzzy number if its complement is a fuzzy number. We denote the set of all complement fuzzy number by $F_C(R)$

$$\mu_{\tilde{A}'}(x)=\begin{cases}\dfrac{10-x}{10} & if\ 0\le x<10\\ \dfrac{x-10}{10} & if\ 10\le x<20\\ 1 & otherwise\end{cases}$$

2.5. Complement of a Trapezoidal Fuzzy Number

A complement of Trapezoidal fuzzy number $\bar{A}=(a_1, a_2, a_3\ a_4)$ is defined by the membership function as

$$\mu_{\tilde{A}'}(x)=\begin{cases}\dfrac{(a_2-x)}{(a_2-a_1)} & if\ a_1\prec x\le a_2\\ 0 & if\ a_2\le x\le a_3\\ \dfrac{(x-a_3)}{(a_4-a_3)} & if\ a_3\le x\prec a_4\\ 1 & otherwise\end{cases}$$

It can be characterized by defining the interval of confidence at level α. Thus for all $\alpha\in[0, 1]$

$\bar{A}_\alpha = [(a_2 - \alpha(a_2-a_1), a_3 + \alpha(a_4-a_3)]$

III. Distinct Approximation Of Trapezoidal Fuzzy Number (DATFN)

3.1 Definition: Distinct approximation of Trapezoidal fuzzy Number (DATFN)

Let $\tilde{A}'=(a_1', a_2', a_3', a_4')$ be a Distinct Approximation of $\tilde{A}=(a_1, a_2, a_3, a_4)$. where $a_1'=\frac{3a_1-a_2}{2}$, $a_2'=\frac{a_1+a_2}{2}$, $a_3'=\frac{a_4+a_3}{2}$, $a_4'=\frac{3a_4-a_3}{2}$

Then the membership function of Distinct Approximation C-fuzzy number is

$$\mu_{\tilde{A}'}(x)=\begin{cases}\dfrac{(a_2+a_1)-2x}{2(a_2-a_1)} & \text{if } \dfrac{3a_1-a_2}{2}\prec x\le\dfrac{a_1+a_2}{2}\\ 0 & \text{if } \dfrac{a_1+a_2}{2}\le x\le\dfrac{a_3+a_4}{2}\\ \dfrac{2x-(a_3+a_4)}{2(a_4-a_3)} & \text{if } \dfrac{a_3+a_4}{2}\le x\prec\dfrac{3a_4-a_3}{2}\\ 1 & otherwise\end{cases}$$

3.2. Example:

Let $\tilde{A}'=(1,3,7,9)$ be a Distinct Approximation of Trapezoidal Fuzzy Number $\tilde{A}=(2,4,6,8)$ whose membership function is given by

$$\mu_{\tilde{A}'}(x)=\begin{cases}\dfrac{(a_2+a_1)-2x}{2(a_2-a_1)} & \text{if } \dfrac{3a_1-a_2}{2}< x\le\dfrac{a_1+a_2}{2}\\ 0 & \text{if } \dfrac{a_1+a_2}{2}\le x\le\dfrac{a_3+a_4}{2}\\ \dfrac{2x-(a_3+a_4)}{2(a_4-a_3)} & \text{if } \dfrac{a_3+a_4}{2}\le x\prec\dfrac{3a_4-a_3}{2}\\ 1 & otherwise\end{cases}$$

The corresponding numerical value of membership function $\tilde{A}'$ is

$$\mu_{\tilde{A}'}(x)=\begin{cases}\dfrac{(2+4)-2x}{4} & \text{if } 1\prec x\le 3\\ 0 & \text{if } 3\le x\le 7\\ \dfrac{2x-(8+6)}{4} & \text{if } 7\le x\prec 9\\ 1 & otherwise\end{cases}$$

3.3 Addition of Two-Complement of Distinct Approximation of trapezoidal Fuzzy Numbers.

Let $\tilde{X}'=(a_1', a_2', a_3', a_4')$ and $\tilde{Y}'=(b_1', b_2', b_3', b_4')$ are Distinct Approximation of Trapezoidal fuzzy numbers $\tilde{X}=(a_1, a_2, a_3, a_4)$ and $\tilde{Y}=(b_1, b_2, b_3, b_4)$ respectively. where

$a_1'=\frac{3a_1-a_2}{2}$, $a_2'=\frac{a_1+a_2}{2}$, $a_3'=\frac{a_4+a_3}{2}$, $a_4'=\frac{3a_4-a_3}{2}$.

And Similarly

$b_1'=\frac{3b_1-b_2}{2}$, $b_2'=\frac{b_1+b_2}{2}$, $b_3'=\frac{b_4+b_3}{2}$, $b_4'=\frac{3b_4-b_3}{2}$ whose membership function of $\tilde{X}'$ and $\tilde{Y}'$ is

$$\mu_{\tilde{X}'}(x)=\begin{cases}\dfrac{(a_2+a_1)-2x}{2(a_2-a_1)} & \text{if } \dfrac{3a_1-a_2}{2}\prec x\le\dfrac{a_1+a_2}{2}\\ 0 & \text{if } \dfrac{a_1+a_2}{2}\le x\le\dfrac{a_3+a_4}{2}\\ \dfrac{2x-(a_3+a_4)}{2(a_4-a_3)} & \text{if } \dfrac{a_3+a_4}{2}\le x\prec\dfrac{3a_4-a_3}{2}\\ 1 & otherwise\end{cases}$$

$$\mu_{\tilde{Y}'}(x)=\begin{cases}\dfrac{(b_2+b_1)-2x}{2(b_2-b_1)} & \text{if } \dfrac{3b_1-b_2}{2}\prec x\le\dfrac{b_1+b_2}{2}\\ 0 & \text{if } \dfrac{b_1+b_2}{2}\le x\le\dfrac{b_3+b_4}{2}\\ \dfrac{2x-(b_4+b_3)}{2(b_4-b_3)} & \text{if } \dfrac{b_3+b_4}{2}\le x\prec\dfrac{3b_4-b_3}{2}\\ 1 & otherwise\end{cases}$$

then, $\tilde{X}_\alpha'=\left[\dfrac{(a_2+a_1)-2\alpha(a_2-a_1)}{2},\dfrac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$ and

$\tilde{Y}_\alpha'=\left[\dfrac{(b_2+b_1)-2\alpha(b_2-b_1)}{2},\dfrac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right]$

are the α-cuts of C-fuzzy numbers $\tilde{X}'$ and $\tilde{Y}'$respectively. To calculate addition of fuzzy numbers $\tilde{X}$ and $\tilde{Y}$. We first add the α-cuts of $\tilde{X}'$ and $\tilde{Y}'$ using interval arithmetic.

$$\tilde{X}_\alpha'+\tilde{Y}_\alpha'=\left[\begin{array}{l}\left[\dfrac{(a_2+a_1)-2\alpha(a_2-a_1)}{2},\dfrac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]+\\ \left[\dfrac{(b_2+b_1)-2\alpha(b_2-b_1)}{2},\dfrac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right]\end{array}\right]$$

$$\tilde{X}_\alpha'+\tilde{Y}_\alpha'=\left[\frac{(a_2+a_1)+(b_2+b_1)-2\alpha(a_2-a_1+b_2-b_1)}{2},\frac{(a_3+a_4)+(b_3+b_4)+2\alpha(a_4-a_3+b_4-b_3)}{2}\right] \quad (1)$$

To find the membership function.

$\mu_{\tilde{X}_\alpha'+\tilde{Y}_\alpha'}(x)$ we equate to x both the first and second component in (1) which gives

X= [$\dfrac{(a_2+a_1)+(b_2+b_1)-2\alpha(a_2-a_1+b_2-b_1)}{2}$] and

X= [$\dfrac{(a_3+a_4)+(b_3+b_4)+2\alpha(a_4-a_3+b_4-b_3)}{2}$]

International Journal of Scientific Research Engineering Technology
Volume 1, Issue 1, ISSN (Online): XX-XX

Now, expressing α in terms of $\tilde{X}'$ and $\tilde{Y}'$ setting α=0 and α=1 in (1). We get α together with the domain of X.

$$X=\left[\frac{(a_2+a_1)+(b_2+b_1)-2\alpha(a_2-a_1+b_2-b_1)}{2}\right]$$

$$2x=(a_2+a_1)+(b_2+b_1)-2\alpha(a_2-a_1+b_2-b_1)$$

$$(a_2+a_1)+(b_2+b_1)-2x=2\alpha(a_2-a_1+b_2-b_1)$$

$$\alpha=\frac{(a_2+a_1)+(b_2+b_1)-2x}{2(a_2-a_1+b_2-b_1)},$$

$$\frac{3(a_1+b_1)-(a_2+b_2)}{2}\prec x\le\frac{(a_1+a_2)+(b_1+b_2)}{2} \text{ and}$$

$$X=\left[\frac{(a_3+a_4)+(b_3+b_4)+2\alpha(a_4-a_3+b_4-b_3)}{2}\right]$$

$$2x-(a_3+a_4)+(b_3+b_4)=2\alpha(a_4-a_3+b_4-b_3)$$

$$\alpha=\frac{2x-(a_4+a_3)+(b_4+b_3)}{2(a_4-a_3+b_4-b_3)}$$

$$;\frac{(a_3+a_4)+(b_3+b_4)}{2}\le x\prec\frac{3(a_4+b_4)-(a_3+b_3)}{2} \text{ and}$$

α=0

$$;\frac{(a_1+a_2)+(b_1+b_2)}{2}\le x\le\frac{(a_3+a_4)+(b_3+b_4)}{2}$$

which gives

$\mu_{\tilde{X}'+\tilde{Y}'}(x)=$

$$\mu_{\tilde{X}'+\tilde{Y}'}(x)=\begin{cases}\frac{(a_2+a_1)+(b_2+b_1)-2x}{2(a_2-a_1+b_2-b_1)} & \text{if } \frac{3(a_1+b_1)-(a_2+b_2)}{2}\prec x\le\frac{(a_1+a_2)+(b_1+b_2)}{2}\\ 0 & \text{if } \frac{(a_1+a_2)+(b_1+b_2)}{2}\le x\le\frac{(a_3+a_4)+(b_3+b_4)}{2}\\ \frac{2x-(a_3+a_4)+(b_3+b_4)}{2(a_4-a_3+b_4-b_3)} & \text{if } \frac{(a_3+a_4)+(b_3+b_4)}{2}\le x\prec\frac{3(a_4+b_4)-(a_3+b_3)}{2}\\ 1 & \text{otherwise}\end{cases}$$

3.4 Subtraction of Two-Complement of Distinct Approximation of trapezoidal Fuzzy Numbers.

Let $\tilde{X}'=(a_1',a_2',a_3',a_4')$ and $\tilde{Y}'=(b_1',b_2',b_3',b_4')$ are Distinct Approximation of Trapezoidal fuzzy numbers $\tilde{X}=(a_1,a_2,a_3,a_4)$ and $\tilde{Y}=(b_1,b_2,b_3,b_4)$ respectively. Then

$$\tilde{X}_\alpha'=\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2},\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right] \text{and}$$

$$\tilde{Y}_\alpha'=\left[\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2},\frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right]$$

are the α-cuts of C-fuzzy numbers $\tilde{X}'$ and $\tilde{Y}'$respectively. To calculate subtraction of fuzzy numbers $\tilde{X}$ and $\tilde{Y}$. We first subtract the α-cuts of $\tilde{X}'$ and $\tilde{Y}'$ using interval arithmetic.

$$\tilde{X}_\alpha'-\tilde{Y}_\alpha'=\left[\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2},\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]-\left[\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2},\frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right]\right]$$

$$\tilde{X}_\alpha'-\tilde{Y}_\alpha'=\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}-\left(\frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right),\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}-\left(\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2}\right)\right]$$

$$\tilde{X}_\alpha'-\tilde{Y}_\alpha'=\left[\frac{(a_2+a_1)-(b_3+b_4)-2\alpha[(a_2-a_1)+(b_4-b_3)]}{2}\right],\left[\frac{(a_4+a_3)-(b_2+b_1)+2\alpha[(a_4-a_3)+(b_2-b_1)]}{2}\right] \quad (2)$$

To find the membership function $\mu_{\tilde{X}'-\tilde{Y}'}(x)$ we equate to x both the first and second component in (2) using gives

$$X=\left[\frac{(a_2+a_1)-(b_4+b_3)-2\alpha(a_2-a_1+b_4-b_3)}{2}\right] \text{ and}$$

$$X=\left[\frac{(a_3+a_4)-(b_2+b_1)+2\alpha(a_4-a_3+b_1-b_2)}{2}\right]$$

Now, expressing α in terms of $\tilde{X}_\alpha'$ and $\tilde{Y}_\alpha'$ setting α=0 and α=1
in (2). We get α together with the domain of X.

$$X=\left[\frac{(a_2+a_1)-(b_4+b_3)-2\alpha(a_2-a_1+b_4-b_3)}{2}\right]$$

$$2x=(a_2+a_1)-(b_4+b_3)-2\alpha(a_2-a_1+b_4-b_3)$$

$$\alpha=\frac{(a_2+a_1)-(b_4+b_3)-2x}{2(a_2-a_1+b_4-b_3)}$$

$$\frac{3(a_1-b_4)-(a_2-b_3)}{2}\prec x\le\frac{(a_1+a_2)-(b_4+b_3)}{2} \text{ and}$$

$$X=\left[\frac{(a_3+a_4)-(b_2+b_1)+2\alpha(a_4-a_3+b_1-b_2)}{2}\right]$$

$$2x=(a_3+a_4)-(b_2+b_1)+2\alpha(a_4-a_3+b_1-b_2)$$

$$\alpha=\frac{2x-(a_4+a_3)-(b_2+b_1)}{2(a_4-a_3-b_1+b_2)},$$

$$\frac{(a_3+a_4)-(b_2+b_1)}{2}\le x\prec\frac{3(a_4-b_1)-(a_3-b_2)}{2} \text{ and}$$

α=0,

$$\frac{(a_1+a_2)+(b_1+b_2)}{2}\le x\le\frac{(a_3+a_4)+(b_3+b_4)}{2}$$

which gives

$$\mu_{\tilde{X}'-\tilde{Y}'}(x)=\begin{cases}\frac{(a_2+a_1)-(b_4+b_3)-2x}{2(a_2-a_1+b_3+b_4)} & \text{if } \frac{3(a_1-b_1)-(a_2-b_3)}{2}\prec x\le\frac{(a_2+a_1)-(b_4+b_3)}{2}\\ 0 & \text{if } \frac{(a_1+a_2)-(b_4+b_3)}{2}\le x\le\frac{(a_3+a_4)-(b_2+b_1)}{2}\\ \frac{2x-(a_3+a_4)-(b_2+b_1)}{2(a_4-a_3-b_1+b_2)} & \text{if } \frac{(a_3+a_4)-(b_2+b_1)}{2}\le x\prec\frac{3(a_4-b_1)-(a_3-b_1)}{2}\\ 1 & \text{otherwise}\end{cases}$$

3.5 Multiplication of Two-Complement of Distinct Approximation of trapezoidal Fuzzy Numbers.

Let $\tilde{X}'=(a_1',a_2',a_3',a_4')$ and $\tilde{Y}'=(b_1',b_2',b_3',b_4')$ be two positive Distinct Approximation of Trapezoidal fuzzy numbers $\tilde{X}=(a_1,a_2,a_3,a_4)$ and $\tilde{Y}=(b_1,b_2,b_3,b_4)$

respectively. Then $\tilde{X}_\alpha{}' = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$ and

$$\tilde{Y}_\alpha{}' = \left[\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2}, \frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right]$$

are the α-cuts of C-fuzzy numbers $\tilde{X}'$ and $\tilde{Y}'$ respectively. To calculate multiplication of fuzzy numbers $\tilde{X}$ and $\tilde{Y}$. We first multiply the α-cuts of $\tilde{X}'$ and $\tilde{Y}'$ using interval arithmetic.

$$\tilde{X}_\alpha{}' * \tilde{Y}_\alpha{}' = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right] * \left[\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2}, \frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right]$$

$$\left(\tilde{X}'_\alpha * \tilde{Y}'_\alpha\right)^L_\alpha = \left(\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}\right) * \left(\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2}\right)$$

$$= \frac{(a_2+a_1)(b_2+b_1)-2\alpha\left[(a_2+a_1)(b_2-b_1)+(b_2+b_1)(a_2-a_1)\right]+4\alpha^2\left[(a_2-a_1)(b_2-b_1)\right]}{4}$$

$$;\left(\frac{3a_1-a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right) \prec x \le \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right)$$

Similarly, $\left(\tilde{X}'_\alpha * \tilde{Y}'_\alpha\right)^U_\alpha =$

$$\frac{-\left[\frac{(a_3+a_4)(b_4-b_3)+(b_4+b_3)(a_4-a_3)}{2}\right]+\sqrt{\left[\frac{(a_3+a_4)(b_4-b_3)+(b_4+b_3)(a_4-a_3)}{2}\right]^2-4\left[(a_4-a_3)(b_4-b_3)\right]\left[\frac{[(a_3+a_4)(b_3+b_4)]}{4}-x\right]}}{2\left[(a_4-a_3)(b_4-b_3)\right]}$$

$$;\left(\frac{a_3+a_4}{2}\right)\left(\frac{b_3+b_4}{2}\right) \le x \prec \left(\frac{3a_4-a_3}{2}\right)\left(\frac{3b_4-b_3}{2}\right)$$

$$\left(\tilde{X}'_\alpha * \tilde{Y}'_\alpha\right)_\alpha = 0$$

$$;\left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right) \le x \le \left(\frac{a_3+a_4}{2}\right)\left(\frac{b_3+b_4}{2}\right)$$

$$\left(\tilde{X}'_\alpha * \tilde{Y}'_\alpha\right)_\alpha = 1 \qquad ;\ \text{otherwise}$$

Which gives,

$$\mu_{\tilde{X}'_\alpha * \tilde{Y}'_\alpha}(x) = \begin{cases} \dfrac{\left[\frac{(a_2+a_1)(b_2-b_1)+(b_2+b_1)(a_2-a_1)}{2}\right]-\sqrt{\left[\frac{(a_2+a_1)(b_2-b_1)+(b_2+b_1)(a_2-a_1)}{2}\right]^2-4\left[(a_2-a_1)(b_2-b_1)\right]\left[\frac{[(a_2+a_1)(b_2+b_1)]}{4}-x\right]}}{2\left[(a_2-a_1)(b_2-b_1)\right]} & \text{if } \left(\frac{3a_1-a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right) \prec x \le \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right) \\ 0 & \text{if } \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right) \le x \le \left(\frac{a_3+a_4}{2}\right)\left(\frac{b_3+b_4}{2}\right) \\ \dfrac{-\left[\frac{(a_3+a_4)(b_4-b_3)+(b_4+b_3)(a_4-a_3)}{2}\right]+\sqrt{\left[\frac{(a_3+a_4)(b_4-b_3)+(b_4+b_3)(a_4-a_3)}{2}\right]^2-4\left[(a_4-a_3)(b_4-b_3)\right]\left[\frac{[(a_3+a_4)(b_3+b_4)]}{4}-x\right]}}{2\left[(a_4-a_3)(b_4-b_3)\right]} & \text{if } \left(\frac{a_3+a_4}{2}\right)\left(\frac{b_3+b_4}{2}\right) \le x \prec \left(\frac{3a_4-a_3}{2}\right)\left(\frac{3b_4-b_3}{2}\right) \\ 1 & \textit{otherwise} \end{cases}$$

3.6 Division of Two-Complement of Distinct Approximation of Trapezoidal Fuzzy Numbers.

Let $\tilde{X}' = (a'_1, a'_2, a'_3, a'_4)$ and $\tilde{Y}' = (b'_1, b'_2, b'_3, b'_4)$ be two Distinct Approximation of Trapezoidal fuzzy numbers $\tilde{X} = (a_1, a_2, a_3, a_4)$ and $\tilde{Y} = (b_1, b_2, b_3, b_4)$ respectively.

Then $\tilde{X}_\alpha{}' = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$ and

$$\tilde{Y}_\alpha{}' = \left[\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2}, \frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right]$$

are the α-cuts of C-fuzzy numbers $\tilde{X}'$ and $\tilde{Y}'$ respectively. To calculate division of fuzzy numbers $\tilde{X}$ and $\tilde{Y}$. We first divide the α-cuts of $\tilde{X}'$ and $\tilde{Y}'$ using interval arithmetic.

$$\frac{\tilde{X}'_\alpha}{\tilde{Y}'_\alpha} = \left[\frac{\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}}{\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2}, \frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}}\right]$$

$$= \left[\frac{\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}}{\frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}}, \frac{\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}}{\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2}}\right]$$

$$= \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{(b_3+b_4)+2\alpha(b_4-b_3)}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{(b_2+b_1)-2\alpha(b_2-b_1)}\right] \quad (3)$$

To find the membership function $\mu_{X/Y}(x)$ we equate to X both the first and second component in (3)using gives

$$X = \frac{(a_2+a_1)-2\alpha(a_2-a_1)}{(b_3+b_4)+2\alpha(b_4-b_3)} \text{ and}$$

$$X = \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{(b_2+b_1)-2\alpha(b_2-b_1)}$$

Now, expressing α-in terms of X setting α=0 and α=1 in (3), we get α together with the domain of X.

$$\left(\tilde{X}'_\alpha / \tilde{Y}'_\alpha\right)^L_\alpha = \frac{(a_2+a_1)-x(b_3+b_4)}{2(x(b_4-b_3)+(a_2-a_1))} \quad ;$$

$$\frac{3a_2-a_1}{3b_4-b_3} \prec x \le \frac{a_1+a_2}{b_3+b_4}$$

$$\left(\tilde{X}'_\alpha / \tilde{Y}'_\alpha\right)^U_\alpha = \frac{x(b_2+b_1)-(a_4-a_3)}{2(x(b_2-b_1)+(a_4-a_3))}$$

$$;\ \frac{a_3+a_4}{b_2+b_1} \le x \prec \frac{3a_4-a_3}{3b_1-b_2}$$

and $\left(\tilde{X}'_\alpha / \tilde{Y}'_\alpha\right)_\alpha = 0 \qquad ;\ \frac{a_2+a_1}{b_4+b_3} \le x \le \frac{a_3+a_4}{b_2+b_1}$

$$\left(\tilde{X}'_\alpha / \tilde{Y}'_\alpha\right)_\alpha = 1 \qquad ;\ \text{otherwise}$$

Which gives,

$$\mu_{\tilde{X}'_\alpha / \tilde{Y}'}(x) = \begin{cases} \dfrac{(a_2+a_1)-x(b_4+b_3)}{2(x(b_4-b_3)+(a_2-a_1))} & \text{if } \dfrac{3a_1-a_2}{3b_4-b_3} \prec x \le \dfrac{a_2+a_1}{b_3+b_4} \\ 0 & \text{if } \dfrac{a_1+a_2}{b_3+b_4} \le x \le \dfrac{a_3+a_4}{b_2+b_1} \\ \dfrac{2x-(a_3+a_4)-(b_2+b_1)}{2(a_4-a_3-b_1+b_2)} & \text{if } \dfrac{a_3+a_4}{b_2+b_1} \le x \prec \dfrac{3a_4-a_3}{3b_1-b_2} \\ 1 & \textit{otherwise} \end{cases}$$

IV. Some Special Operations on Complement of Distinct Approximations of Trapezoidal Fuzzy Numbers

4.1 Inverse of Complement of Distinct Approximation of Trapezoidal Fuzzy Numbers.

Let $\tilde{X}' = (a'_1, a'_2, a'_3, a'_4)$ be a Distinct Approximation of Trapezoidal fuzzy numbers $\tilde{X} = (a_1, a_2, a_3, a_4)$. Then

$$\tilde{X}_\alpha{}' = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

is the α-cut of C-fuzzy number $\tilde{X}'$. To calculate inverse of fuzzy numbers $\tilde{X}$. We first take inverse of $\tilde{X}'$ and using interval arithmetic.

$$\frac{1}{\tilde{X}'_\alpha} = \left[\frac{1}{\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}}, \frac{1}{\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}}\right]$$

$$\frac{1}{\tilde{X}'_\alpha} = \left[\frac{2}{(a_3+a_4)+2\alpha(a_4-a_3)}, \frac{2}{(a_2+a_1)-2\alpha(a_2-a_1)}\right] \quad (4)$$

To find the membership function $\mu_{1/\tilde{X}'_\alpha}(x)$ we equate to x both the first and second component in (4) using gives

$$X = \frac{2}{(a_3+a_4)+2\alpha(a_4-a_3)}$$

and $$X = \frac{2}{(a_2+a_1)-2\alpha(a_2-a_1)}$$

Now, expressing α-in terms of X setting α=0 and α=1 in (4), we get α together with the domain of X.

$$\left(1/\tilde{X}'_\alpha\right)^L_\alpha = \frac{2-x(a_3+a_4)}{2x(a_4-a_3)} \;;\; \frac{2}{3a_4-a_3} \prec x \le \frac{2}{a_3+a_4}$$

$$\left(1/\tilde{X}'_\alpha\right)^U_\alpha = \frac{x(a_2-a_1)-2}{2x(a_2-a_1)} \;;\; \frac{2}{a_2+a_1} \le x \prec \frac{2}{3a_1-a_2}$$

$$\left(1/\tilde{X}'_\alpha\right)_\alpha = 0 \quad ; \frac{2}{a_3+a_4} \le x \le \frac{2}{a_2+a_1}$$

$$\left(1/\tilde{X}'_\alpha\right)_\alpha = 1 \quad ; \text{otherwise}$$

Which gives,

$$\mu_{1/\tilde{X}'_\alpha}(x) = \begin{cases} \frac{2-x(a_4+a_3)}{2x(a_4-a_3)} & \text{if } \frac{2}{3a_4-a_3} \prec x \le \frac{2}{a_3+a_4} \\ 0 & \text{if } \frac{2}{a_3+a_4} \le x \le \frac{2}{a_2+a_1} \\ \frac{x(a_2-a_1)-2}{2x(a_4-a_1)} & \text{if } \frac{2}{a_2+a_1} \le x \prec \frac{2}{3a_1-a_2} \\ 1 & \textit{otherwise} \end{cases}$$

4.2 Exponential of Complement of Distinct Approximation of Trapezoidal fuzzy Numbers.

Let $\tilde{X}'=(a'_1, a'_2, a'_3, a'_4)$ >0 be a Distinct Approximation of Trapezoidal fuzzy numbers $\tilde{X}=(a_1, a_2, a_3, a_4)$. Then

$$\tilde{X}'_\alpha = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

is the α-cut of C-fuzzy number $\tilde{X}'$. To calculate exponential of fuzzy numbers $\tilde{X}$. We first take the exponential of the α-cut of $\tilde{X}'$ and using interval arithmetic.

$$\tilde{X}'_\alpha = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

$$Exp(\tilde{X}'_\alpha) = \exp\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}\right], \exp\left[\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right] \quad (5)$$

To find the membership function.

$\mu_{\exp(\tilde{X}'_\alpha)}(x)$ we equate to x both the first and second component in (5) which gives

$$X = \exp\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

$$X = \exp\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}\right] \text{and}$$

$$X = \exp\left[\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

Now, expressing α-in terms of X setting α=0 and α=1 in (5), we get α together with the domain of X.

To find the membership function

$$\exp\left(\tilde{X}'_\alpha\right)^L_\alpha = \frac{(a_2+a_1)-2\ln(x)}{2(a_2-a_1)} \;; \exp\left(\frac{3a_1-a_2}{2}\right) \prec x \le \exp\left(\frac{a_1+a_2}{2}\right)$$

and

$$\exp\left(\tilde{X}'_\alpha\right)^U_\alpha = \frac{2\ln(x)-(a_4+a_3)}{2(a_4-a_3)} \;; \exp\left(\frac{a_4+a_3}{2}\right) \le x \prec \exp\left(\frac{3a_4-a_3}{2}\right)$$

$$\exp\left(\tilde{X}'_\alpha\right)_\alpha = 0 \quad ; \exp\left(\frac{a_1+a_2}{2}\right) \le x \le \exp\left(\frac{a_4+a_3}{2}\right)$$

$$\exp\left(\tilde{X}'_\alpha\right)_\alpha = 1 \quad ; \text{otherwise}$$

Which gives,

$$\mu_{\exp(\tilde{X}'_\alpha)}(x) = \begin{cases} \frac{(a_2+a_1)-2\ln(x)}{2(a_2-a_1)} & \text{if } \exp\left(\frac{3a_1-a_2}{2}\right) \prec x \le \exp\left(\frac{a_1+a_2}{2}\right) \\ 0 & \text{if } \exp\left(\frac{a_1+a_2}{2}\right) \le x \le \exp\left(\frac{a_4+a_3}{2}\right) \\ \frac{2\ln(x)-(a_4+a_3)}{2(a_4-a_3)} & \text{if } \exp\left(\frac{a_4+a_3}{2}\right) \le x \prec \exp\left(\frac{3a_4-a_3}{2}\right) \\ 1 & \textit{otherwise} \end{cases}$$

4.3 Logarithmic of Complement of Distinct Approximation of Trapezoidal fuzzy Numbers.

Let $\tilde{X}'=(a'_1, a'_2, a'_3, a'_4)$ >0 be a Distinct Approximation of Trapezoidal fuzzy number $\tilde{X}=(a_1, a_2, a_3, a_4)$. Then

$$\tilde{X}'_\alpha = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

is the α-cut of C-fuzzy number $\tilde{X}'$. To calculate logarithmic of fuzzy numbers $\tilde{X}'$. We first take the logarithmic of the α-cut of $\tilde{X}'$ and using interval arithmetic.

$$\tilde{X}'_\alpha = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

$$\ln(\tilde{X}'_\alpha) = \ln\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}\right], \ln\left[\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right] \quad (6)$$

To find the membership function.

$\mu_{\ln(\tilde{X}'_\alpha)}(x)$. We equate to x both the first and second component in (6) which gives

$$X = \ln\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

International Journal of Scientific Research Engineering Technology
Volume 1, Issue 1, ISSN (Online): XX-XX

$$X = \ln\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}\right] \text{and}$$

$$X = \ln\left[\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

Now, expressing α-in terms of X setting α=0 and α=1 in (6), we get α together with the domain of X.

To find the membership function

$$\ln\left(\tilde{X}'_\alpha\right)^L_\alpha = \frac{(a_2+a_1)-2\ln(x)}{2(a_2-a_1)} \quad ;\ln\left(\frac{3a_1-a_2}{2}\right) < x \le \ln\left(\frac{a_1+a_2}{2}\right) \text{ and}$$

$$\ln\left(\tilde{X}'_\alpha\right)^U_\alpha = \frac{2\ln(x)-(a_4+a_3)}{2(a_4-a_3)} \quad ;\ln\left(\frac{a_4+a_3}{2}\right) \le x \prec \ln\left(\frac{3a_4-a_3}{2}\right)$$

$$\exp\left(\tilde{X}'_\alpha\right)_\alpha = 0$$

$$;\ln\left(\frac{a_1+a_2}{2}\right) \le x \le \ln\left(\frac{a_4+a_3}{2}\right)$$

$$\exp\left(\tilde{X}'_\alpha\right)_\alpha = 1 \qquad ;\text{ otherwise}$$

Which gives,

$$\mu_{\ln(\tilde{X}'_\alpha)}(x) = \begin{cases} \dfrac{(a_2+a_1)-2\ln(x)}{2(a_2-a_1)} & \text{if } \ln\left(\dfrac{3a_1-a_2}{2}\right) \prec x \le \ln\left(\dfrac{a_1+a_2}{2}\right) \\ 0 & \text{if } \ln\left(\dfrac{a_1+a_2}{2}\right) \le x \le \ln\left(\dfrac{a_4+a_3}{2}\right) \\ \dfrac{2\ln(x)-(a_4+a_3)}{2(a_4-a_3)} & \text{if } \ln\left(\dfrac{a_4+a_3}{2}\right) \le x \prec \ln\left(\dfrac{3a_4-a_3}{2}\right) \\ 1 & \text{otherwise} \end{cases}$$

4.4 Square root of Complement of Distinct Approximation of Trapezoidal fuzzy Numbers.

Let $\tilde{X}' = (a'_1, a'_2, a'_3, a'_4)$ >0 be a Distinct Approximation of Trapezoidal fuzzy number $\tilde{X} = (a_1, a_2, a_3, a_4)$. Then

$$\tilde{X}'_\alpha = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

is the α-cut of C-fuzzy number $\tilde{X}'$. To calculate square root of fuzzy numbers $\tilde{X}'$ .We first take the square root of the α-cut of $\tilde{X}'$ and using interval arithmetic.

$$\sqrt{\tilde{X}'} = \sqrt{\left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]}$$

$$= \sqrt{\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}}, \sqrt{\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}} \tag{7}$$

To find the membership function $\mu_{\sqrt{\tilde{X}'}}(x)$. we equate to x both the first and second component in (7). Which gives

$$X = \sqrt{\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}} \quad \text{and}$$

$$X = \sqrt{\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}}$$

Now, expressing α-in terms of X setting α=0 and α=1 in (7),
we get α together with the domain of X.

To find the membership function

$$\left(\sqrt{\tilde{X}'}\right)^L_\alpha = \frac{(a_2+a_1)-2x^2}{2(a_2-a_1)} \quad ; \sqrt{\frac{3a_1-a_2}{2}} \prec x \le \sqrt{\frac{a_1+a_2}{2}}$$

$$\left(\sqrt{\tilde{X}'}\right)^U_\alpha = \frac{2x^2-(a_3+a_4)}{2(a_4-a_3)} \quad ; \sqrt{\frac{a_4+a_3}{2}} \le x \prec \sqrt{\frac{3a_4-a_3}{2}}$$

$$\left(\sqrt{\tilde{X}'}\right)_\alpha = 0 \quad ; \sqrt{\frac{a_1+a_2}{2}} \le x \le \sqrt{\frac{a_4+a_3}{2}}$$

$$\left(\sqrt{\tilde{X}'}\right)_\alpha = 1 \quad ;\text{ otherwise}$$

Which gives

$$\mu_{\sqrt{\tilde{X}'}}(x) = \begin{cases} \dfrac{(a_2+a_1)-2x^2}{2(a_2-a_1)} & \text{if } \sqrt{\dfrac{3a_1-a_2}{2}} \prec x \le \sqrt{\dfrac{a_1+a_2}{2}} \\ 0 & \text{if } \sqrt{\dfrac{a_1+a_2}{2}} \le x \le \sqrt{\dfrac{a_4+a_3}{2}} \\ \dfrac{2x^2-(a_4+a_3)}{2(a_4-a_3)} & \text{if } \sqrt{\dfrac{a_4+a_3}{2}} \le x \prec \sqrt{\dfrac{3a_4-a_3}{2}} \\ 1 & \text{otherwise} \end{cases}$$

4.5 n^{th} Root of Complement of Distinct Approximation of Trapezoidal fuzzy Numbers.

Let $\tilde{X}' = (a'_1, a'_2, a'_3, a'_4)$ >0 be a Distinct Approximation of Trapezoidal fuzzy number $\tilde{X} = (a_1, a_2, a_3, a_4)$. Then

$$\tilde{X}'_\alpha = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]$$

is the α-cut of C-fuzzy number $\tilde{X}'$. To calculate n^{th} root of fuzzy numbers $\tilde{X}'$ .We first take the n^{th} root of the α-cut of $\tilde{X}'$ and using interval arithmetic.

$$\left(\tilde{X}'_\alpha\right)^{1/n} = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]^{1/n}$$

$$= \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}\right]^{1/n}, \left[\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]^{1/n} \tag{8}$$

To find the membership function $\mu_{\sqrt[n]{\tilde{X}'}}(x)$. we equate to x both the first and second component in (8). Which gives

$$X = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}\right]^{1/n} \quad \text{and}$$

$$X = \left[\frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right]^{1/n}$$

Now, expressing α-in terms of X setting α=0 and α=1 in (8), we get α together with the domain of X.

To find the membership function

$$\left(\sqrt[n]{\tilde{X}'}\right)^L_\alpha = \frac{(a_2+a_1)-2x^n}{2(a_2-a_1)} \quad ; \sqrt[n]{\frac{3a_1-a_2}{2}} < x \le \sqrt[n]{\frac{a_1+a_2}{2}}$$

$$\left(\sqrt[n]{\tilde{X}'}\right)^U_\alpha = \frac{2x^n-(a_3+a_4)}{2(a_4-a_3)} \quad ; \sqrt[n]{\frac{a_4+a_3}{2}} \le x \prec \sqrt[n]{\frac{3a_4-a_3}{2}}$$

$$\left(\sqrt{\tilde{X}'}\right)_\alpha = 0 \quad ; \sqrt[n]{\frac{a_1+a_2}{2}} \le x \le \sqrt[n]{\frac{a_4+a_3}{2}}$$

$$\left(\sqrt{\tilde{X}'}\right)_\alpha = 1 \quad ;\text{ otherwise}$$

Which gives

$$\mu_{\sqrt[n]{\tilde{X}'}}(x) = \begin{cases} \dfrac{(a_2+a_1)-2x^n}{2(a_2-a_1)} & \text{if } \sqrt[n]{\dfrac{3a_1-a_2}{2}} \prec x \le \sqrt[n]{\dfrac{a_1+a_2}{2}} \\ 0 & \text{if } \sqrt[n]{\dfrac{a_1+a_2}{2}} \le x \le \sqrt[n]{\dfrac{a_4+a_3}{2}} \\ \dfrac{2x^n-(a_4+a_3)}{2(a_4-a_3)} & \text{if } \sqrt[n]{\dfrac{a_4+a_3}{2}} \le x \prec \sqrt[n]{\dfrac{3a_4-a_3}{2}} \\ 1 & otherwise \end{cases}$$

V. Numerical Application

5.1 Multiplication of Two-Complement of Distinct Approximation of trapezoidal Fuzzy Numbers.

Let $\tilde{X}'$= (1,3,7,9) and $\tilde{Y}'$= (0,2,6,8) be two positive Distinct Approximation of Trapezoidal fuzzy numbers $\tilde{X}$ =(2,4,6,8) and $\tilde{Y}$ =(1,3,5,7) respectively. Then

$$\tilde{X}_\alpha' = \left[\frac{(a_2+a_1)-2\alpha(a_2-a_1)}{2}, \frac{(a_3+a_4)+2\alpha(a_4-a_3)}{2}\right] \text{ and}$$

$$\tilde{Y}_\alpha' = \left[\frac{(b_2+b_1)-2\alpha(b_2-b_1)}{2}, \frac{(b_3+b_4)+2\alpha(b_4-b_3)}{2}\right]$$

are the α-cuts of C-fuzzy numbers $\tilde{X}'$ and $\tilde{Y}'$respectively. To calculate multiplication of fuzzy numbers $\tilde{X}$ and $\tilde{Y}$. We first multiply the α-cuts of $\tilde{X}'$ and $\tilde{Y}'$ using interval arithmetic.

$$\tilde{X}_\alpha' * \tilde{Y}_\alpha' = \left[\frac{(2+4)-2\alpha(4-2)}{2}, \frac{(9+7)+2\alpha(8-6)}{2}\right] * \left[\frac{(2+1)-2\alpha(2-0)}{2}, \frac{(8+6)+2\alpha(7-5)}{2}\right]$$

$$\left(\tilde{X}_\alpha' * \tilde{Y}_\alpha'\right)_\alpha^L = \left(\frac{(2+4)-2\alpha(4-2)}{2}\right) * \left(\frac{(2+1)-2\alpha(2-0)}{2}\right)$$

$$= \frac{(6)(3)-2\alpha[(6)(2)+(4)(2)]+4\alpha^2[(2)(2)]}{4}$$

$$= \frac{\left[\frac{20}{2}\right] - \sqrt{\left[\frac{20}{2}\right]^2 - 4[4]\left[\frac{[24]}{4} - x\right]}}{2[4]} \qquad ;0 < x \le 6$$

Similarly,

$$\left(\tilde{X}_\alpha' * \tilde{Y}_\alpha'\right)_\alpha^U = \frac{-\left[\frac{52}{2}\right] + \sqrt{\left[\frac{52}{2}\right]^2 - 4[4]\left[\frac{[(168]}{4} - x\right]}}{2[4]} \quad ;42 \le x \prec 72$$

$$\left(\tilde{X}_\alpha' * \tilde{Y}_\alpha'\right)_\alpha = 0 \qquad ;6 \le x \le 42$$

$$\left(\tilde{X}_\alpha' * \tilde{Y}_\alpha'\right)_\alpha = 1 \qquad ; \text{otherwise}$$

Which gives

$$\mu_{\tilde{X}' * \tilde{Y}'}(x) = \begin{cases} \dfrac{\left[\frac{20}{2}\right] - \sqrt{\left[\frac{20}{2}\right]^2 - 4[4]\left[\frac{[24]}{4} - x\right]}}{2[4]} & if\ ;0 \prec x \le 6 \\ 0 & if\ ;6 \le x \le 42. \\ \dfrac{-\left[\frac{52}{2}\right] + \sqrt{\left[\frac{52}{2}\right]^2 - 4[4]\left[\frac{[(168]}{4} - x\right]}}{2[4]} & if\ ;42 \le x \prec 72 \\ 1 & otherwise \end{cases}$$

VI. Conclusion

We have investigated the complement of Approximation of trapezoidal fuzzy numbers and different algebraic operations. We have investigated the algebraic operations on such fuzzy quantities and their properties using α-cut method is also discussed. The same method can be utilized in different complement of fuzzy numbers also.

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